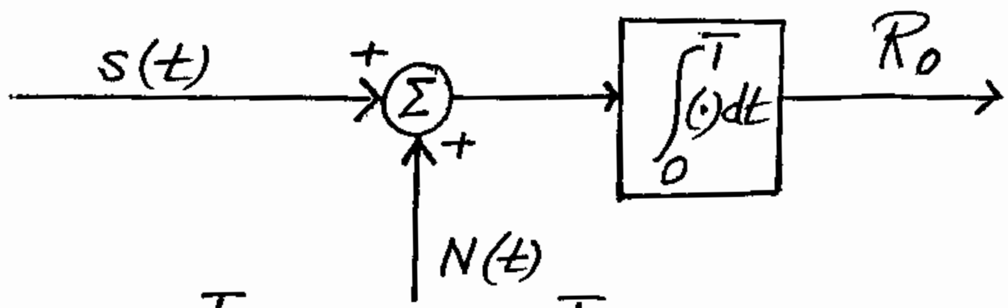
Problem #1



$$R_0 = \int_0^T s(t)\,dt + \int_0^T N(t)\,dt$$

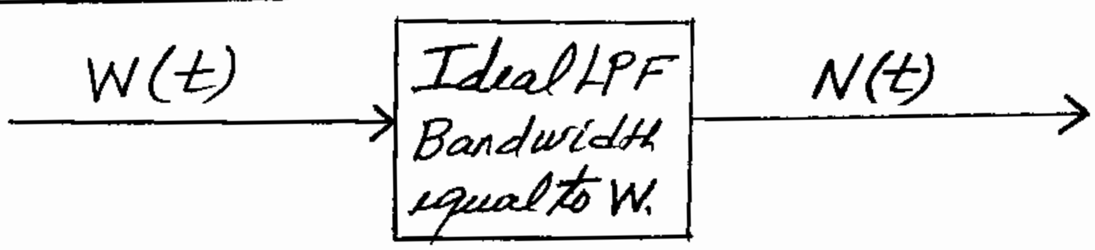
$$E(R_0) = \int_0^T s(t)\,dt + \int_0^T E[N(t)]\,dt = \int_0^T s(t)\,dt.$$

$$\sigma_{R_0}^2 = E\left[(R_0 - \overline{R_0})^2\right] = E\left[\left(\int_0^T N(t)\,dt\right)^2\right]$$

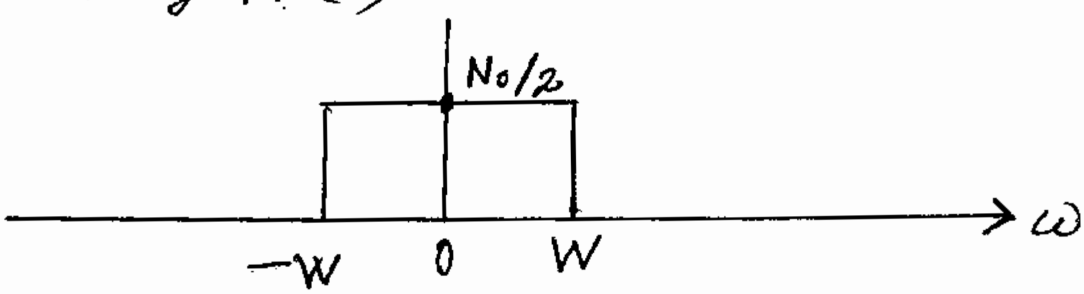
$$= E\left[\int_0^T N(t)\,dt \int_0^T N(u)\,du\right] = \int_0^T dt \int_0^T E[N(t)N(u)]\,du$$

$$= \int_0^T dt \int_0^T \frac{N_0}{2}\delta(t-u)\,du = \frac{N_0}{2}\int_0^T dt = \frac{N_0}{2}T.$$

