EECE 442 Final Exam Solution Summer 2005

Problem # 1

(a) The bit rate at the output of the PCM transmitter is given by:

$$\text{Bit rate} = \frac{\log_2 L_1}{T_s} = 2 \times 4 \times 10^3 \times \log_2(256 = 2^8)$$

$$= 64{,}000 \text{ bits/sec.}$$

(b) The bit rate at the output of the DPCM transmitter is:

$$\text{Bit rate} = 64{,}000 - \frac{64000}{100} \times 25 = 48{,}000 \text{ bits/sec.}$$

$$48{,}000 = \frac{\log_2 L_2}{T_s} = 2 \times 4 \times 10^3 \times \log_2 L_2$$

$$\Rightarrow \log_2 L_2 = \frac{48{,}000}{8{,}000} = 6$$

$$\Rightarrow L_2 = 2^6 = 64 \text{ levels.}$$

The step size in PCM = step size in DPCM

$$= \frac{10}{256} = 0.039 \text{ volts.}$$

The dynamic range of the input to the DPCM quantizer is:

$$DR_{DPCM} = \frac{64 \times 10}{256} = 2.5 \text{ volts.}$$

Problem # 2

$$m(t) = A_m \cos(\omega_m t) \text{ Volts.}$$

(a) $A_m = 5$ volts.

$$\omega_m = 2\pi \times 4 \times 10^3 = 8 \times \pi \times 10^3 \text{ rad/sec.}$$

(b) $$(SNR)_{o\,DPCM} = \frac{\sigma_M^2}{\frac{\Delta_{DPCM}^2}{12}} = 100 \times \frac{\sigma_M^2}{\frac{\Delta_{PCM}^2}{12}}$$

$$= 100 \times (SNR)_{o\,PCM}$$

$$\Rightarrow \Delta_{DPCM}^2 = \frac{\Delta_{PCM}^2}{100} \text{ or } \Delta_{DPCM} = \frac{\Delta_{PCM}}{10}$$

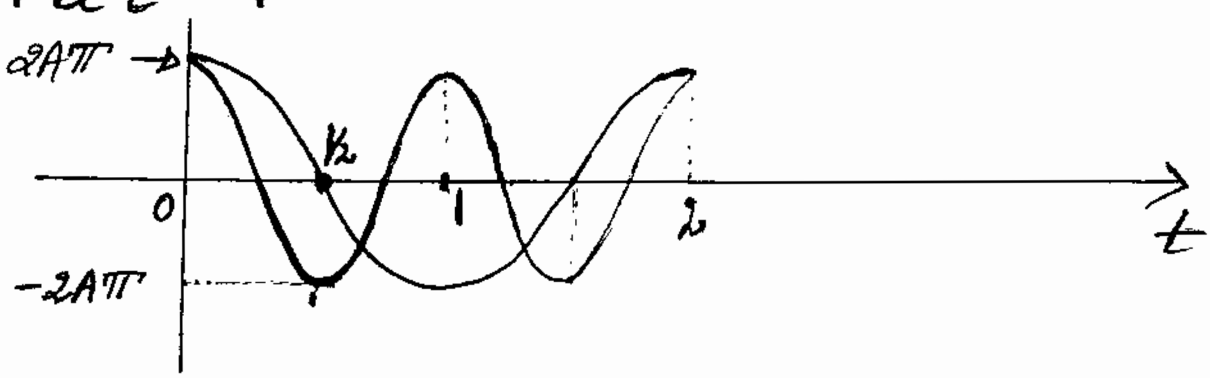
$$\Rightarrow \frac{DR_{DPCM}}{L} = \frac{DR_{PCM}}{10L} = \frac{10}{10L}$$

