AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT
EECE 442 – Communications Systems
FINAL EXAM
Closed book
EIGHT SHEETS OF FORMULAS WITH NO PROBLEM SOLUTIONS ARE ALLOWED
TIME: 2 hours
Monday, September 18, 2006
INSTRUCTOR: Dr. JEAN J. SAADE

NAME ____________________ **ID #:** ____________________

INSTRUCTIONS

- WRITE YOUR ID # AND NAME ON THE COMPUTER CARD, ON THIS SHEET AND ON THE SCRATCH BOOKLET IN THE PROVIDED SPACES.
- PROVIDE YOUR ANSWER ON THE COMPUTER CARD **and solution of each problem on the scratch booklet**
- **Random checking will be done to find out about any inconsistency between the problem solutions and the provided answers on the computer card.**
- RETURN THE COMPUTER CARD ATTACHED ON TOP OF THE QUESTION SHEET AND SCRATCH BOOKLET.
- **ONLY YOUR ANSWER PROVIDED ON THE COPMUTER CARD WILL BE CONSIDERED IN GRADING.**
- All QUESTIONS ARE EQUALLY WEIGHTED IN GRADING.

PROBLEM # 1

Consider PAM and suppose that we want the bandwidth of the transmitted PAM signal to be the smallest possible. Determine the duration, T, of each PAM pulse if the bandwidth of the analog message signal is 5KHz and it is sampled at the Nyquist rate. Let the bandwidth of a rectangular pulse be given by the first zero crossing of its Fourier spectrum with the frequency axis.

(a) T=0.2 ms (b) T=0.1 ms (c) T=0.4 ms (d) T=0.6 ms (e) T=0.8ms

PROBLEM # 2

Consider an analog message signal m(t). This signal is instantaneously sampled at a rate equal to 1/Ts, which is much bigger that the Nyquist rate, and then the samples are entered into a device whose output is a PDM signal. Let the device implement the following exponential law:

$$PD = Ke^{[m(nTs)/\max(m(nTs))]}$$

PD is the pulse duration and K is a multiplicative constant. The dynamic range of the message signal is between –2 Volts and 2 Volts. Determine K if the maximum pulse duration is equal to Ts and also determine the minimum pulse duration, $(PD)_{min}$, in the PDM signal.

$(a)\ K = \frac{Ts}{e^2},\ (PD)_{\min} = \frac{Ts}{e^4}$ $(b)\ K = Ts,\ (PD)_{\min} = \frac{Ts}{e}$

$(c)\ K = \frac{Ts}{e},\ (PD)_{\min} = \frac{Ts}{e^2}$ $(d)\ K = 2Ts,\ (PD)_{\min} = \frac{1}{2}Ts$

$(e)\ K = 3Ts,\ (PD)_{\min} = \frac{1}{3}Ts$

PROBLEM # 3

Consider uniform quantization with the quantization level, v_k, placed in decision cell $(m_k, m_{k+1}]$ in such a way that $(m_{k+1} - v_k) = (2/3)\ \Delta = 2(v_k - m_k)$ with Δ being the step size of the quantizer. The quantization noise, represented by Q=M-V, is still uniformly distributed but not in $[-\Delta/2\ ,\ \Delta/2]$ range now since v_k is not the mid-point of the decision cell. Determine first the range of the random variable Q and then compute the average power of Q; i.e., $E(Q^2)$.

$(a)\ E[Q^2] = \frac{\Delta^2}{9}$ $(b)\ E[Q^2] = \frac{\Delta^2}{12}$ $(c)\ E[Q^2] = \frac{\Delta^2}{16}$

$(d)\ E[Q^2] = \frac{\Delta^2}{6}$ $(e)\ E[Q^2] = \frac{\Delta^2}{18}$

PROBLEM # 4

Consider again Problem # 3 and determine based on the result you obtained whether it is better to place the quantization level at the mid-point of the decision cell or somewhere else as specified below:

(a) Mid-point of decision cell is better $(b)\ (m_{k+1} - v_k) = 2(v_k - m_k)$ is better

$(c)\ (m_{k+1} - v_k) = 3(v_k - m_k)$ is better $(d)\ (m_{k+1} - v_k) = 4(v_k - m_k)$ is better

$(e)\ (m_{k+1} - v_k) = 5(v_k - m_k)$ is better

PROBLEM # 5

Non-uniform quantization has the objective of reducing the bit rate compared to uniform quantization. It can be implemented by first compressing the analog message signal using either the μ-law or the A-law and then applying the compressed signal to a uniform quantizer. The bit rate reduction in the non-uniform quantization scheme is achieved because the uniform quantizer used requires less number of levels. This, in turn, is due to the fact that:

(a) The compression law compresses high amplitude values of the message more than low amplitude values.
(b) The compressed signal has a smaller dynamic range that the original message signal.
(c) The compression law compresses low amplitude values of the message more than high amplitude values.
(d) The compression law compresses all amplitude values of the message in the same way.
(e) The compressed signal has a larger dynamic range than the original message signal.