Problem #2



The PSD of $N(t)$ is:



$$R_N(0) = E[N^2(t)] = \frac{1}{2\pi}\int_{-W}^{W} \frac{N_0}{2}\, d\omega = \frac{N_0 W}{2\pi}$$

Hence, from among the given auto correlation functions, $R_N(\tau) = \frac{N_0 W}{2\pi} e^{-2|\tau|}$ can be the autocorrelation function of $N(t)$. Also, knowing that $R_N(\tau)$ is even; i.e., $R_N(\tau) = R_N(-\tau)$, then $R_N(\tau) = \frac{N_0 W}{2\pi} e^{-2\tau}$ cannot be the autocorrelation function of $N(t)$.

Problem#3

$E[N(t)] = E[W(t)] \times H(0)$, where $H(0)$ is the value of the LPF frequency response at $\omega = 0$.

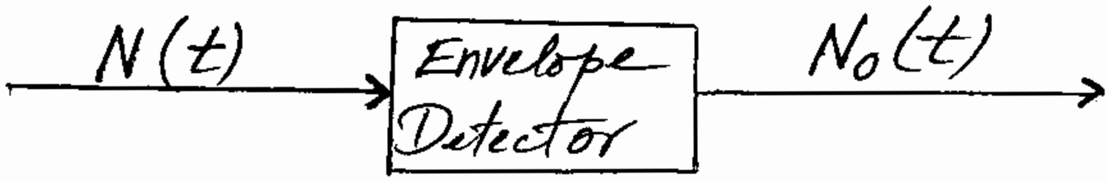
Hence, $E[N(t)] = 0$ since $E[W(t)] = 0$.

$$\sigma_N^2 = E\left[\left(N(t) - \overline{N}(t)\right)^2\right] = E[N^2(t)] = R_N(0)$$

$$= \frac{N_0 W}{2\pi}$$

Problem # 4

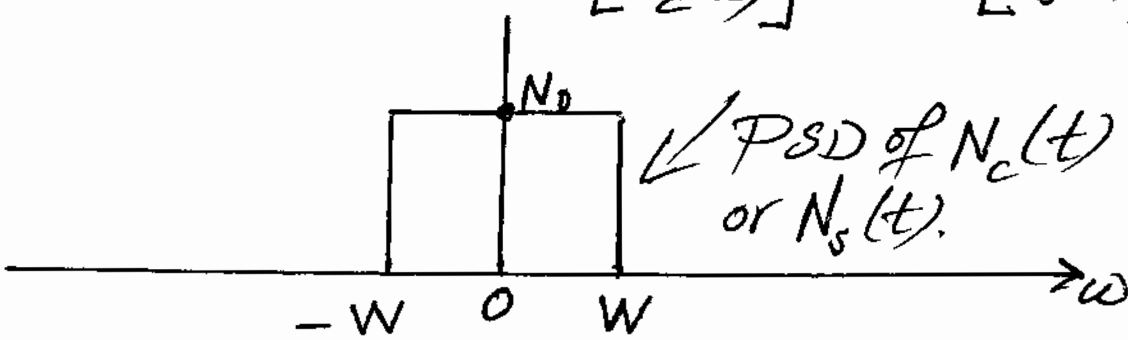
$$N(t) = N_c(t)\cos(\omega_c t) - N_s(t)\sin(\omega_c t)$$
$$= R(t)\cos[\omega_c t + \Psi(t)]$$



Average power of $N_0(t)$ is equal to $E[N_0^2(t)]$.

But, $N_0(t) = [N_c^2(t) + N_s^2(t)]^{1/2}$.