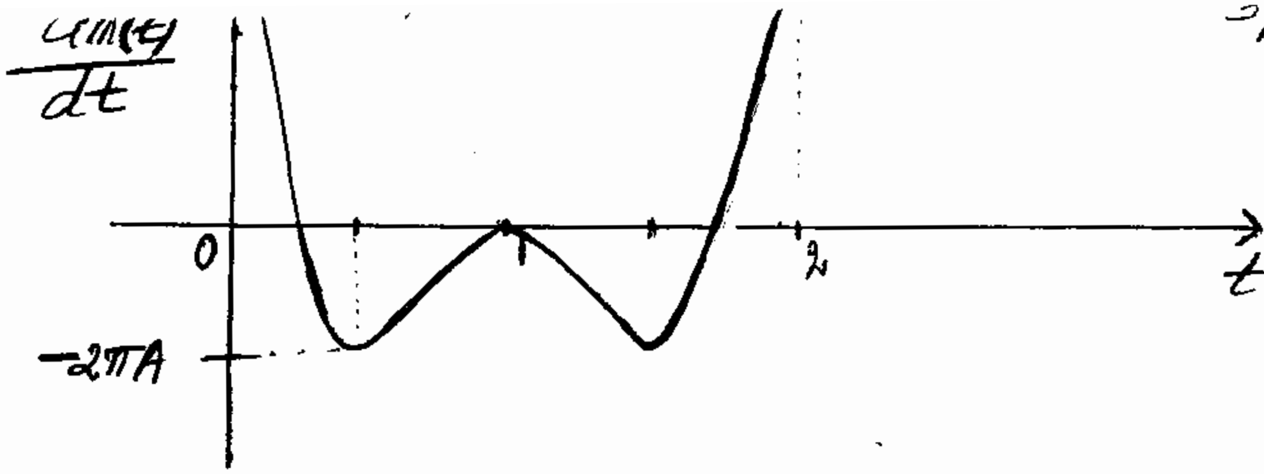
$$\Rightarrow DR_{DPCM} = 1 \text{ Volts.}$$

Problem # 3

$$m(t) = A\sin(2\pi t) + 2A\sin(\pi t) \text{ V.}$$

(a) $$\left|\frac{dm(t)}{dt}\right| = \left|2A\pi\cos(2\pi t) + 2A\pi\cos(\pi t)\right|$$





$$\Rightarrow \max\left|\frac{dm(t)}{dt}\right| = 4\pi A$$

(b) The condition of no slope overload distortion is:

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

$$\Rightarrow \Delta f_s \geq 4\pi A$$

$$\Rightarrow A \leq \frac{\Delta f_s}{4\pi}$$

The maximum usable value of A is $\frac{\Delta f_s}{4\pi}$.

(c) $A_{max} = 2.5A \leq \frac{2.5\,\Delta f_s}{4\pi} = \frac{\Delta f_s}{4\pi \times \frac{2}{5}}$

$$\Rightarrow A_{max} \leq \frac{\Delta f_s}{1.6\pi}$$

The highest usable value of A_{max} is $\frac{\Delta f_s}{1.6\pi}$

Highest $A_{max} = \frac{\Delta f_s}{W \times \frac{1.6}{2}} = \frac{\Delta f_s}{0.8W}$

Where $W = 2\pi$ is the bandwidth of $m(t)$.

Note The result in Part (C) can be used to explain the reason for which the maximum usable amplitude for a voice signal to avoid slope overload distortion is given by $A_{max} = \frac{\Delta f_s}{2\pi \times 800}$ Not $\frac{\Delta f_s}{2\pi \times 3.2 10^3}$,

Problem #4

$$g(t) = \begin{cases} A, & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}$$

$$h(t) = g(T-t) \Leftrightarrow H(\omega) = G^*(\omega) e^{-j\omega T}$$

(a) $g_0(T) = \int_0^T g^2(t)dt = \int_0^T A^2 dt = A^2 T$

$$\Rightarrow |g_0(T)|^2 = A^4 T^2$$

(b) $E[n_0^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N_0}{2} |H(\omega)|^2 d\omega$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N_0}{2} |G(\omega)|^2 d\omega$$

$= \frac{N_0}{2} \times E_g$ where E_g is the energy of $g(t)$. $E_g = A^2 T$

$$\Rightarrow E(n_0^2(t)) = \frac{N_0 A^2 T}{2}$$

$$(SNR)_0 = \frac{|g_0(T)|^2}{E[n^2(t)]} = \frac{(A^2T)^2}{\frac{N_0 A^2 T}{2}} = \frac{2A^2T}{N_0}$$

$$= \frac{2E_g}{N_0},$$

(c) $$g_{new}(t) = \begin{cases} A, & 0 \le t \le T/2 \\ -A, & T/2 < t \le T \\ 0, & \text{elsewhere} \end{cases}$$