PROBLEM # 6

Delta Modulation (DM) is a technique aimed at reducing the complexity of the A/D conversion (Quantization + Coding) in PCM. The bit rate at the output of the DM transmitter is supposed to be comparable to that at the output of the PCM transmitter. Let the PCM system use a sampling rate equal to the Nyquist rate for a message, m(t), having a bandwidth equal to 5KHz and 256 quantization levels. Determine the sampling rate that needs to be used in the DM system so that the bit rate in DM is equal to that in PCM.

(a) 160,000 samples/sec.
(b) 40,000 samples/sec.
(c) 10,000 samples/sec.
(d) 20,000 samples/sec
(e) 80,000 samples/sec.

PROBLEM # 7

DPCM is a technique basically aimed at reducing the bit rate compared to PCM. This can be achieved by having the dynamic range of the quantizer input (error signal) in DPCM smaller than that of the message signal and, thus, requiring a smaller number of quantization levels compared to PCM if the step size, Δ, is the same in the PCM and DPCM quantizers. Consider a message signal, m(t), with bandwidth equal to 5 KHz. The sampling rate is the same in PCM and DPCM and it is equal to the Nyquist rate. The dynamic range of m(t) is 16 Volts and Δ is equal to 0.0625 Volts. Determine the dynamic range of the quantizer input in DPCM if the DPCM bit rate is equal to 3/4 of the bit rate in PCM.

(a) 2 Volts
(b) 4 Volts
(c) 3 Volts
(d) 8 Volts
(e) 12 Volts

PROBLEM # 8

Consider again Problem # 7 but with DPCM used to increase the quantizer output signal-to-noise ratio while keeping the same bit rate compared to PCM. In this case, therefore, the same number of levels is used in PCM and DPCM and, thus, the quantizer step size in DPCM is smaller than that in PCM. Let the dynamic range of the quantizer input in DPCM be equal to 8 Volts and determine the SNR improvement. Use $\sigma_Q^2 = \Delta^2 / 12$ as the average power of the quantization noise in both PCM and DPCM.

(a) $(SNR)_{DPCM}=4(SNR)_{PCM}$
(b) $(SNR)_{DPCM}=2(SNR)_{PCM}$
(c) $(SNR)_{DPCM}=6(SNR)_{PCM}$
(d) $(SNR)_{DPCM}=8(SNR)_{PCM}$
(e) $(SNR)_{DPCM}=3(SNR)_{PCM}$

PROBLEM # 9

Consider the binary baseband transmission of a sequence of bits under the unipolar non-return to zero (UNRZ) line code representation. The receiver is formed by a matched filter followed by a sampler and a decision-making device that uses threshold comparison in the detection of the binary digits. The input to the decision-making device is represented as follows:

$$Y = \begin{cases} \sqrt{E} + W, & \textit{if bit 1 is transmitted} \\ W, & \textit{if bit 0 is transmitted} \end{cases}$$

W is a zero mean Gaussian random variable having a variance equal to $N_0/2$. E is the energy of the pulse representing the bit 1. Apply the minimum probability of error criterion and determine the optimum threshold, λ, that needs to be used in the decision making process. The bits 0 and 1 have equal a priori probabilities.

$(a)\ \lambda = 0$ $(b)\ \lambda = \sqrt{\frac{E}{2}}$ $(c)\ \lambda = \frac{\sqrt{E}}{2}$ $(d)\ \lambda = \frac{\sqrt{E}}{4}$ $(e)\ \lambda = \sqrt{\frac{E}{6}}$

PROBLEM # 10

Consider the summation given below and representing the received signal term +ISI term when the receiver output is sampled at $t=iT_b$ for i=1, 2, 3, …

$$y(iT_b) = \sum_{k=1}^{i} A_k p[(i-k)T_b]$$

Let the received digits be 1010 with the zero on the right-hand side being the first received bit. The signal p(t) is given by

$$p(t) = e^{-|t|} \text{ for all } t.$$

Determine the ISI term in the receiver output at $t=3T_b$; i.e., in $y(3T_b)$. T_b is the bit duration. Use $A_k = A$ if bit 1 is received and $A_k = -A$ if bit 0 is received.

