Problem 3.1

Let $2W$ denote the bandwidth of a narrowband signal with carrier frequency f_c. The in-phase and quadrature components of this signal are both low-pass signals with a common bandwidth of W. According to the sampling theorem, there is no information loss if the in-phase and quadrature components are sampled at a rate higher than $2W$. For the problem at hand, we have

$f_c = 100$ kHz
$2W = 10$ kHz

Hence, $W = 5$ kHz, and the minimum rate at which it is permissible to sample the in-phase and quadrature components is 10 kHz.

pages 1-12 are pages retrieved from the solution manual of Haykin's book: Communications Systems, 4th Edition

Problem 3.2

(a) Consider a periodic train c(f) of rectangular pulses, each of duration T. The Fourier series expansion of c(t) (assuming that a pulse of the train is centered on the origin) is given by

$$c(t) = \sum_{n=-\infty}^{\infty} f_s \, \mathrm{sinc}(nf_s T)\exp(j2\pi n f_s t)$$

where f_s is the repetition frequency, and the amplitude of a rectangular pulse is assumed to be 1/T (i.e., each pulse has unit area). The assumption that $f_s T >> 1$ means that the spectral lines (i.e., harmonics) of the periodic pulse train c(t) are well separated from each other.

Multiplying a message signal g(t) by c(t) yields

$$\begin{aligned} s(t) &= c(t)g(t) \\ &= \sum_{n=-\infty} f_s \, \mathrm{sinc}(nf_s T)\, g(t) \exp(j2\pi n f_s(t) \end{aligned} \tag{1}$$

Taking the Fourier transform of both sides of Eq. (1) and using the frequency-shifting property of the Fourier transform:

$$S(f) = \sum_{n=-\infty}^{\infty} f_s \, \mathrm{sinc}(nf_s T)\, G(f - nf_s) \tag{2}$$

where G(f) = F[g(t)]. Thus, the spectrum S(f) consists of frequency-shifted replicas of the original spectrum G(f), with the nth replica being scaled in amplitude by the factor $f_s \mathrm{sinc}(nf_s T)$.

(b) In accordance with the sampling theorem, let it be assumed that

- The signal g(t) is band-limited with

$$G(f) = 0 \quad \text{for} \quad -W < f < W$$

- The sampling frequency f_s is defined by

$$f_s > 2W$$

Then, the different frequency-shifted replicas of G(f) involved in the construction of S(f) will <u>not</u> overlap. Under the conditions described herein, the original spectrum G(f), and therefore the signal g(t), can be recovered exactly (except for a trivial amplitude scaling) by passing s(t) through a low-pass filter of bandwidth W.

Problem 3.3

(a) $g(t) = \text{sinc}(200t)$

This sinc pulse corresponds to a bandwidth W = 100 Hz. Hence, the Nyquist rate is 200 Hz, and the Nyquist interval is 1/200 seconds.

(b) $g(t) = \text{sinc}^2(200t)$

This signal may be viewed as the product of the sinc pulse sinc(200t) with itself. Since multiplication in the time domain corresponds to convolution in the frequency domain, we find that the signal g(t) has a bandwidth equal to twice that of the sinc pulse sin(200t), that is, 200 Hz. The Nyquist rate of g(t) is therefore 400 Hz, and the Nyquist interval is 1/400 seconds.

(c) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

The bandwidth of g(t) is determined by the highest frequency component of sinc(200t) or $\text{sinc}^2(200t)$, whichever one is the largest. With the bandwidth (i.e., highest frequency component of) the sinc pulse sinc(200 t) equal to 100 Hz and that of the squared sinc pulse $\text{sinc}^2(200t)$ equal to 200 Hz, it follows that the bandwidth of g(t) is 200 Hz. Correspondingly, the Nyquist rate of g(t) is 400 Hz, and its Nyquist interval is 1/400 seconds.

Problem 3.5

The spectrum of the flat-top pulses is given by

$$H(f) = T\,\text{sinc}(fT)\exp(-j\pi fT)$$

$$= 10^{-4}\,\text{sinc}(10^{-4}f)\exp(-j\pi f10^{-4})$$

Let $s(t)$ denote the sequence of flat-top pulses:

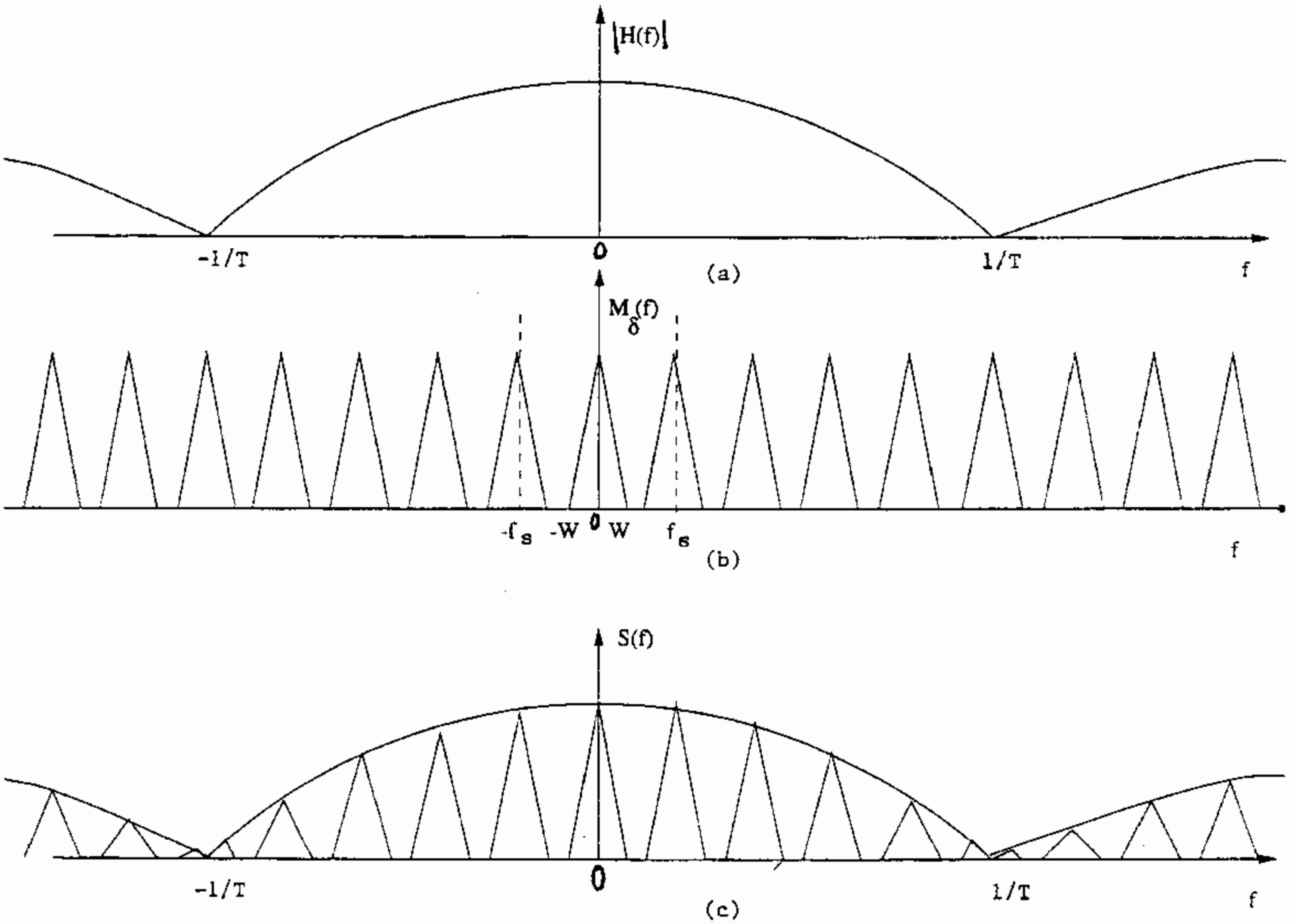
$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s)$$

The spectrum $S(f) = F[s(t)]$ is as follows:

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)H(f)$$

$$= f_s H(f) \sum_{k=-\infty}^{\infty} M(f-kf_s)$$

The magnitude spectrum $|S(f)|$ is thus as shown in Fig. 1c.



$1/T = 10{,}000\text{Hz}$
$f_s = 1{,}000\text{Hz}$
$W = 400\text{Hz}$

Figure 1

Problem 3.6

At $f = 1/2T_s$, which corresponds to the highest frequency component of the message signal for a sampling rate equal to the Nyquist rate, we find from Eq. (6-19) that the amplitude response