$$g_0(T) = \int_0^T g^2_{new}(t)\,dt = A^2T.$$

$$|g_0(T)|^2 = (A^2T)^2.$$

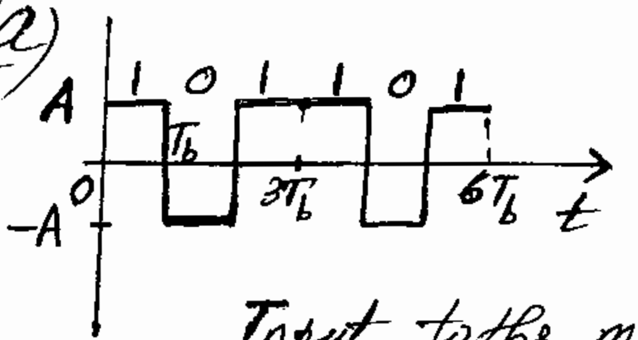
$$E[(n_0^2(t)]^2 = \frac{N_0}{2} E_{g_{new}} = \frac{N_0 A^2 T}{2}.$$

$(SNR)_0 = \dfrac{2A^2T}{N_0}$ equal to the $(SNR)_0$ obtained in part (b).

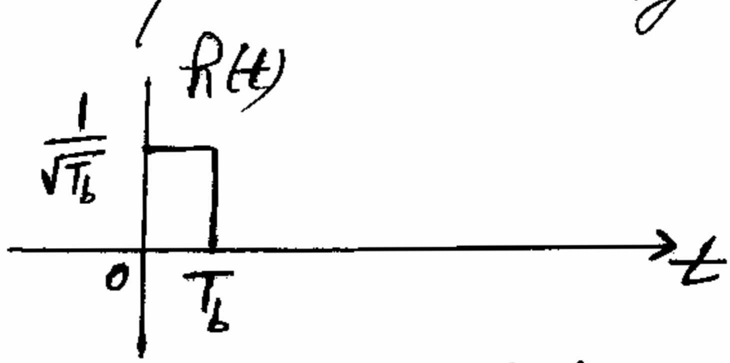
The matched filter output (SNR) is independent of the shape of the signal $g(t)$ as long as the energy of $g(t)$ does not change.

Problem # 5

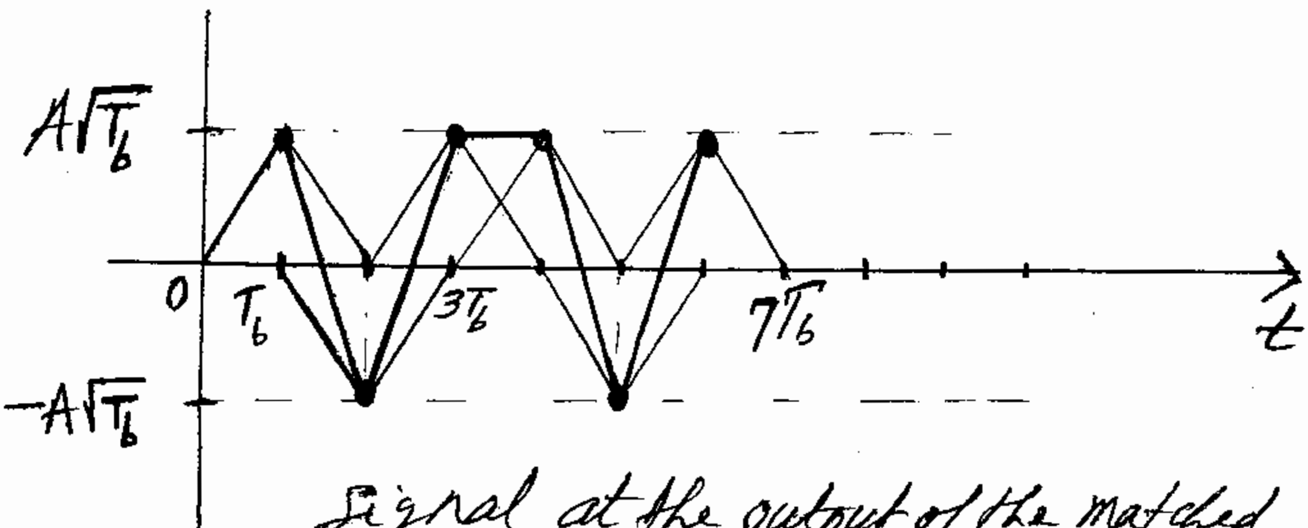
(a)



Input to the matched filter.



