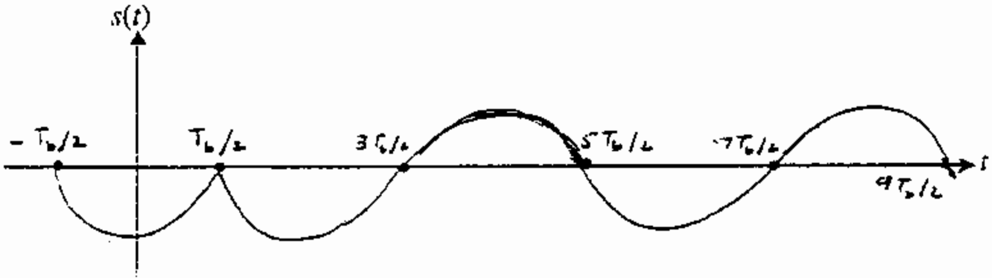
Problem 3.13

(a)



(b) $g(t) = \begin{cases} \cos\left(\dfrac{\pi t}{T_b}\right), & \dfrac{-T_b}{2} < t \leq \dfrac{T_b}{2} \\ 0, & \text{otherwise} \end{cases}$

Equivalently, we may write

$$g(t) = \cos\left(\frac{\pi t}{T_b}\right) A\text{rect}\left(\frac{t}{T_b}\right)$$

where rect(t) is a rectangular function of unit amplitude and unit duration. The Fourier transform of $g(t)$ is given by

$$G(f) = \frac{AT_b}{2}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] \ast \text{sinc}(fT_b)$$

where A denotes the pulse amplitude and $\ast$ denotes convolution in the frequency domain.

These pages are retrieved from Haykin's Solution Manual

Using the replication property of the delta function $\delta(f)$, we get

$$G(f) = \frac{AT_b}{2}\left[\text{sinc}\left(T_b\left(f - \frac{2}{T_b}\right)\right) + \text{sinc}\left(T_b\left(f + \frac{2}{T_b}\right)\right)\right]$$

Using Eq. (1.52) of the textbook, the power spectral density of the binary data stream is

$$S(f) = \frac{|G(f)|^2}{T_b}$$

$$= \frac{A^2T_b}{4}\left[\text{sinc}^2\left(T_b\left(f - \frac{2}{T_b}\right)\right) + \text{sinc}^2\left(T_b\left(f + \frac{2}{T_b}\right)\right)\right.$$

$$\left. + 2\,\text{sinc}\left(T_b\left(f - \frac{2}{T_b}\right)\right)\text{sinc}\left(T_b\left(f + \frac{2}{T_b}\right)\right)\right] \qquad (1)$$

Note that the two spectral components $\text{sinc}\left(T_b\left(f - \frac{2}{T_b}\right)\right)$ and $\text{sinc}\left(T_b\left(f + \frac{2}{T_b}\right)\right)$ overlap in the frequency interval $-(1/T_b) \le f \le (1/T_b)$, hence the presence of cross-product terms in Eq. (1).

Figure 1 plots the normalized power spectral density $S(f)/(A^2T_b/4)$ versus the normalized frequency fT_b. The interesting point to note in this figure is the significant reduction in the power spectrum of the pulse-shaped data stream $x(t)$ in the interval $-1/T_b \le f \le 1/T_b$.

(c) The power spectral density of the standard form of polar NRZ signaling is

$$S(f) = A^2T_b\,\text{sinc}^2(fT_b) \qquad (2)$$

Comparing this expression with that of Eq. (1), we observe the following differences:

	Polar NRZ signals using cosine pulses	Polar NRZ signals using rectangular pulses
$f = 0$	0	A^2T_b
$f = \pm 2/T_b$	$A^2T_b/4$	0

