

Homework 3 (Due October 22, 2013)

Objective of this HW: get familiar with Fourier transform, Autocorrelation Function, AM modulation; hands-on experience on Matlab and signal processing for communications.

Estimated time: ??? minutes

Problem 1: consider the signal $x(t) = \cos(2\pi f_0 t)$

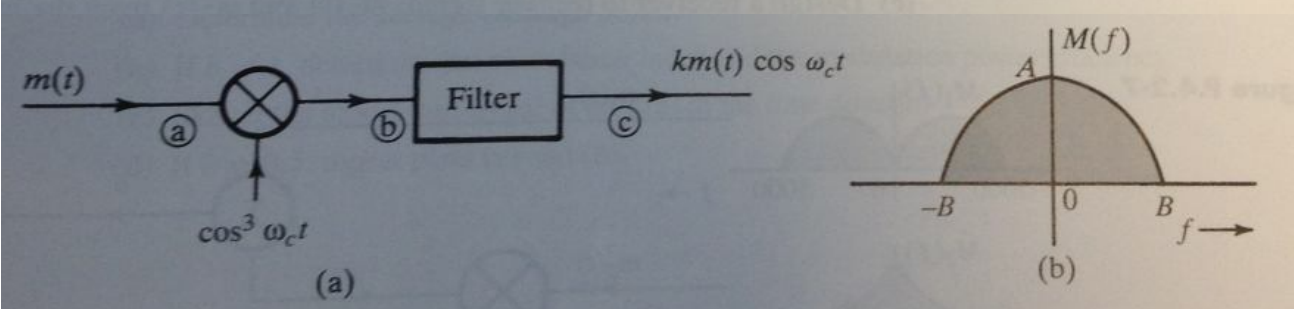
- Derive the Fourier transform of $x(t)$;
- Suppose $f_0 = 10\text{Hz}$, and that the signal is sampled at sampling rate of 30 samples per second. Sketch the frequency response of the sampled signal and show how a low-pass filter can recover the original signal.
- Now suppose that the signal is sampled at a rate of 15 samples per second; sketch the frequency response of the sampled signal.

Note: In your sketches, you can limit yourselves to a bandwidth of 40Hz.

Problem 2:

You are asked to design a DSB-SC modulator to generate a modulated signal $km(t) \cos(\omega_c t + \theta)$, where $m(t)$ is a signal band-limited to B Hz. Figure P4.2-3 shows a DSB-SC modulator available in the stockroom. The carrier generator available generates not $\cos \omega_c t$, but $\cos^3 \omega_c t$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

- What kind of filter is required in Fig. P4.2-3?
- Determine the signal spectra at points b and c , and indicate the frequency bands occupied by these spectra.
- What is the minimum usable value of ω_c ?
- Would this scheme work if the carrier generator output were $\sin^3 \omega_c t$? Explain.
- Would this scheme work if the carrier generator output were $\cos^n \omega_c t$ for any integer $n \geq 2$?



Problem 3: Solve problem 4.11 of the textbook

Problem 4: Solve Problem 4.18 of the textbook

Problem 5: Solve Problem 4.22 of the textbook

Problem 6: Solve Problem 4.26 of the textbook

Problem 7: Let the random process $X(t)$ be defined by $X(t) = A + Bt$, where A and B are independent random variables each uniformly distributed on $[-1, 1]$. Find $m_X(t)$ and $R_X(t_1, t_2)$.

Problem 8: Given a random variable X with a Rayleigh distribution

$$f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, x \geq 0$$

Calculate the mean of X. Show the steps of your calculation

Problem 9: Given random variable X with a Gaussian distribution having mean μ and variance σ^2 .

- a- Determine the distribution of random variable $Y = 2X$.
- b- Find its mean