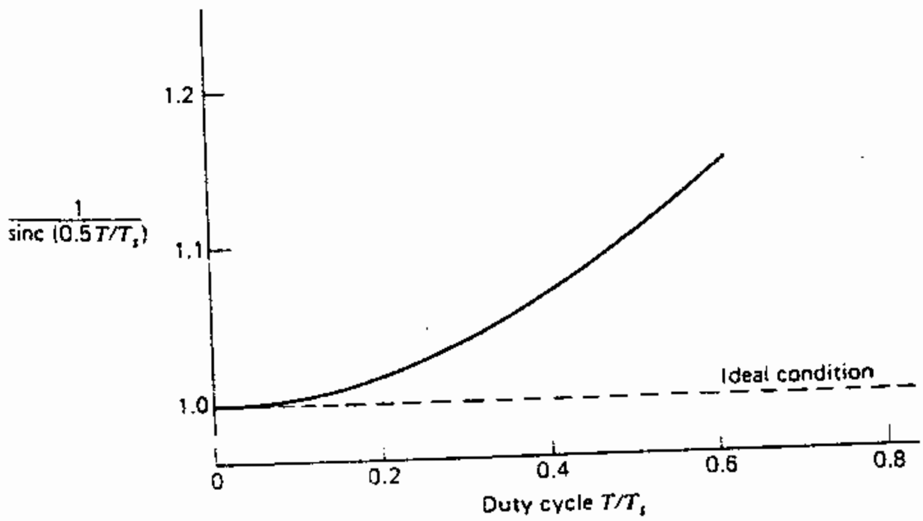


Figure 1

of the equalizer normalized to that at zero frequency is equal to

$$\frac{1}{\text{sinc}(0.5T/T_s)} = \frac{(\pi/2)(T/T_s)}{\sin[(\pi/2)(T/T_s)]}$$

where the ratio T/T_s is equal to the duty cycle of the sampling pulses. In Fig. 1, this result is plotted as a function of T/T_s. Ideally, it should be equal to one for all values of T/T_s. For a duty cycle of 10 percent, it is equal to 1.0041. It follows therefore that for duty cycles of less than 10 percent, the aperture effect becomes negligible, and the need for equalization may be omitted altogether.

Problem 3.8

(a) The sampling interval is T_s = 125 μs. There are 24 channels and 1 sync pulse, so the time alloted to each channel is $T_c = T_s/25$ = 5 μs. The pulse duration is 1 μs, so the time between pulses is 4 μs.

(b) If sampled at the nyquist rate, 6.8 kHz, then T_s = 147 μs, T_c = 5.88 μs, and the time between pulses is 4.88 μs.

Problem 3.9

(a) The bandwidth required for each single sideband channel is 10 kHz. The total bandwidth for 12 channels is 120 kHz.

(b) The Nyquist rate for each signal is 20 kHz. For 12 TDM signals, the total data rate is 240 kHz. By using a sinc pulse whose amplitude varies in accordance with the modulation, and with zero crossings at multiples of (1/240) ms, we need a minimum bandwidth of 120 kHz.

For the case of rectangular pulses, the minimum bandwidth needed is 240 kHz if we define the bandwidth of sinc function by its first zero crossing with the frequency axis.

Problem 3.16

The minimum number of bits per sample is 7 for a signal-to-quantization noise ratio of 40 dB. Hence,

$$\begin{pmatrix}\text{The number of samples} \\ \text{in a duration of } 10s\end{pmatrix} = 8000 \times 10$$
$$= 8 \times 10^4 \text{ samples}$$

The minimum storage is therefore

$$= 7 \times 8 \times 10^4$$
$$= 5.6 \times 10^5$$
$$= 560 \text{ kbits}$$

Problem 3.17

Suppose that baseband signal m(t) is modeled as the sample function of a Gaussian random rocess of zero mean, and that the amplitude range of m(t) at the quantizer input extends from $-4A_{rms}$ to $4A_{rms}$. We find that samples of the signal m(t) will fall outside the amplitude range $8A_{rms}$ with a probability of overload that is less than 1 in 10^4. If we further assume the use of a binary code with each code word having a length n, so that the number of quantizing levels is 2", we find that the resulting quantizer step size is

$$\delta = \frac{8\,A_{rms}}{2^{\mathcal{R}}} \tag{1}$$

Substituting Eq. (1) to the formula for the output signal-to-quantization noise ratio, we get

$$(\text{SNR})_o = \frac{3}{16}(2^{2\mathcal{R}}) \tag{2}$$

Expressing the signal-to-noise ratio in decibels:

$$10\log_0(\text{SNR})_o = 6\mathcal{R} - 7.2 \tag{3}$$

This formula states that each bit in the code word of a PCM system contributes 6dB to the signal-to-noise ratio. It gives a good description of the noise performance of a PCM system, provided that the following conditions are satisfied:

1. The system operates with an average signal power above the error threshold, so that the effect of transmission noise is made negligible, and performance is thereby limited essentially by quantizing noise alone.
2. The quantizing error is uniformly distributed.
3. The quantization is fine enough (say $\mathcal{R} > 6$) to prevent signal-correlated patterns in the quantizing error waveform.
4. The quantizer is aligned with the amplitude range from $-4A_{rms}$ to $4A_{rms}$.

In general, conditions (1) through (3) are true of toll quality voice signals. However, when demands on voice quality are not severe, we may use a coarse quantizer corresponding to $\mathcal{R} \leq 6$. In such a case, degradation in system performance is reflected not only by a lower signal-to-noise ratio, but so by an undesirable presence of signal-dependent patterns in the waveform of quantizing error.

