EECE 442 Communication Systems
American University of Beirut, Spring 2010–2011

February 25, 2011
Handout #2
Due: Friday, Mar. 4, 2011

Problem Set 1

Reminder on Collaboration Policy

The following is an acceptable form of collaboration: discuss with your classmates possible approaches to solving the problems, and then have *each one* fill in the details and *hand-write her/his own* solutions *independently*.

An unacceptable form of dealing with homework is to copy a solution that someone else has written.

At the top of each homework you turn in, list all sources of information you used, apart of course from the text, books on reserve for this course or discussions with the Prof. A brief note such as "did problem 7 with May Berite in study group" would be sufficient.

In general, we expect students to adhere to basic, common sense concepts of academic honesty. Presenting another's work as if it were your own, or cheating in exams will not be tolerated.

Problem 1.1 (Fourier Transforms)

We define the Fourier Transform (FT) of a deterministic signal $s(t)$ as,

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft}\, dt.$$

Note that this is a function of the frequency "f" and not of the pulsation "$\omega = 2\pi f$". By adopting this definition, the inverse FT, the transforms of "classical" function, as well as some of the famous relationships become easier, as the normalization factors disappear.

(a) Using tools acquired in previous classes, prove that with this definition, the Inverse Fourier Transform (IFT) is

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft}\, df.$$

(b) Let $s(t)$ be a Dirac delta function, more specifically,

$$s(t) = \delta(t - \tau),$$

for a given fixed value of τ. Find the FT of $s(t)$ for all possible values of τ.

(c) Let $s(t)$ and $u(t)$ be a cosine and a sine function respectively, i.e.,

$$s(t) = \cos(2\pi f_c t), \qquad u(t) = \sin(2\pi f_c t).$$

Find the FT of $s(t)$ and $u(t)$.

(d) Now let $s(t)$ be a "rect" function, i.e.,

$$s(t) = \text{rect}(t) = \begin{cases} 1 & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Find the FT of $s(t)$.

(e) Let $s(t)$ be a "sinc" function, i.e.,

$$s(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

Find the FT of $s(t)$.

(f) If we denote by $v(f)$ the FT of $u(t)$ prove that the FT of $v(t)$ is $u(-f)$.

Problem 1.2

Consider a baseband signal $\tilde{s}(t)$ with Fourier Transform (FT) $\tilde{S}(f)$ shown in Figure 1

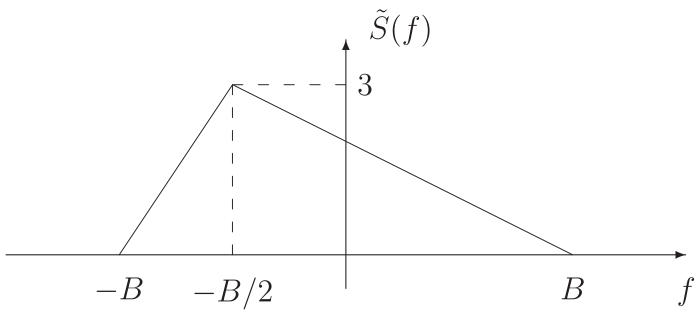


Figure 1: Fourier Transform of $\tilde{s}(t)$.

(a) You decide to sample $\tilde{s}(t)$ every T seconds. How small should T be in order not to lose information through the sampling process?

(b) What is the energy E_s of $\tilde{s}(t)$?

You form the signal $s(t)$ as follows:

$$s(t) = \Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right],$$

where the carrier frequency f_c is a multiple of B.

(c) If E_s is the energy of $\tilde{s}(t)$, what is the energy of $s(t)$?

(d) You decide to sample $s(t)$ every T seconds. How small should T be in order not to lose information through the sampling process?

(e) Is your result consistent with (a)? Explain.

Consider now the reverse case, where you have a passband signal $s(t)$ and you would like to define its "baseband" equivalent $\tilde{s}(t)$ such that

$$s(t) = \Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right].$$

(f) Explain how you would obtain $\tilde{s}(t)$ from $s(t)$.

Problem 1.3 (Implementation of Envelope Detectors)

Figure 2 shows the circuit diagram of an envelope detector. It consists simply of a diode and resistor-capacitor (RC) filter. On a positive half-cycle of the input signal, the diode is forward-biased and the capacitor C charges up rapidly to the peak value of the input signal. When the input signal falls below this value, the diode becomes reverse-biased and the capacitor discharges slowly through the load resistor R_l. The discharging process continues until the next positive half-cycle. Thereafter, the charging-discharging routine is continued.

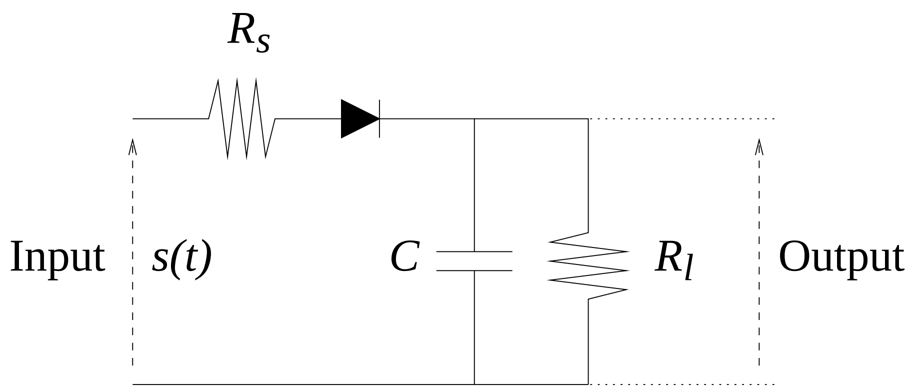


Figure 2: Envelope Detector.

