Problem 4.12

The rectangular pulse given in Fig. P4.12 is defined by

$g(t) = \text{rec}(t/T)$

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-T/2}^{T/2} \exp(-j2\pi f t)\,dt$$

$$= T\,\text{sinc}\,(fT)$$

We thus have the Fourier-transform pair

$$\text{rec}(t/T) \rightleftharpoons T\,\text{sinc}\,(fT)$$

The magnitude spectrum $|G(f)|/T$ is plotted as the solid line in Fig. 1, shown on the next page.

Consider next a Nyquist pulse (raised cosine pulse with a rolloff factor of zero). The magnitude spectrum of this second pulse is a rectangular function of frequency, as shown by the dashed curve in Fig. 1.

Comparing the two spectral characteristics of Fig. 1, we may say that the rectangular pulse of Fig. P4.12 provides a crude approximation to the Nyquist pulse.

These pages are retrieved from the solution manual of Haykin's book : Comm. Systems, 4th Edition, Wiley.

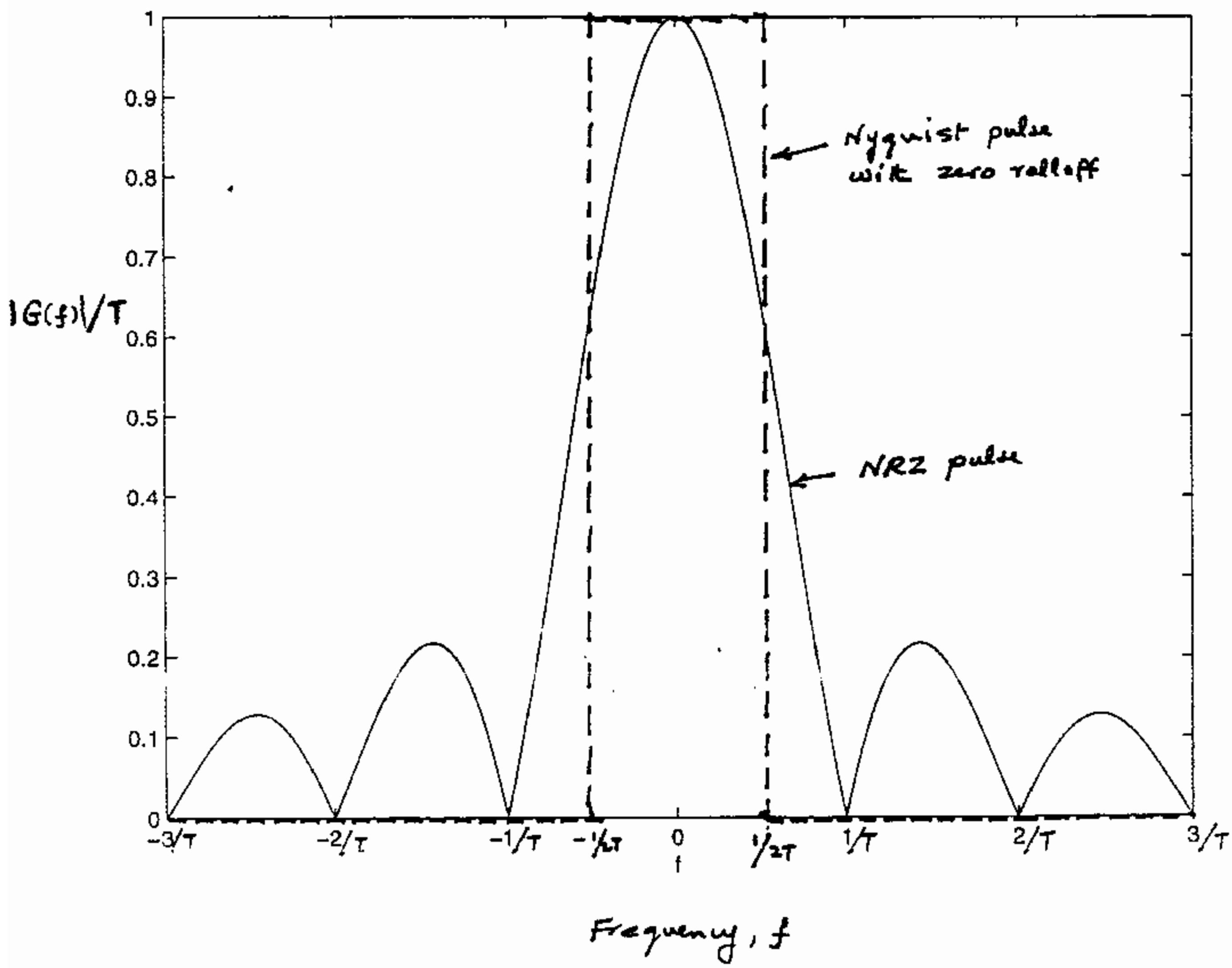

Figure 1 Spectral characteristics

$$* \cos\left[\frac{\pi(|f|-f_1)}{2W-2f_1}\right] = \cos\left[\frac{\pi|f|}{2W-2f_1} - \frac{\pi f_1}{2W-2f_1}\right]$$

$$= \cos\left[\frac{\pi|f|}{2W-2f_1} - \frac{\pi W(1-\alpha)}{2W-2f_1}\right] = \cos\left[\frac{\pi(|f|-W)}{2W-2f_1} + \frac{\alpha\pi W}{2W-2W(1-\alpha)}\right]$$

$$= \cos\left[\frac{\pi(|f|-W)}{2W-2f_1} + \frac{\pi}{2}\right]$$

$$= -\sin\left(\frac{\pi(|f|-W)}{2W-2f_1}\right)$$

Problem 4.13

Since P(f) is an even real function, its inverse Fourier transform equals

$$p(t) = 2\int_0^{\infty} P(f)\cos(2\pi f t)\,df \tag{1}$$

The P(f) is itself defined by Eq. (7.60) which is reproduced here in the form

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 < |f| < f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2W - 2f_1}\right]\right\}, & f_1 < f < 2W - f_1 \\ 0, & |f| > 2W - f_1 \end{cases} \quad * \text{(See above)} \tag{2}$$

