Math 219 — Fall 2014
Quiz I, September 30. TIME: 70 minutes

GRADES (each problem is worth 12 points):

1	2	3	4	5	6	7	TOTAL/84

This exam booklet is also your answer sheet; **please answer question 0 on this page, and answer questions 1–7 inside the booklet.** The problems are repeated inside the booklet. There are extra blank sheets at the end for solutions. You can also use the back of any page for solutions or scratchwork. If you need to continue a problem beyond the first page where it appears, please INDICATE CLEARLY where the solution continues.

Read through the problems before starting, and decide which of them you wish to work on first. Do as much of the exam as you can, and budget your time wisely. Each of questions 1–7 counts for 12 points. The exam is closed book, and calculators are not allowed.

Make sure to communicate your ideas by explaining all of your steps precisely and clearly.

Good luck!

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0. a) Your name: b) Your AUB ID#:

1. (12 pts) Given the matrices $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ -2 & -2 \\ 0 & 0 \end{pmatrix}$. Find AB and BA.

2. (4 pts each part, total 12 pts) Given a linear transformation $T : V \to W$.
a) Carefully define $\operatorname{Ker} T$ and $\operatorname{Image} T$.
b) Show that $\operatorname{Ker} T$ is a subspace of V. (Don't just quote the theorem from class; prove it.)
c) Show that if $\operatorname{Ker} T = \{\vec{0}\}$, then T is injective. (Same comment as for part (b).)

3. (6 pts each part, total 12 pts) In $\mathcal{P}^3$, consider the subspace $W = \operatorname{Span}\{x^3 + 1,\ 4x^3 + x,\ x - 4\}$.
a) Is $3x^2 + x - 1 \in W$? Justify your answer.
b) Is $3x^3 + x - 1 \in W$? Justify your answer.

4. (12 pts) Let V be a vector space, and suppose $\vec{x}, \vec{y}, \vec{z} \in V$ satisfy the equation

$$2\vec{x} + 3\vec{y} + 4\vec{z} = \vec{0}.$$

Prove that $\operatorname{Span}\{\vec{x}, \vec{y}\} = \operatorname{Span}\{\vec{y}, \vec{z}\}$.

5. (6 pts each part, total 12 pts) Let $W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \in \mathbf{R}^5 \mid x_1 = x_2 + 2x_4 \text{ and } x_3 = x_5 \right\}$.
a) Show that W is a subspace of $\mathbf{R}^5$.
b) Find finitely many vectors such that W is the span of these vectors.

6. (6 pts each part, total 12 pts) Given a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ with the following properties:

$$T(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad T(\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad T(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

a) Show that T is surjective.
b) Find the matrix A_T of T.

7. (12 pts) Given two linear transformations $T : V \to W$ and $S : W \to Z$. Assume that T is a bijection. Show that $\operatorname{Image}(S \circ T) = \operatorname{Image} S$.

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