(a) Specify the condition that must be satisfied by the capacitor C for it to charge rapidly and thereby follow the input voltage up to the positive peak when the diode is conducting.

(b) Specify the condition which the load resistor R_l must satisfy so that the capacitor C discharges slowly between positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave.

(c) Draw a schematic diagram of the input and output signals under the conditions in (a) and (b), when the input $s(t)$ is an AM signal.

(d) Can you propose a way to make the output in (c) "smoother" and closer to the envelope?

Note: Envelope detectors fall under the category of *non-coherent* detectors as the phase of the transmitted signal is not necessary for demodulation, as opposed to the "multiplier/low pass filter" presented in class where the exact phase is needed.

Corollary Note: For envelope detectors to work (recover the message signal), it is necessary to have the amplitude modulating the cosine to be always positive.

Problem 1.4 (Power Consumption in DSB-SC)

We investigate in this problem the power consumption of a DSB-SC modulated signal. Feel free to assume that the information-baring signal $m(t)$ is of "very low frequency content" when compared to f_c.

Consider a DSB-SC wave

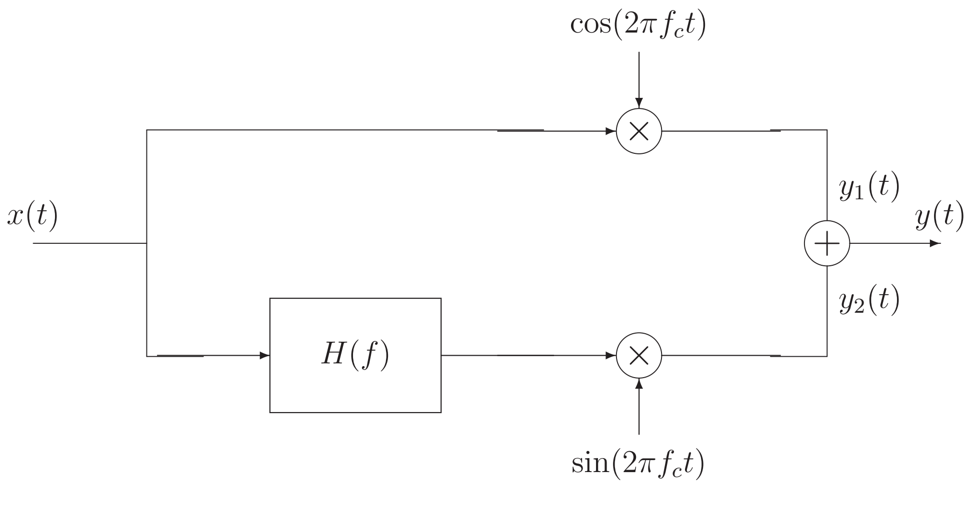
$$s(t) = A_c m(t) \cos(2\pi f_c t),$$

where the bandwidth W of $m(t)$ is such that $W << f_c$.

Assume that the duration of $s(t)$ is T and compute the power of the signal $s(t)$. Feel free to make appropriate assumptions on the relations between T, W and f_c, and express your answer function of these parameters, A_c and the power of the message P_m.

Problem 1.5

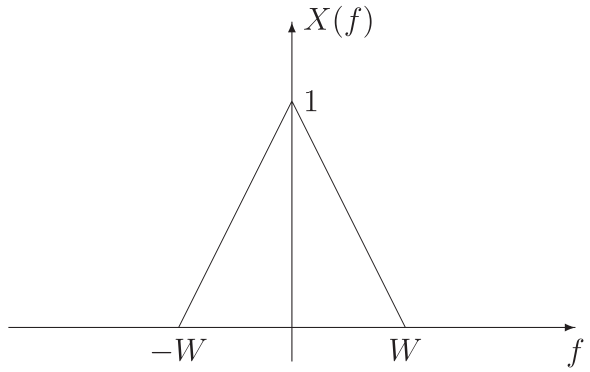
Consider the following system:



where,

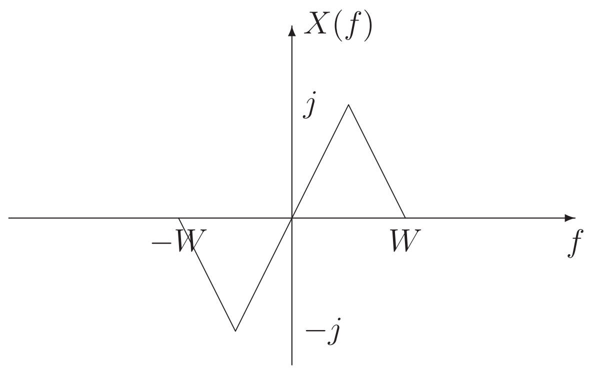
$$H(f) = \begin{cases} j & f \geq 0 \\ -j & f < 0 \end{cases}$$

(a) Consider the following signal,



Sketch $Y_1(f)$, $Y_2(f)$ and $Y(f)$, and demonstrate that only the upper sidebands are retained.

(b) Now consider the imaginary signal



Sketch $Y_1(f)$, $Y_2(f)$ and $Y(f)$, and demonstrate that also only the upper sidebands are retained.