

FACULTY OF ENGINEERING AND ARCHITECTURE  
AMERICAN UNIVERSITY OF BEIRUT  
FALL 2013 - 2014  
**FINAL EXAM**  
**Communication Systems (EECE442)- Dr. Youssef Nasser**

December 16, 2013

NAME: \_\_\_\_\_

ID: \_\_\_\_\_

**CLOSED BOOK, OPEN LECTURES (120 MINUTES)**

WRITE YOUR NAME AND ID NUMBER IN THE SPACE PROVIDED ABOVE.

PROVIDE YOUR ANSWERS IN THE SPACE PROVIDED ON THE QUESTION SHEET.

THE SCRATCH BOOKLET WILL NOT BE CONSIDERED IN GRADING.

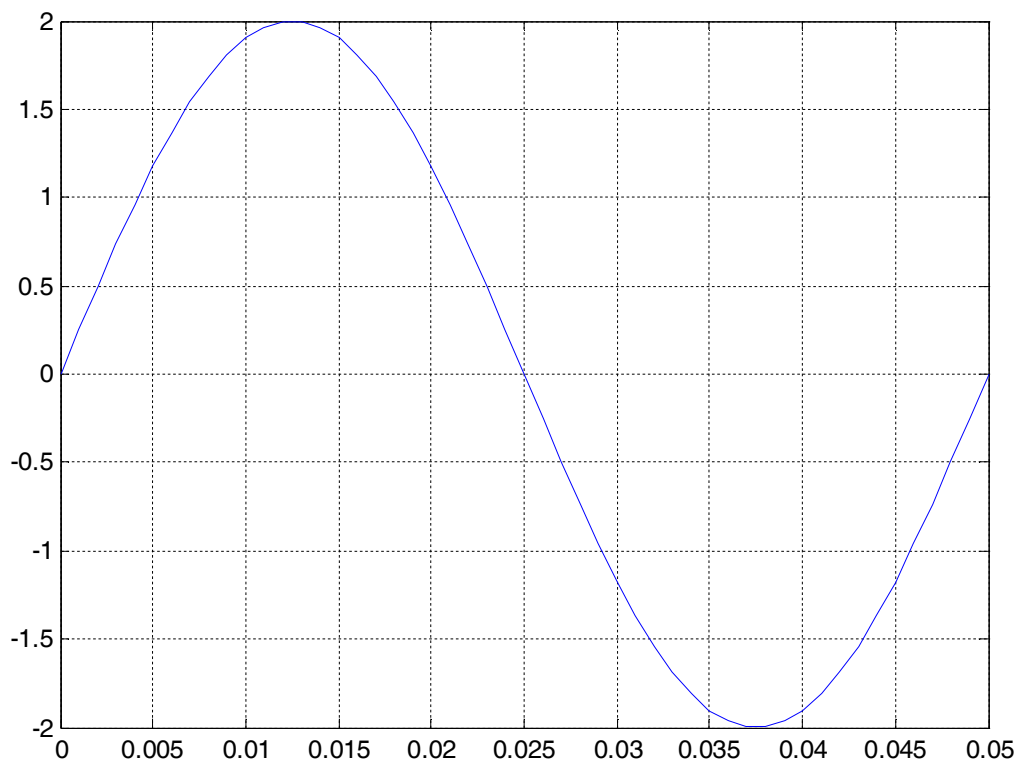
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<b>Problem</b>	<b>Total Points</b>	<b>Earned Points</b>
<b>1</b>	<b>26</b>	
<b>2</b>	<b>20</b>	
<b>3</b>	<b>24</b>	
<b>4</b>	<b>15</b>	
<b>3</b>	<b>15</b>	
<b>Total</b>	<b>100</b>	



- 4- Repeat the same question above if the required signal should be a band-limited signal. Justify (2pts)

- 5-**Quantization:** Consider the signal  $x(t) = A\sin(2\pi f_0 t)$  where  $A=2$  and  $f_0 = 20\text{Hz}$ . It is required to plot the quantized signal at the output of the quantizer, on the figure below, when:
- An 8-levels Uniform quantizer is used assuming that the sampling rate is  $f_s = 400\text{Hz}$ . Use a dot (.) to represent a sample and a dash (-) to represent a quantized sample (6pts)



- 6- Consider the binary sequence of 12 bits: 010101011111. It is required to **express** (mathematical expression) the bandpass signal at each symbol duration  $k.T_s$  corresponding to the transmission of this sequence when:
- 8-PSK digital modulation is used (4 pts)
  - QPSK modulation is used. (4 pts)
  - Assume that the duration of one transmitted bit is  $T_b$ ; deduce the duration  $T_s$  of the transmitted symbol in each case a and b above. (2 pts)

**Problem 2 [20 points]: Log Likelihood ratio**

In communications systems, a soft decision is usually required at the input of the channel decoding due to its efficient. The soft decision is expressed as log likelihood ratio (LLR) defined for each transmitted bit as:

$$LLR(b) = \ln \left( \frac{P(b_1/y)}{P(b_0/y)} \right) \xrightarrow{\text{Bayes rule}} \ln \left( \frac{P(y/b_1)}{P(y/b_0)} \right)$$

where  $y$  is the received signal,  $b_0$  is the transmitted bit '0' and  $b_1$  is the transmitted bit 1.