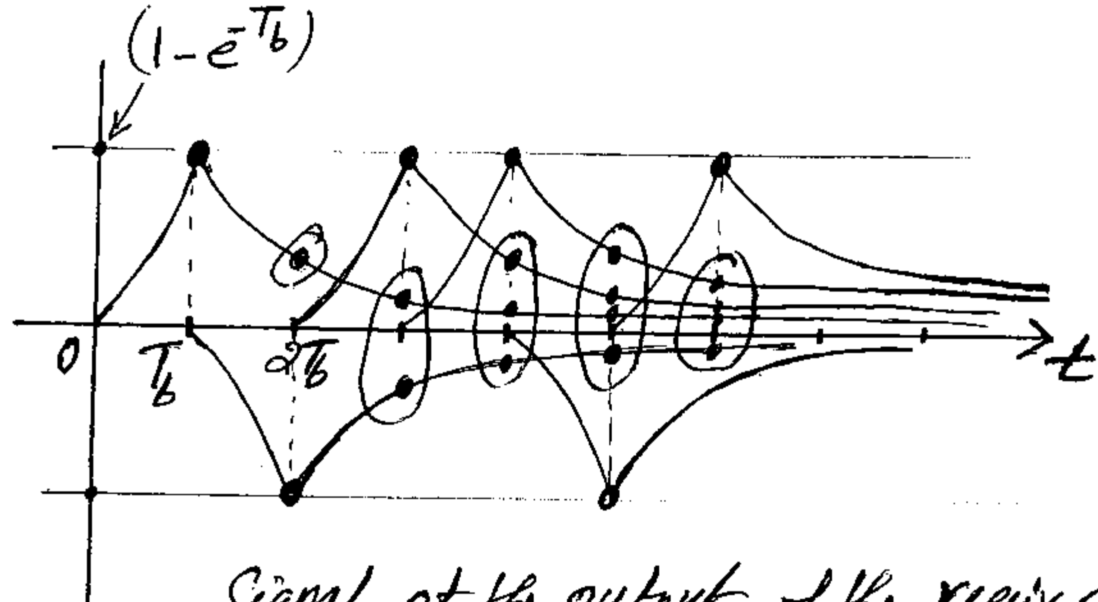
Normalized matched filter impulse response.



Signal at the output of the matched filter before sampling.

The sample at $t = iT_b$ is obtained from the output pulse that corresponds directly to the input pulse with no contribution from other previously transmitted pulses. Hence, $\nexists$ ISI.

(b)



Signal at the output of the receiver before sampling and the ISI effect shown as sample values within the circles.

Problem # 6

(a) The 4 rectangular pulses are linearly dependent.

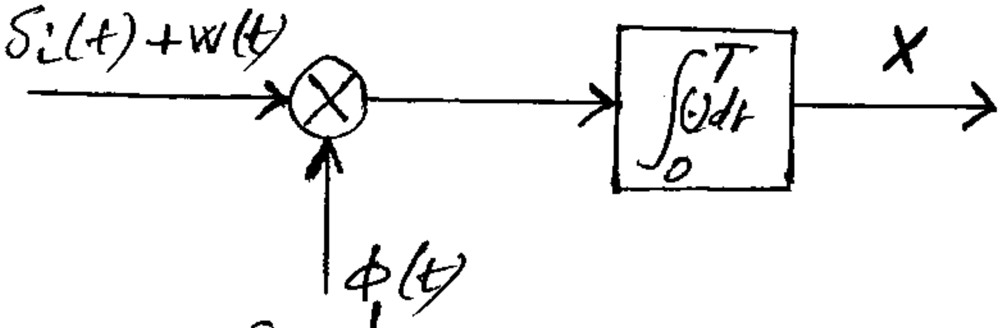
$\Rightarrow$ Gram-Schmidt procedure gives only one orthonormal function.

$$\phi_1(t) = \frac{A}{\sqrt{A^2 T}} = \frac{1}{\sqrt{T}}, \quad 0 \le t \le T$$

Let $s_1(t) = \begin{cases} 2A, & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$ $\quad s_2(t) = \begin{cases} A, & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$