Hence, $E[N_0^2(t)] = E[N_c^2(t) + N_s^2(t)]$
$$= 2E[N_c^2(t)] = 2E[N_s^2(t)].$$



Hence, $E[N_c^2(t)] = E[N_s^2(t)] = \dfrac{N_0 W}{\pi}$

$$\Rightarrow E[N_0^2(t)] = \frac{2N_0 W}{\pi}.$$

Problem #5

$$N(t) = N_c(t)\cos(\omega_c t) - N_s(t)\sin(\omega_c t)$$
$$= R(t)\cos[\omega_c t + \psi(t)]$$
$$= R(t)\cos\psi(t)\cos(\omega_c t) - R(t)\sin\psi(t)\sin(\omega_c t).$$

Hence, $N_c(t) = R(t)\cos\psi(t)$
$N_s(t) = R(t)\sin\psi(t).$

$$\Rightarrow R^2(t) = N_c^2(t) + N_s^2(t) = N_0^2(t)$$
$$\Rightarrow R(t) = \left[N_c^2(t) + N_s^2(t)\right]^{1/2}$$

Hence, the average power of $R(t)$ is:

$$E[R^2(t)] = E[N_0^2(t)] = \frac{2N_0W}{\pi}$$

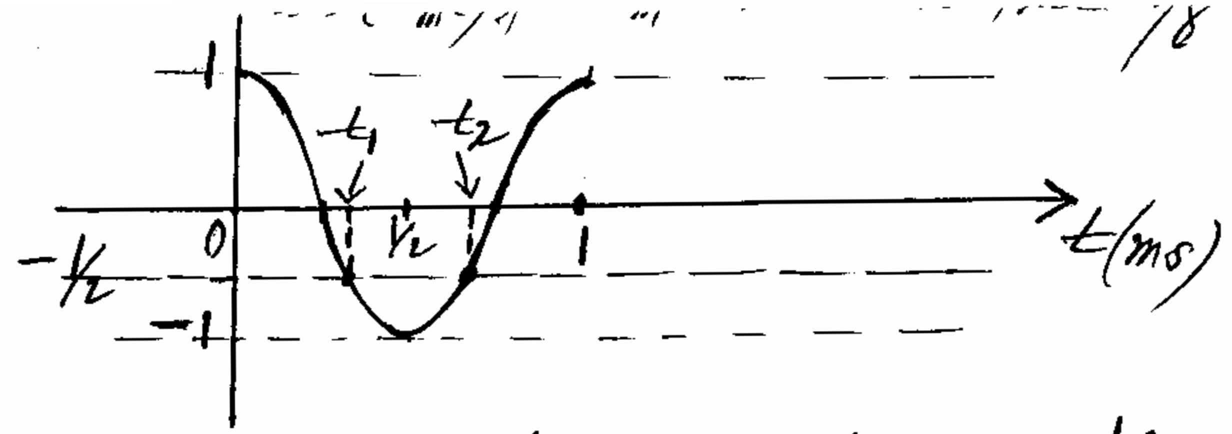
Problem #6

$$S(t) = A_c[1 + 2\cos(\omega_m t)]\cos(\omega_c t)$$

The output of the envelope detector is:

$$\begin{cases} -A_c[1+2\cos(\omega_m t)] & \text{when } (1+2\cos(\omega_m t)) < 0 \\ A_c[1+2\cos(\omega_m t)] & \text{when } (1+2\cos(\omega_m t)) > 0. \end{cases}$$