- B- We consider a modified BPSK modulation where the bit  $b_0$  is represented by  $(-A_0)$ , the bit  $b_1$  is represented by  $A_1$  where  $A_0$  and  $A_1$  two arbitrary deterministic values. Using the same methodology of part A, find the LLR of a bit  $b$ . (8 pts)

**Problem 3 [24 points]: Probability of error**

We consider a modified BPSK modulation where the bit  $b_0$  is represented by  $A_0$ , the bit  $b_1$  is represented by  $A_1$  where  $A_0$  and  $A_1$  two arbitrary deterministic values. We consider an AWGN transmission model with zero mean and variance equal to  $N_0/2$ .

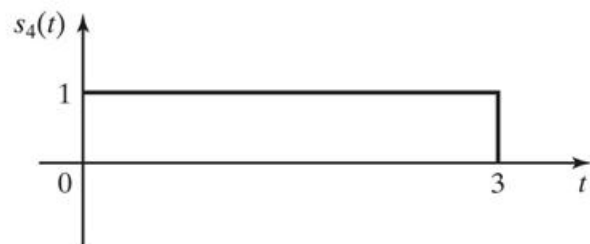
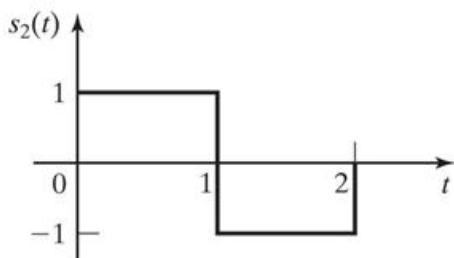
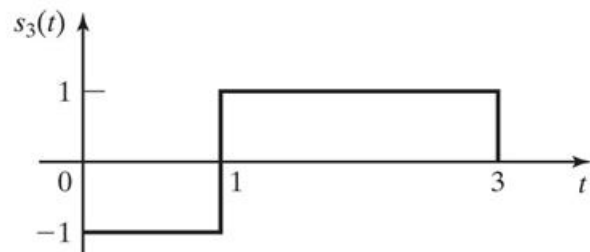
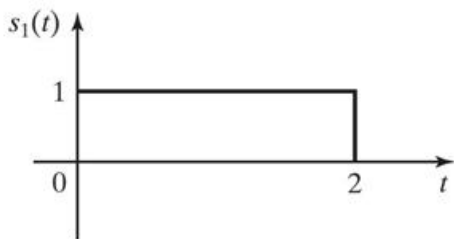
- a- Find the average energy per transmitted bit  $E_b$  in terms of  $A_1$ ,  $A_0$  and  $T_b$ . (3pts)
- b- It is required to find the probability of error of this modified BPSK modulation. (15pts)

c- Deduce the probability of error when  $A_1=A_0$  and compare your results with the derivation done in class. (3pts)

- d- We assume that  $A_1=2$  and  $A_0=1$ . We define the cross energy  $E_{bc} = A_1 A_0 T_b$ . Using the look-up table in the appendix (page 11), find the probability of error of this modified BPSK when  $E_b/N_0=10\text{dB}$  and  $E_{bc}/N_0=4.7\text{dB}$  (3pts)

**Problem 4 [15 points]: Gram-Schmidt Orthogonalization**

**Find and plot** the orthonormal basis functions of the following set of symbols using the Gram-Schmidt Orthogonalization







**Problem 5 [15 points]: Matched filter**

Consider the following pulse shaping filter defined by:

$$g(\tau) = \begin{cases} 1 & \text{if } 0 \leq \tau \leq 1 \\ -\tau + 2 & \text{if } 1 \leq \tau \leq 2 \end{cases}$$

- a- Plot the pulse shape of this filter  $g(\tau)$ . (1 pt)
- b- Derive and plot the expression of the output of the matched filter. (8 pts)
- c- At which time, this (absolute value) output is maximized. Find this maximum (2 pts)

- d- We assume that this filter is used for a signal of bandwidth  $B=10\text{MHz}$  and that the noise power is  $50\text{dBw}$ . Find the maximum SNR value at the output of the matched filter. (4 pts)

**Appendix: The Q-function Look-up table**

$z$	$Q(z)$	$z$	$Q(z)$
0.0	0.50000	2.0	0.02275
0.1	0.46017	2.1	0.01786
0.2	0.42074	2.2	0.01390
0.3	0.38209	2.3	0.01072
0.4	0.34458	2.4	0.00820
0.5	0.30854	2.5	0.00621
0.6	0.27425	2.6	0.00466
0.7	0.24196	2.7	0.00347
0.8	0.21186	2.8	0.00256
0.9	0.18406	2.9	0.00187
1.0	0.15866	3.0	0.00135
1.1	0.13567	3.1	0.00097
1.2	0.11507	3.2	0.00069
1.3	0.09680	3.3	0.00048
1.4	0.08076	3.4	0.00034
1.5	0.06681	3.5	0.00023
1.6	0.05480	3.6	0.00016
1.7	0.04457	3.7	0.00011
1.8	0.03593	3.8	0.00007
1.9	0.02872	3.9	0.00005

The definition of  $Q$  function is:

$$Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$