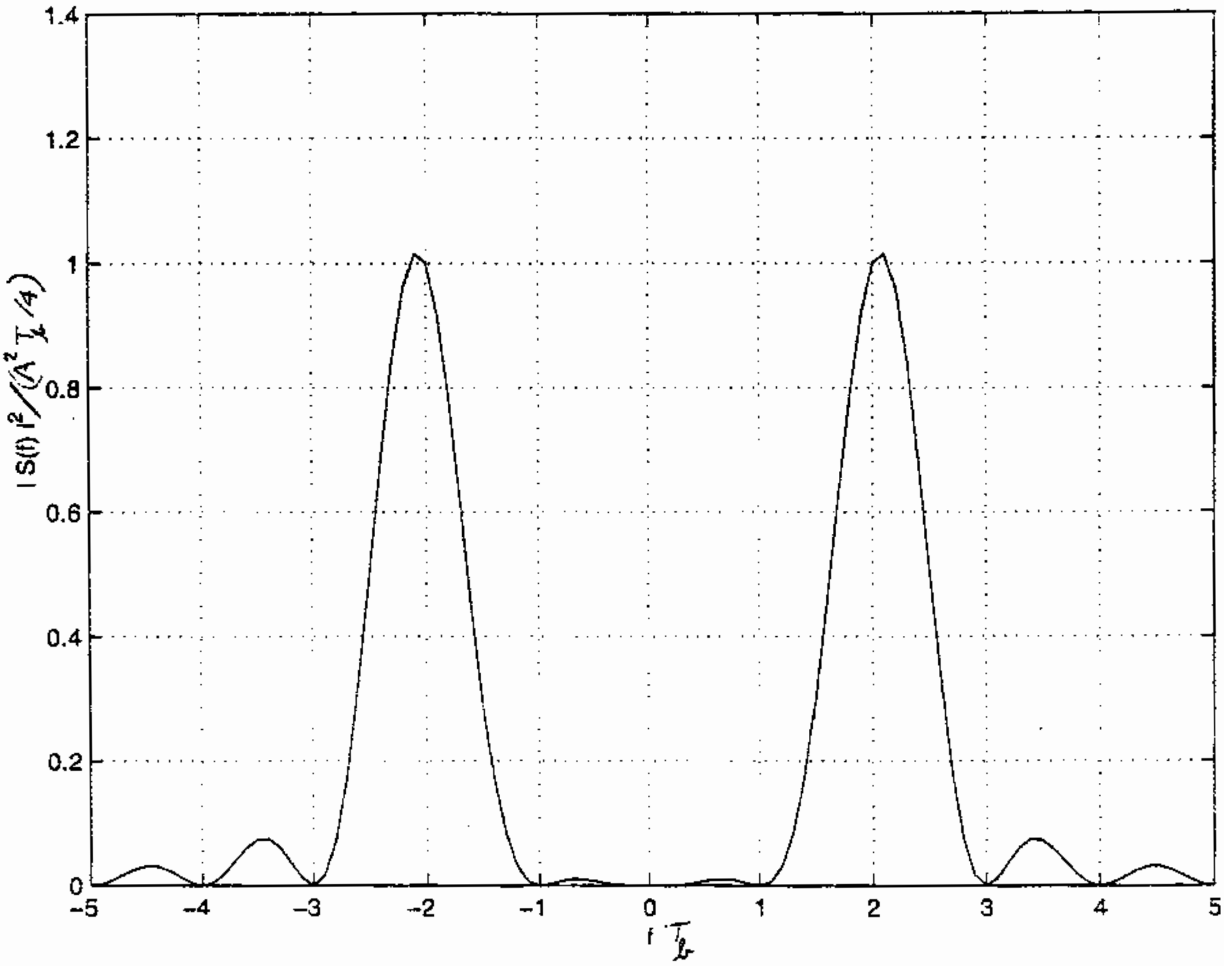
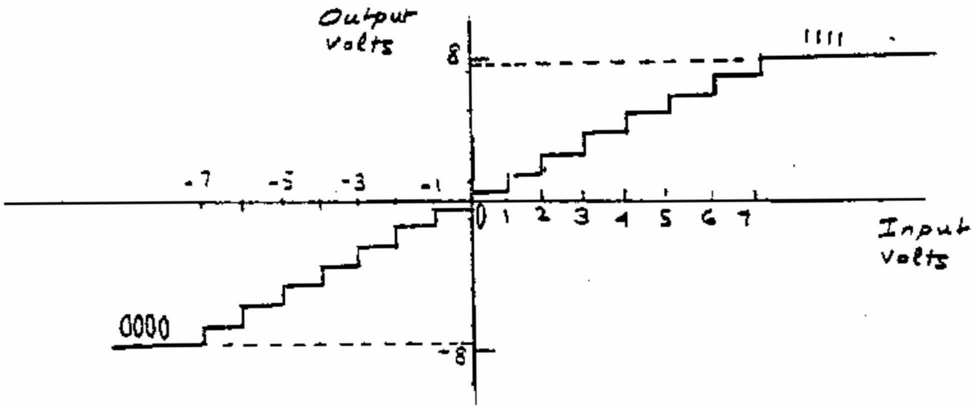


