ECCE 442

AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

QUIZ # 1

FALL 2003-2004

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TIME: 1.5 HOURS

CLOSED BOOK EXAM

Two sheets of formulas only are permitted

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NAME:____________________ **ID #:** ________________

Problem #1

Consider the following finite duration sinusoidal pulse:

$$s(t) = A\cos(\omega_c t),\ 0 \le t \le T$$

Let s(t) be sampled at the Nyquist rate; i.e., fs=2B, where B Hz is the bandwidth of s(t). The first sample from s(t) is taken at t=0. The sampled signal is then quantized to 5 levels located at A, 0.5A, 0, -0.5A and -A. Determine the following:

(a) The sampling rate.
(b) The energy of the signal at the quantizer output.
(c) The quantizer output signal to noise ratio.

Assume that $\omega_c=\pi/T$ and that the quantization noise energy is approximated by the difference between the energy of s(t) and the quantized signal energy. Also, the bandwidth of a "sinc" function is determined by the first zero crossing of its spectrum with the frequency axis.

Problem # 2

In a binary digital communications system, the bit error probability at the receiver output (PCM decoder input) is equal to 10^{-6}. The bit duration is equal to 2μsec. Consider fixed length coding with the number of quantization levels being equal to 256.

(a) Determine the maximum number of codewords that could be decoded to the wrong quantization levels in a received binary sequence of duration 40 secs.
(b) Determine the minimum number of codewords that could be decoded to the wrong quantization levels in a received binary sequence of duration 40 secs.

Problem # 3

(a) Consider a TDM-DM system that transmits the multiplexed digital pulses over a base-band channel. Twenty messages are to be multiplexed with each sampled at 8 times the Nyquist rate. Each message has a bandwidth equal to 4 KHz. Assume that no synchronization pulse is used and determine the needed transmission bandwidth.
(b) Consider again Part (a) of this problem with the TDM-DM system replaced by a TDM-PCM system. Determine the message sampling rate that results in the transmission bandwidth being the same as the one in the TDM-DM system. The number of quantization levels in PCM is 256.

Problem # 4

Consider the transmission of a digital signal represented in the form of UNRZ (On-Off) line code. Let bit 1 be represented by a rectangular pulse. The amplitude of this pulse at the receiver is A volts. A zero level pulse represents the bit 0. The bit duration is T_b. Assume that the detection of the binary sequence in additive noise W(t) is done by sampling each received pulse (which could be 0 or 1) in noise at t= kT_b, k=1, 2, 3, ...and comparing the sample value to a threshold. Consider the detection of the nth bit as follows:

$$\text{If } r(nT_b) > A/2, \text{decide } 1$$

$$\text{If } r(nT_b) < A/2, \text{decide } 0,$$

with

$$r(nT_b) = \begin{cases} A + W(nT_b), \text{if bit } 1 \text{ is transmitted} \\ W(nT_b), \text{if bit } 0 \text{ is transmitted.} \end{cases}$$

Let W(t) be a zero mean Gaussian and stationary noise process with average power σ_W^2.

(a) Determine *Prob(decide 0 given 1 is transmitted)* and *Prob(decide 1 given 0 is transmitted)* in terms of their relationship to $W(nT_b)$ and its probability density function.

(b) Determine the expected number of 0's detected as 1's and the expected number of 1's detected as 0's in a binary sequence containing 1,000,000 bits and having 10^{-4} as a BER. The transmitted sequence is supposed to contain equal number of 1's and 0's.

Problem # 5

Assume that the received pulse energy in a digital communication link is related to the transmitted pulse energy by the following formula:

$$E_{br} = \frac{K}{d^2} E_{bt},$$

where d is the distance between the transmitter and receiver and K is a constant whose value is determined by the gain of the used antennas and by the used frequency. Let the bit error rate (BER) at the receiver be a function of the receiver signal-to-noise ratio represented by E_{br} / N_0, where N_0 represents the average power of the receiver noise. Let $K = 2.5 \times 10^{-4}\ m^2$, $N_0 = 10^{-10}\ V^2 s$, the bit rate used in the digital transmission, which is done according to PNRZ, be 64,000 bits/sec. and $E_{bt} = 20\ V^2 s$. Determine the minimum number of repeaters needed between the transmitter and receiver if the distance that separates them is equal to 20Km.

Problem #6

(a) A message signal that uses 16 quantization levels is compressed by some compression law to make its dynamic range reduced to half of its original one. The same step size and the same sampling rate are used for the compressed and original signal. Determine the ratio of the bit rate resulting from the use of the compressed signal over the bit rate resulting from the use of the uncompressed signal. In both cases, uniform quantization and fixed-length coding are used.

(b) Instead of using a compression law, consider the use of variable-length coding. That is, the probability of each of the 16 quantization levels is assessed by some means and then these probabilities are used to devise a code where the length of each codeword is related to the probability value of the corresponding level. Write down the average codeword length of the code and then determine the equation that needs to be satisfied in order to make the variable-length coding scheme equivalent to the compression scheme in Part (a).