$$1 + 2\cos(\omega_m t) < 0 \Rightarrow \cos(\omega_m t) < \frac{-1}{2}$$



t_1 can be obtained using $\cos(\omega_m t_1) = -1/2$

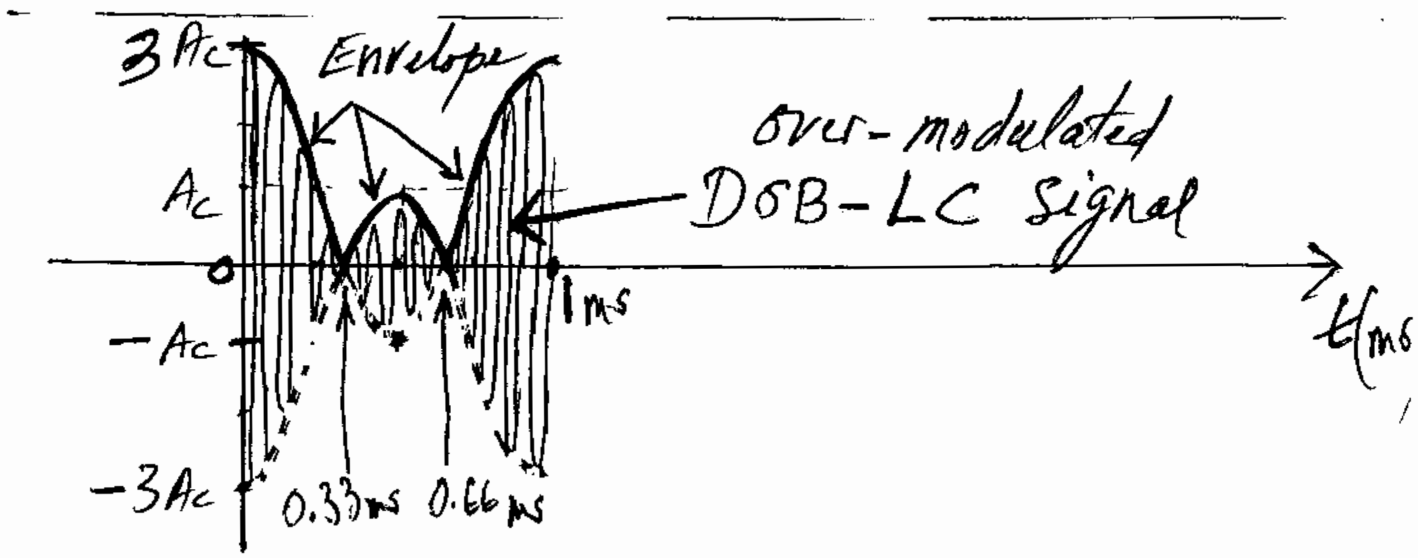
$$\Rightarrow t_1 = \frac{\cos^{-1}(-1/2)}{\omega_m} = \frac{2\pi}{3} \times \frac{1}{2\pi \times 10^3} = 0.33\,ms.$$

$$t_2 = 1 - \frac{1}{3} = 0.66\,ms.$$

Hence, the output of the envelope detector is:

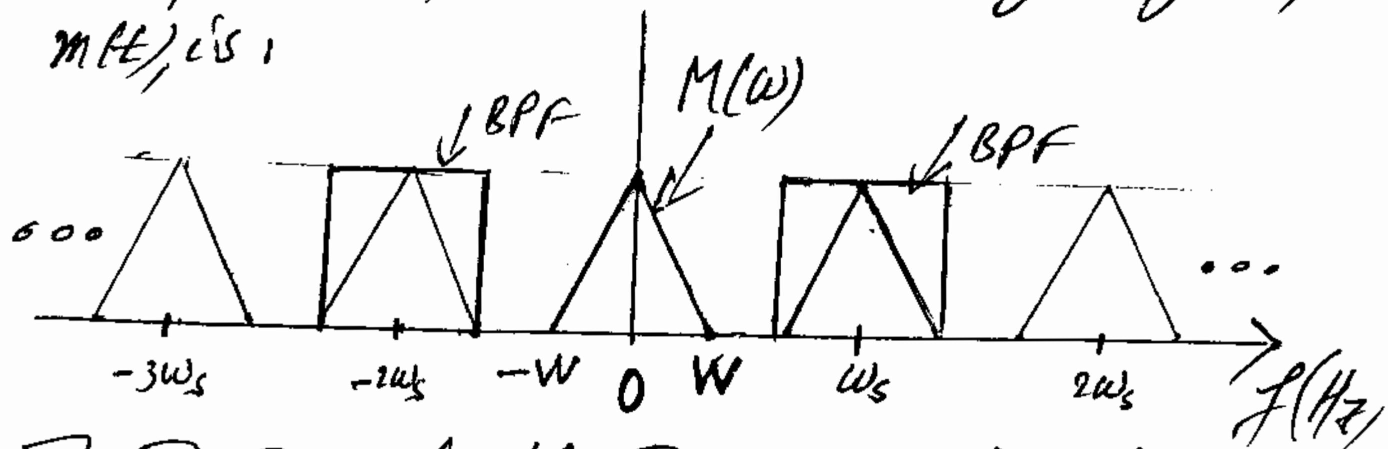
$$\begin{cases} -A_c\left[1 + 2\cos(\omega_m t)\right], & 0.33ms \leq t \leq 0.66ms \\ A_c\left[1 + 2\cos(\omega_m t)\right], & \text{otherwise} \end{cases}$$