Hence, using Eq. (2) in (1):

$$p(t) = \frac{1}{W}\int_0^{f_1}\cos(2\pi f t)\,df + \frac{1}{2W}\int_{f_1}^{2W-f_1}\left[1 + \cos\left(\frac{\pi(f-f_1)}{2W\alpha}\right)\right]\cos(2\pi f t)\,df$$

$$= \left[\frac{\sin(2\pi f t)}{2\pi W t}\right]_0^{f_1} + \left[\frac{\sin(2\pi f t)}{4\pi W t}\right]_{f_1}^{2W-f_1}$$

$$+ \frac{1}{4W}\left[\frac{\sin\left(2\pi f t + \dfrac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t + \pi/2W\alpha}\right]_{f_1}^{2W-f_1} + \frac{1}{4W}\left[\frac{\sin\left(2\pi f t - \dfrac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t - \pi/2W\alpha}\right]_{f_1}^{2W-f_1}$$

$$= \frac{\sin(2\pi f_1 t)}{4\pi W t} + \frac{\sin[2\pi t(2W-f_1)]}{4\pi W t}$$

$$- \frac{1}{4W}\left[\frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]}{2\pi t + \pi/2W\alpha} + \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]}{2\pi t - \pi/2W\alpha}\right]$$

$$= \frac{1}{W}[\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]]\left[\frac{1}{4\pi t} - \frac{\pi t}{(2\pi t)^2 - (\pi/2W\alpha)^2}\right]$$

Note: $I = \sin(2\pi Wt)\cos(2\pi\alpha Wt) = \frac{1}{2}\left[\sin 2\pi Wt(1-\alpha) + \sin 2\pi t(W+W\alpha\right.$

But: $f_1 = W(1-\alpha)$ and $2W - f_1 = 2W - W(1-\alpha) = W + W\alpha$

$\Rightarrow I = \frac{1}{2}\left[\sin 2\pi f_1 t + \sin 2\pi t(2W - f_1)\right]$

$$= \frac{2}{W}\left[\sin(2\pi Wt)\cos(2\pi\alpha Wt)\right]\left[\frac{-(\pi/2W\alpha)^2}{4\pi t\,[(2\pi t)^2 - (\pi/2W\alpha)^2]}\right]$$

$$= \operatorname{sinc}(2Wt)\cos(2\pi\alpha Wt)\left[\frac{1}{1 - 16\,\alpha^2 W^2 t^2}\right]$$

Problem 4.14

The minimum bandwidth, B_T, is equal to $1/2T$, where T is the pulse duration. For 64 quantization levels, $\log_2 64 = 6$ bits / Sample. $\Rightarrow$ Bit rate $= 6 \times 8\,kHz = 48\,KHz = \frac{1}{T_b}$

For 2 levels, $T = T_b \Rightarrow B_T = \frac{1}{2T_b} = \frac{48}{2} = 24\,KHz$.

For 4 levels, $T = 2T_b \Rightarrow B_T = \frac{1}{4T_b} = \frac{48}{4} = 12\,KHz$. For 8 levels: $T = 3T_b$ and $B_T = \frac{1}{6T_b} = \frac{48}{6} = 8\,KHz$

Problem 4.16

The Bandwidth B of a raised cosine pulse spectrum is $2W - f_1$, where $W = 1/2T_b$ and $f_1 = W(1-\alpha)$. Thus $B = W(1+\alpha)$. For a data rate of 56 kilobits per second, W = 28 kHz.

(a) For $\alpha = 0.25$,

$$B = 28 \text{ kHz} \times 1.25$$
$$= 35 \text{ kHz}$$

(b) $$B = 28 \text{ kHz} \times 1.5$$
$$= 42 \text{ kHz}$$

(c) B = 49 kHz

(d) B = 56 kHz

Problem 4.17

The use of eight amplitude levels ensures that 3 bits can be transmitted per pulse. The symbol period can be increased by a factor of 3. All four bandwidths in problem 7.12 will be reduced to 1/3 of their binary PAM values.

Problem 4.18

(a) For a unity rolloff, raised cosine pulse spectrum, the bandwidth B equals 1/T, where T is the pulse length. Therefore, T in this case is 1/12kHz. Quarternary PAM ensures 2 bits per pulse, so the rate of information is

$$\frac{2 \text{ bits}}{T} = 24 \text{ kilobits per second.}$$

(b) For 128 quantizing levels, 7 bits are required to transmit an amplitude. The additional bit for synchronization makes each code word 8 bits. The signal is transmitted at 24 kilobits/s, so it must be sampled at

$$\frac{24 \text{ kbits/s}}{8 \text{ bits/sample}} = 3 \text{ kHz.}$$

The maximum possible value for the signal's highest frequency component is 1.5 kHz, in order to avoid aliasing.

Problem 4.19

The raised cosine pulse bandwidth $B_T = 2W - f_1$, where $W = 1/2T_b$. For this channel, $B_T = 75$ kHz. For the given bit duration, $W = 50$ kHz. Then,

$$f_1 = 2W - B_T$$
$$= 25 \text{ kHz}$$
$$\alpha = 1 - f_1/W$$
$$= 0.5$$