$s_3(t) = \begin{cases} -A, & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$ $\quad s_4(t) = \begin{cases} -2A, & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$

$$\Rightarrow s_1(t) = 2A\sqrt{T}\,\phi_1(t)$$
$$s_2(t) = A\sqrt{T}\,\phi_1(t)$$
$$s_3(t) = -A\sqrt{T}\,\phi_1(t)$$
$$s_4(t) = -2A\sqrt{T}\,\phi_1(t)$$



$$X = \begin{cases} 2A\sqrt{T} + W_1, & \text{if } s_1(t) \text{ is sent} \\ A\sqrt{T} + W_1, & \text{if } s_2(t) \text{ is sent} \\ -A\sqrt{T} + W_1, & \text{if } s_3(t) \text{ is sent} \\ -2A\sqrt{T} + W_1, & \text{if } s_4(t) \text{ is sent.} \end{cases}$$

$$W_1 = \int_0^T w(t)\phi_1(t)\,dt.$$

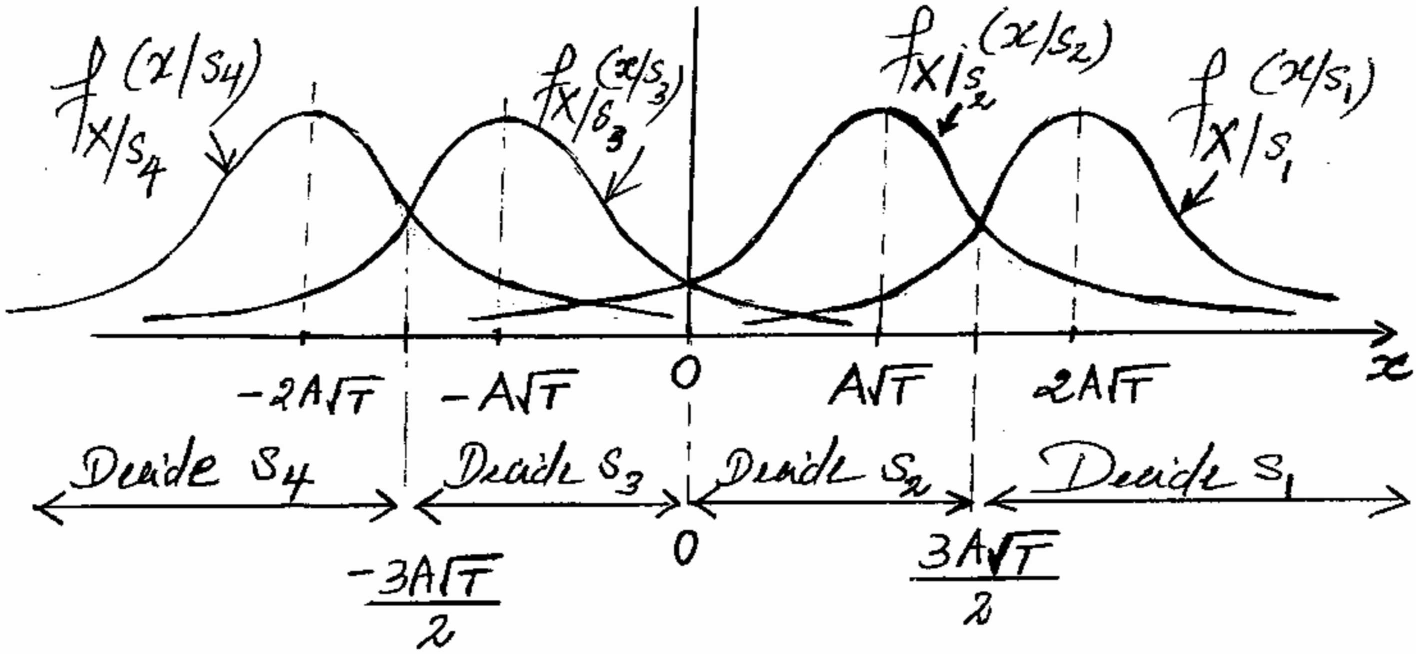
$$E(W_1) = 0, \quad \sigma_{W_1}^2 = \frac{N_0}{2}$$

$$f_{X|s_1}(x/s_1) = \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{2}\frac{(x-2A\sqrt{T})^2}{N_0/2}\right\}$$

$$f_{X|s_2}(x/s_2) = \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{2}\frac{(x-A\sqrt{T})^2}{N_0/2}\right\}$$

$$f_{X|s_3}(x/s_3) = \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{2}\frac{(x+A\sqrt{T})^2}{N_0/2}\right\}$$

$$f_{X|s_4}(x/s_4) = \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{2}\frac{(x+2A\sqrt{T})^2}{N_0/2}\right\}$$



$$P_e = 1 - \frac{1}{4}\sum_{i=1}^{4} \text{Prob.}(\text{decide } s_i / s_i \text{ is transmitted})$$

