AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

QUIZ # 1
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TIME: 2 HOURS
CLOSED BOOK EXAM
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NAME:____________________ **ID #:** ________________

All problems are equally graded

QUESTION # 1

Consider an unfair die where the 6 faces, 1, 2, 3, …, 6, have the following probability values: 1/12, 1/6, 1/3, 1/3, 1/24, 1/24. Consider the random variable X(i)=10i, i=1, 2, 3, …, 6. X maps the probability space of the die rolling experiment to the real line (set of real numbers, $\Re$).

(a) Plot the probability distribution function $F_X(x)$ of the random variable X, for all $x \in \Re$, as well as $f_X(x)$.

(b) Consider the event $A = \{2,3\} \cup \{2,4,5\}$ and determine its probability value. Determine also this probability value using $F_X(x)$ and the integration of $f_X(x)$ over the appropriate limits, which you need to specify.

QUESTION # 2

Consider a stationary white and Gaussian noise process X(t) with zero mean and spectral height equal to $N_0/2$. Let X_1 and X_2 be given by:

$$X_1 = \int_0^T X(t)\phi_1(t)dt,\ X_2 = \int_0^T X(t)\phi_2(t)dt$$

where $\phi_1(t)$ and $\phi_2(t)$ are orthonormal over [0,T]. That is,

$$\int_0^T \phi_1(t)\phi_2(t)dt = 0 \text{ and } \int_0^T \phi_i^2(t)dt = 1,\ i = 1,2.$$

(a) Determine the mean, variance and covariance of X_1 and X_2.
(b) Determine the joint probability density function of X_1 and X_2.

QUESTION # 3

Consider pulse amplitude modulation (PAM) and let $H(\omega)$ be non-negligible only in the frequency range between 0 and the first zero crossing of $H(\omega)$ with the frequency axis. Let the sampling frequency be equal to 15 KHz. The analog message has a bandwidth equal to 5 KHz and it is assumed strictly band-limited.

(a) Let h(t) be of duration equal to 25 μsecs. and determine the transmission bandwidth needed for the PAM signal.
(b) What is the largest duration of h(t) that makes the transmission bandwidth of the PAM signal equal to 50 KHz?

QUESTION # 4

Consider the following sinusoidal message signal:

$$m(t) = A\sin(\omega_m t) = 2\sin(\pi \times 10^4 t).$$

Let m(t) be sampled at $f_s = 8 f_m$. Consider the use of PDM and PPM where: In PDM, the pulse width D is related to the sample value of m(t); i.e., m(kTs), by the following formula:

$$D = \frac{m(kT_s)}{24 \times 10^4} + 0.1 \times 10^{-4}\ secs.$$

In PPM, the pulse width is equal to 10^{-6} $secs.$

(a) Show the plots of the sampled m(t) and also of the PDM and PPM signals.

(b) Determine the bandwidth required for the transmission of the PDM and PPM signals. Use the first zero crossing for bandwidth definition.

QUESTION # 5

Consider uniform quantization applied to a specific type of an analog signal. Let Δ be the quantizer step-size. Upon observing a large number of sample values and the quantization levels to which they are mapped, it has been concluded that the quantization noise is appropriately represented by a probability density function $f_Q(q)$ having a triangular shape and symmetric about the vertical axis passing by q=0. The base of the triangular density is between $-\Delta/2$ and $\Delta/2$.

(a) Conclude based on the specified $f_Q(q)$ about the location of the majority of the sample values with respect to the quantization levels and the boundaries of the amplitude cells; i.e., the decision thresholds.
(b) Determine the mathematical expression of $f_Q(q)$ and use it to obtain the mean-square value of the quantization noise. Use the result you obtained to devise a way by which the quantization noise can be reduced. Also, comment on whether the principle of bandwidth-noise trade off still holds in the considered case.

QUESTION # 6

Consider non-uniform quantization consisting of increasing the density of the quantization levels at low amplitude values of the message signal and decreasing the density at higher amplitude values. This principle can be realized by passing the signal to a compressor and then using uniform quantization.

(a) We want the principle and its realization to be equivalent. Based on this requirement, provide an explanation and justification about which amplitude values of the message signal are to be compressed more and which need to be compressed less.
(b) Consider the $\mu-$law of compression and show its consistency with the explanation you provided in part (a) above.

QUESTION # 7

Consider binary PNRZ transmission technique and a bit rate of 64000 bits/sec. Let the relationship between the transmitted pulse energy and the received one be given by:

$$E_{br} = \frac{K}{d^2} E_{bt}$$

where $K = 9 \times 10^{-2} m^2$, and d is the distance (in meters) between the transmitter and the receiver or the first repeater. Let the noise corrupting the received signal be white with a PSD height $(N_0/2) = 10^{-12} V^2.sec$. Consider the transmitted pulse amplitude $A=8\ V$ and determine the largest d that results in an acceptable regeneration of the sequence of binary digits at the first repeater.

QUESTION # 8

Consider the following sinusoidal signal:

$$m(t) = 2cos(2\pi \times 10^3 t)\ Volts.$$

(a) Determine the smallest value of Δ needed to avoid slope overload distortion if:
(a.1) m(t) is sampled and then applied to a delta modulator.
(a.2) m(t) is integrated first, sampled and then applied to a delta modulator.

The sampling is done at 4 times the Nyquist rate.

(b) Compare the $\Delta's$ you obtained in (a.1) and (a.2) above and determine the signal (from among m(t) and its integrated version) which is less sloppy. Justify your answer based on the needed $\Delta's$.

QUESTION # 9

In a DPCM system, it is assumed that the dynamic range of the quantizer input has been reduced to 1/20 of the dynamic range of the message signal. Determine the relationship between the SNR's (in dB) in DPCM and PCM if both techniques use the same number of qunatization levels.