Figure 1

.oblem 3.21

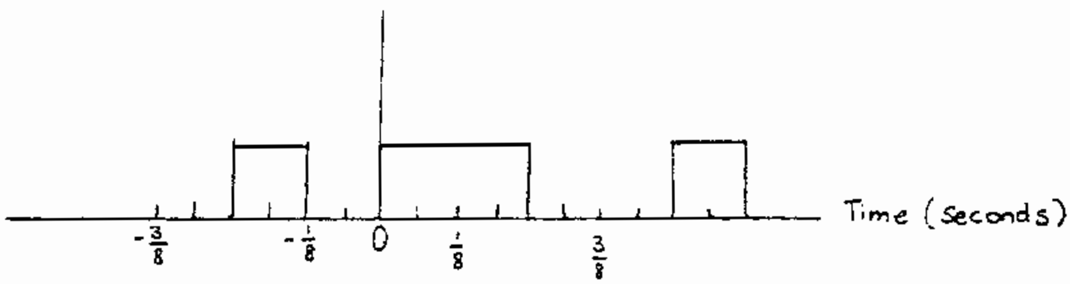
The quantizer has the following input-output curve:



At the sampling instants we have:

t	m(t)	code
-3/8	-3√2	0011
-1/8	-3√2	0011
+1/8	3√2	1100
+3/8	3√2	1100

. . the coded waveform is (assuming on-off signaling):

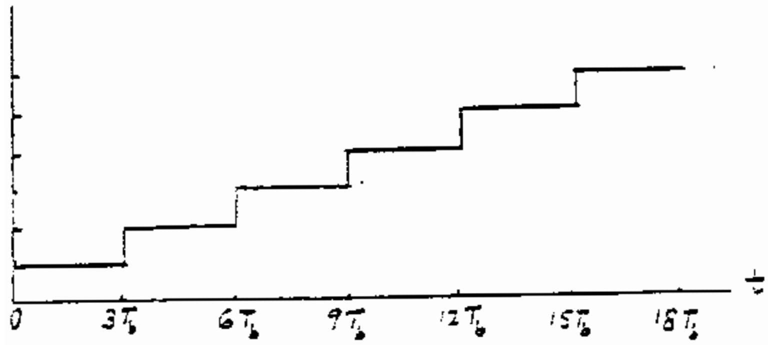


Problem 3.22

The transmitted code words are:

t/T_b	code
1	001
2	010
3	011
4	100
5	101
6	110

The sampled analog signal is



Problem 3.25

$$m(t) = A\tanh(\beta t)$$

To avoid slope overload, we require

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right| \qquad (1)$$

$$\frac{dm(t)}{dt} = A\beta\,\text{sech}^2(\beta t) \qquad (2)$$

Hence, using Eq. (2) in (1):

$$\Delta \geq \max(A\beta\,\text{sech}^2(\beta t)) \times T_s \qquad (3)$$

Since $\text{sech}(\beta t) = \dfrac{1}{\cosh(\beta t)}$

$$= \frac{2}{e^{+\beta t} + e^{-\beta t}}$$

it follows that the maximum value of $\text{sech}(\beta t)$ is 1, which occurs at time $t = 0$. Hence, from Eq. (3) we find that $\Delta \geq A\beta T_s$.

Problem 3.26

The modulating wave is

$$m(t) = A_m \cos(2\pi f_m t)$$

The slope of m(t) is

$$\frac{dm(t)}{dt} = -2\pi f_m A_m \sin(2\pi f_m t)$$

The maximum slope of m(t) is equal to $2\pi f_m A_m$.

The maximum average slope of the approximating signal $m_a(t)$ produced by the delta modulator is δ/T_s, where δ is the step size and T_s is the sampling period. The limiting value of A_m is therefore given by

$$2\pi f_m A_m > \frac{\delta}{T_s}$$

or

$$A_m > \frac{\delta}{2\pi f_m T_s},$$

Assuming a load of 1 ohm, the transmitted power is $A_m^2/2$. Therefore, the maximum power that may be transmitted without slope-overload distortion is equal to $\delta^2/8\pi^2 f_m^2 T_s^2$.