$$= 1 - \frac{1}{4}\left[2\int_{-\infty}^{-\frac{3A\sqrt{T}}{2}} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{2}\frac{(x+2A\sqrt{T})^2}{N_0/2}\right\}dx + 2\int_{0}^{\frac{3A\sqrt{T}}{2}} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{1}{2}\frac{(x-A\sqrt{T})^2}{N_0/2}\right\}dx\right]$$

$$= 1 - \frac{1}{4}\left[2\,\mathrm{erf}_*\left(\frac{A\sqrt{T}}{\sqrt{2N_0}}\right) + 2\,\mathrm{erf}_*\left(\frac{A\sqrt{T}}{\sqrt{2N_0}}\right) - 2\,\mathrm{erf}_*\left(\frac{-2A\sqrt{T}}{\sqrt{2N_0}}\right)\right]$$

$$P_e = 1 - \mathrm{erf}_*\left(\frac{A\sqrt{T}}{\sqrt{2N_0}}\right) + \frac{1}{2}\,\mathrm{erf}_*\left(\frac{-2A\sqrt{T}}{\sqrt{2N_0}}\right)$$
$$= \mathrm{erf}_*\left(\frac{-A\sqrt{T}}{\sqrt{2N_0}}\right) + \frac{1}{2}\,\mathrm{erf}_*\left(\frac{-2A\sqrt{T}}{\sqrt{2N_0}}\right)$$

(b) $s_1(t) = 2A\cos(\omega_c t), \quad 0 \le t \le T$
$s_2(t) = A\cos(\omega_c t), \quad 0 \le t \le T$
$s_3(t) = -A\cos(\omega_c t), \quad 0 \le t \le T$
$s_4(t) = -2A\cos(\omega_c t), \quad 0 \le t \le T$

$$\phi_1(t) = \frac{s_2(t)}{\sqrt{E_2}} = \frac{A\cos(\omega_c t)}{\sqrt{\frac{1}{2}A^2 T}}$$
$$= \sqrt{\frac{2}{T}}\cos(\omega_c t), \quad 0 \le t \le T$$

$\Rightarrow s_1(t) = 2A\sqrt{\frac{T}{2}}\,\phi_1(t)$
$s_2(t) = A\sqrt{\frac{T}{2}}\,\phi_1(t)$
$s_3(t) = -A\sqrt{\frac{T}{2}}\,\phi_1(t)$
$s_4(t) = -2A\sqrt{\frac{T}{2}}\,\phi_1(t)$

As in Part (a), one correlator is to be used here as well with a multiplying function $\phi_1(t)$ as expressed above. The probability density functions are also as in part (a), but with $\sqrt{T}$ replaced by $\sqrt{\frac{T}{2}}$.

The decision thresholds are at $-\frac{3A\sqrt{\frac{T}{2}}}{2}$, 0, and $\frac{3A\sqrt{T/2}}{2}$. Hence, the symbol probability of error is:

$$P_e = \operatorname{erf}_*\left(\frac{-A\sqrt{T/2}}{\sqrt{2N_0}}\right) + \frac{1}{2}\operatorname{erf}_*\left(\frac{-2A\sqrt{T/2}}{\sqrt{2N_0}}\right)$$

(c) With $E_1 = A^2T$, then

$$P_e = \operatorname{erf}_*\left(-\sqrt{\frac{E_1}{2N_0}}\right) + \frac{1}{2}\operatorname{erf}_*\left(-\sqrt{\frac{2E_1}{N_0}}\right)$$

in the baseband case.

With $E_2 = \frac{1}{2}A^2T$, then

$$P_e = \operatorname{erf}_*\left(-\sqrt{\frac{E_2}{2N_0}}\right) + \frac{1}{2}\operatorname{erf}_*\left(-\sqrt{\frac{2E_2}{N_0}}\right)$$ band-pass case

If $E_1 = E_2$; i.e, the same pulse energy is used or same amount of transmission power, then the error probabilities are equal and we have the same signal quality at the receiver output.

(d) As was seen in Part (c), the sinusoidal modulation by a carrier does not provide an improvement in performance over baseband data transmission. Modulation, however, is needed in application areas where the communication link is wireless and done using antennas. Since, when the frequency gets higher, the antenna length gets smaller and more practical. This is the case in mobile phone, satellite and other communications technologies.

Modulation is also needed in areas where the principle of FDM is to be used; such as radio and television broadcasting.

Baseband data transmission, on the other hand, can be used when transmission is done over a transmission line, fiber optic and the like. Time division multiplexing can be used in such a case to transmit more than one signal over the same channel but not FDM.