Problem 3.18

(a) Let the message bandwidth be W. Then, sampling the message signal at its Nyquist rate, and using an R-bit code to represent each sample of the message signal, we find that the bit duration is

$$T_b = \frac{T_s}{R} = \frac{1}{2WR}$$

The bit rate is

$$\frac{1}{T_b} = 2WR$$

The maximum value of message bandwidth is therefore

$$W_{max} = \frac{50 \times 10^6}{2 \times 7}$$

$$= 3.57 \times 10^6 \text{ Hz}$$

(b) The output signal-to-quantizing noise ratio is given by (see Example 2):

$$10 \log_{10}(SNR)_0 = 1.8 + 6R$$

$$= 1.8 + 6 \times 7$$

$$= 43.8 \text{ dB}$$

Problem 3.19

Let a signal amplitude lying in the range

$$x_i - \frac{1}{2}\delta_i \leq x \leq x_i + \frac{1}{2}\delta_i$$

be represented by the quantized amplitude x_i. The instantaneous square value of the error is $(x-x_i)^2$. Let the probability density function of the input signal be $f_X(x)$. If the step size δ_i is small in relation to the input signal excursion, then $f_X(x)$ varies little within the quantum step and may be approximated by $f_X(x_i)$. Then, the mean-square value of the error due to signals falling within this quantum is

$$E[Q_i^2] = \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} (x-x_i)^2 f_X(x)dx$$

$$= \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} (x-x_i)^2 f_X(x_i)dx$$

$$= f_X(x_i) \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} (x-x_i)^2 dx$$

$$= f_X(x_i) \int_{-\frac{1}{2}\delta_i}^{\frac{1}{2}\delta_i} x^2 \, dx$$

$$= \frac{1}{12} \delta_i^3 f_X(x_i) \qquad (1)$$

The probability that the input signal amplitude lies within the ith interval is

$$p_i = \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} f_X(x)dx = f_X(x_i) \int_{x_i - \frac{1}{2}\delta_i}^{x_i + \frac{1}{2}\delta_i} dx = f_X(x_i)\delta_i \qquad (2)$$

Therefore, eliminating $f_X(x_i)$ between Eqs. (1) and (2), we get

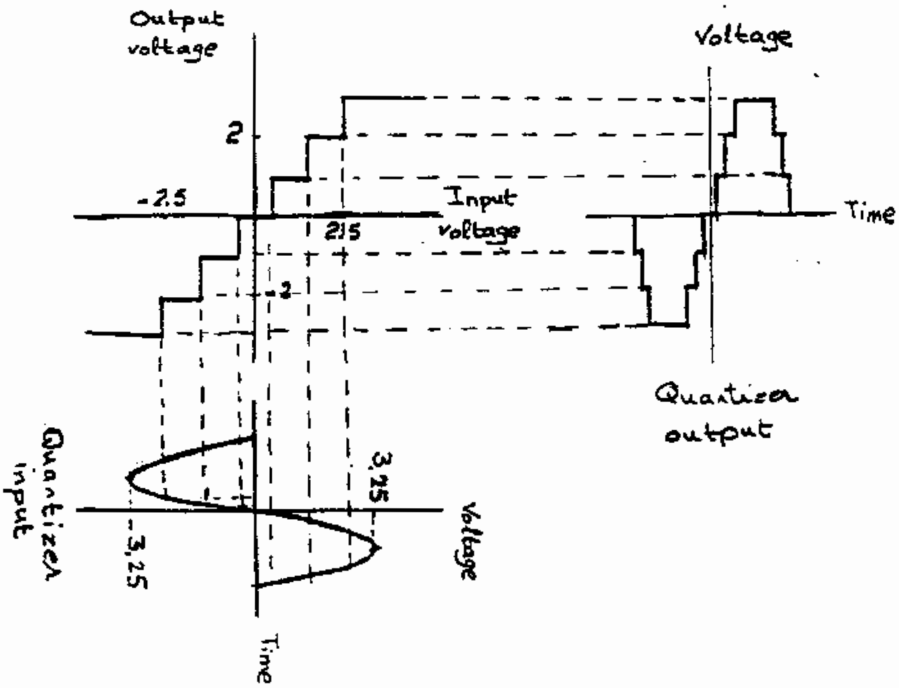
$$E[Q_i^2] = \frac{1}{12} p_i \delta_i^2$$

The total mean-square value of the quantizing error is the sum of that contributed by each of the several quanta. Hence,

$$\sum_i E[Q_i^2] = \frac{1}{12} \sum_i p_i \delta_i^2$$

Problem 3.20

(a)



(b)