Problem 3.27

$f_s = 10 f_{\text{Nyquist}}$

$f_{\text{Nyquist}} = 6.8$ kHz

$f_s = 10 \times 6.8 \times 10^3 = 6.8 \times 10^4$ Hz

$$\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

For the sinusoidal signal $m(t) = A_m \sin(2\pi f_m t)$, we have

$$\frac{dm(t)}{dt} = 2\pi f_m A_m \cos(2\pi f_m t)$$

Hence,

$$\left|\frac{dm(t)}{dt}\right|_{\max} = \left|2\pi f_m A_m\right|_{\max}$$

or, equivalently,

$$\frac{\Delta}{T_s} \geq \left|2\pi f_m A_m\right|_{\max}$$

Therefore,

$$\left|A_m\right|_{\max} = \frac{\Delta}{T_s \times 2\pi \times f_m}$$

$$= \frac{\Delta f_s}{2\pi f_m}$$

$$= \frac{0.1 \times 6.8 \times 10^4}{2\pi \times 10^3}$$

$$= 1.08 \text{V}$$

Problem 3.28

(a) From the solution to Problem 3.27, we have

$$A = \frac{\Delta f_s}{2\pi f_m} \quad \text{or} \quad \Delta = \frac{2\pi f_m A}{f_s} \tag{1}$$

The average signal power $= \dfrac{A^2}{2}$

$$= \frac{1}{2}\left(\frac{\Delta f_s}{2\pi f_m}\right)^2$$

$$S_Q(f) = \begin{cases} \Delta^2/3f_s & -W \le f \le W \\ 0, & \text{otherwise} \end{cases}$$

where W is the bandwidth of the reconstruction filter at the demodulator output. Hence, the average quantization noise power is

$$N = \int_{-W}^{W} S_Q(f)df = \frac{2\Delta^2 W}{3f_s} \tag{2}$$

Substituting Eq. (2) into (1), we get

$$N = 2\left(\frac{2\pi f_m A}{f_s}\right)^2 \frac{W}{3f_s}$$

$$= \frac{8\pi^2 f_m^2 A^2 W}{3f_s^3}$$

(b) Correspondingly, output signal-to-noise ratio is

$$\text{SNR} = \frac{\left(\frac{1}{2}\right)A^2}{(8\pi^2 f_m^2 A^2 W)/3f_s^3}$$

$$= \frac{3f_s^3}{16\pi^2 f_m^2 W}$$

Problem 3.29

(a) $A \le \dfrac{\Delta f_s}{2\pi f_m}$

$$\Delta \ge \frac{2\pi f_m A}{f_s}$$

$$\Delta \geq \frac{2 \times \pi \times 10^3 \times 1}{50 \times 10^3}$$

$$= 0.126\text{V}$$

(b) $$(\text{SNR})_{\text{out}} = \frac{3}{16\pi^2} \frac{f_s^3}{f_m^2 W}$$

$$= \frac{3}{16\pi^2} \times \frac{(50 \times 10^3)^3}{10^6 \times 5 \times 10^3}$$

$$= 475$$

In decibels,

$$(\text{SNR})_{\text{out}} = 10\log_{10} 475$$

$$= 26.8 \quad \text{dB}$$

Problem 3.30

(a) For linear delta modulation, the maximum amplitude of a sinusoidal test signal that can be used without slope-overload distortion is

$$A = \frac{\Delta f_s}{2\pi f_m}$$

$$= \frac{0.1 \times 60 \times 10^3}{2\pi \times 1 \times 10^3} \qquad f_s = 2 \times 3 \times 10^3 \times 10$$

$$= 0.95\text{V}$$

(b) (i) Under the pre-filtered condition, it is reasonable to assume that the granular quantization noise is uniformly distributed between $-\Delta$ and $+\Delta$. Hence, the variance of the quantization noise is

$$\sigma_Q^2 = \int_{-\Delta}^{\Delta} \frac{1}{2\Delta} q^2 dq$$

$$= \frac{1}{6\Delta}[q^3]_{-\Delta}^{\Delta}$$

$$= \frac{\Delta^2}{3}$$

The signal-to-noise ratio under the pre-filtered condition is therefore

$$(\text{SNR})_{\text{prefiltered}} = \frac{A^2/2}{\Delta^2/3}$$

$$= \frac{3A^2}{2\Delta^2}$$

$$= \frac{3 \times 0.95^2}{2 \times 0.1^2}$$

$$= 135$$
$$= 21.3 \text{ dB}$$

(ii)The signal-to-noise ratio under the post-filtered condition is

$$\left(\frac{S}{N}\right)_{\text{postfiltered}} = \frac{3}{16\pi^2} \times \frac{f_s^3}{f_m^2 W}$$

$$= \frac{3}{16\pi^2} \times \frac{(60)^3}{(1)^2 \times 3}$$

$$= 1367$$

$$= 31.3 \text{ dB}$$

The filtering gain in signal-to-noise ratio due to the use of a reconstruction filter at the demodulator output is therefore 31.3 - 21.3 = 10 dB.