and this is for one period of the message signal or $\cos(\omega_m t)$.



Problem # 7

The spectrum of the sampled message signal, $m(t)$, is:



The BPF picks the DSB-SC signal with carrier frequency equal to ω_s.

But, $\omega_s = \dfrac{1}{T_s} = 500\,kHz$.

$$\Rightarrow T_s = \frac{1}{5\times10^5} = 2\times10^{-6}\ \text{secs or } 2\,\mu\text{secs.}$$

The maximum value of W is such that:

$$\omega_s - W = W \text{ or } \omega_s = 2W$$

$$\Rightarrow W = \frac{\omega_s}{2} = \frac{500\,KHz}{2} = 250\,KHz.$$

Problem # 8

By the mixing operation:

$$A_c\, m(t)\cos(\omega_c t)\cos(\omega_p t) =$$

$$\frac{A_c}{2} m(t)\left[\cos(\omega_c+\omega_p)t + \cos(\omega_c-\omega_p)t\right]$$

Since $f_{c_1} = 1200\,KHz$, $f_{c_2} = 900\,KHz$, and $f_{IF} = 500\,KHz$, then the frequency translation is to be done using $\frac{A_c}{2} m(t) \cos(\omega_c - \omega_p)t$.

Hence, $f_{c_1} - f_{P_1} = f_{IF}$ and $f_{c_2} - f_{P_2} = f_{IF}$.

$$\Rightarrow f_{P_1} = f_{c_1} - f_{IF} = 1200 - 500 = 700\,KHz.$$

$$f_{P_2} = f_{c_2} - f_{IF} = 900 - 500 = 400\,KHz.$$

Problem # 9

$$S_{app}(t) = A_c \cos(\omega_c t) - \beta A_c \sin(\omega_m t) \sin(\omega_c t).$$

$$Env[S_{app}(t)] = \left[A_c^2 + \beta^2 A_c^2 \sin^2(\omega_m t)\right]^{1/2}$$

$$= A_c\left[1 + \beta^2 \sin^2(\omega_m t)\right]^{1/2}$$

$$Max\ Env[S_{app}(t)] = A_c\left[1 + \beta^2\right]^{1/2}$$

Max Deviation of $Env[S_{app}(t)]$ from A_c is:

$$A_c - A_c\left[1 + \beta^2\right]^{1/2} = A_c\left[1 - (1 + \beta^2)^{1/2}\right]$$

$$= A_c\left[1 - (1 + (0.1)^2)^{1/2}\right] = 0.005\,A_c$$

Problem #10

By Carson's rule:

$$B_T = 2(\Delta\omega + W)$$

$$\Rightarrow 200\,KHz = 2(\Delta\omega + 52)$$

$$\Rightarrow \Delta\omega = \frac{200 - 104}{2} = 48\,KHz.$$

But, $\Delta\omega = k_f\,|\max m(t)|$

$$\Rightarrow |\max(m(t))| = \frac{\Delta\omega}{k_f} = \frac{48\times10^3}{4\times10^3}$$

$$= 12 \text{ Volts.}$$