$(a)\ ISI = -A\left[e^{-T_b} + e^{-2T_b}\right]$ $(b)\ ISI = A\left[e^{-2T_b} - e^{-T_b}\right]$ $(c)\ ISI = A\left[e^{-T_b} + e^{-2T_b}\right]$

$(d)\ ISI = A\left[e^{-T_b} - e^{-2T_b}\right]$ $(e)\ ISI = A\left[e^{-3T_b} + e^{-2T_b} - e^{-T_b}\right]$

PROBLEM # 11

Consider the quarternary band-pass amplitude shift keying problem where the 4 signals are represented as follows:

$$s_i(t) = i\sqrt{\frac{2E}{T}}\cos(\omega_c t),\ 0 \le t \le T,$$

where i=-2, -1, 1, 2 and E is the energy of the sinusoidal pulse $s_1(t)$. In the time interval [0,T] either one of the 4 pulses is received in additive zero mean white and Gaussian noise process having a spectral height $N_0/2$. Use

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_c t),\ 0 \le t \le T,$$

as obtained by the Gram-Schmidt procedure to obtain the correlation receiver first. Then use the minimum probability of error criterion under equally probable transmitted signals to determine the decision rule (maximum likelihood decision rule), which applies to the correlator output, denoted by x.

(a)
$0 < x < 1.5\sqrt{E}$, decide $s_1(t)$
$-1.5\sqrt{E} < x < 0$, decide $s_{-1}(t)$
$1.5\sqrt{E} < x < \infty$, decide $s_2(t)$
$-\infty < x < -1.5\sqrt{E}$, decide $s_{-2}(t)$

(b)
$0 < x < 1.5\sqrt{E}$, decide $s_1(t)$
$-1.5\sqrt{E} < x < 0$, decide $s_{-1}(t)$
$1.5\sqrt{E} < x < \infty$, decide $s_{-2}(t)$
$-\infty < x < -1.5\sqrt{E}$, decide $s_2(t)$

(c)
$0 < x < 1.5\sqrt{E}$, decide $s_2(t)$
$-1.5\sqrt{E} < x < 0$, decide $s_{-1}(t)$
$1.5\sqrt{E} < x < \infty$, decide $s_1(t)$
$-\infty < x < -1.5\sqrt{E}$, decide $s_{-2}(t)$

(d)
$0 < x < 1.5\sqrt{E}$, decide $s_{-1}(t)$
$-1.5\sqrt{E} < x < 0$, decide $s_1(t)$
$1.5\sqrt{E} < x < \infty$, decide $s_2(t)$
$-\infty < x < -1.5\sqrt{E}$, decide $s_{-2}(t)$

(e)
$0 < x < 1.5\sqrt{E}$, decide $s_{-2}(t)$
$-1.5\sqrt{E} < x < 0$, decide $s_{-1}(t)$
$1.5\sqrt{E} < x < \infty$, decide $s_2(t)$
$-\infty < x < -1.5\sqrt{E}$, decide $s_1(t)$

PROBLEM # 12

Consider the following 4 band-pass signals:

$$s_1(t) = \sqrt{\frac{2E}{T}}\cos(\omega_c t),\ 0 \le t \le T,$$

$$s_2(t) = \sqrt{\frac{2E}{T}}\cos\left(\omega_c t - \frac{\pi}{2}\right),\ 0 \le t \le T,$$

$$s_3(t) = \sqrt{\frac{2E}{T}}\cos\left(\omega_c t + \frac{\pi}{2}\right),\ 0 \le t \le T,$$

$$s_4(t) = \sqrt{\frac{2E}{T}}\cos(\omega_c t + \pi),\ 0 \le t \le T,$$

Use the Gram-Schmidt orthogonalization procedure and determine the number of orthonormal functions that can be constructed using the given set of 4 PSK signals. Determine, as a result, the number of correlators that need to be used in the reception of the given signals in additive white and Gaussian noise process.

(a) 1 correlator needs to be used.
(b) 2 correlators need to be used.
(c) 4 correlators need to be used.
(d) 3 correlators need to be used.
(e) 1 or 2 correlators need to be used.

PROBLEM # 13

In a two-dimensional space (x_1, x_2), consider the following 2 signal vectors:

$$\underline{s}_1 = \begin{bmatrix} \sqrt{E} \\ 0 \end{bmatrix}, \quad \underline{s}_2 = \begin{bmatrix} -\sqrt{E} \\ \sqrt{E} \end{bmatrix}.$$

Use the minimum distance decision rule and determine the equation of the straight line that partitions the (x_1, x_2) space into two decision regions, Z_1 and Z_2, which can be used to decide in favor of signal $\underline{s}_1$ when the received data vector $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ falls in region Z_1 or in favor of $\underline{s}_2$ when $\underline{x}$ falls in region Z_2.

$(a)\ x_2 = 2x_1 - \sqrt{E}/2$ $\quad$ $(b)\ x_2 = x_1 + \sqrt{E}/2$ $\quad$ $(c)\ x_2 = x_1 - \sqrt{E}/2$

$(d)\ x_2 = x_1 - \sqrt{E}/4$ $\quad$ $(e)\ x_2 = 2x_1 + \sqrt{E}/2$