January 31, 2002	MATHEMATICS 219	NAME ------------------
Time : 2 Hours.	FINAL EXAMINATION	ID# -----------------
Prof. H. Abu-Khuzam	FALL 2001-2002	ver

PART I. Answer each of the following problems in the space provided for each problem(Problem 1 to Problem 6).

1. Let $T : P_2(R) \to R^3$ be the function defined by
   $T(a+bx+cx^2)=(a, a+b, 2a-b)$

   (a) Show that T is a linear transformation.

   [4 points]

   (b) Find Ker T. Is T an isomorphism?

   [4 points]

2. Let $A=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) Find the eigenvalues of A

[3 points]

(b) Find bases for the corresponding eigenspaces of A.

[6 points]

2. (c) Deduce that there is a basis of R^3 consisting of eigenvectors of A and find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP=D$. (Do not verify)

[3 points]

3. Let $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be the linear transformation defined by

$$T(x, y, z)= (2y+z, x-4y, 3x)$$

(a) Find the matrix A of T relative to the standard ordered basis $\beta = \{ (1,0,0), (0,1,0), (0,0,1)\}$ of $\mathbf{R}^3$.

[3 points]

3. (b) Find the matrix B of T relative to the ordered basis $ß'=\{(0,0,1), (1,1,0), (1,0,0)\}$ of $\mathbf{R}^3$.

[4 points]

3. (c) Find the change of basis matrix Q from ß′ to ß such that $B=Q^{-1}AQ$. (Do not verify)

[3 points]

4. (a) Show that an orthogonal set of nonzero vectors in an inner product space V is linearly independent.

[6 points]

(b) Let V be the subspace of R^3 with basis { (1,2,0), (0,1,1)}. Use the Gram-Scmidt process to find an orthonormal basis of V.

[4 points]

5. Let $U = \{ p(x) \in P_4(R) \mid p'(0) = 0, \text{ and } p'''(x) = 0 \}$.

(a) Find a basis for the subspace U of $P_4(R)$.

[4 points]

(b) Use linear extension to find a linear transformation $T : P_4(R) \to P_4(R)$ such that $T(p(x)) = p(x)$ for all $p(x) \in U$, and $T(x+1) = 0$.

[6 points]

6. (a) Let A and B be similar $n \times n$ matrices such that A is invertible. Prove that B must be invertible. [4 points]

(b) Suppose that λ is an eigenvalue of an $n \times n$ matrix A. Show that λ^2 is an eigenvalue of the matrix A^2.

[6 points]

PART II. Circle the correct answer for each of the following problems (Problem 7 to Problem 16). [4 points for each correct answer, 0 for no answer, and −1 for each wrong answer].

7. Consider the matrix

$$A=\begin{pmatrix} 0 & 0 & 0 \\ -5 & 3 & 0 \\ 3 & -4 & 1 \end{pmatrix}, \text{ then}$$

a. Rank A =3.
b. A is invertible.
c. A is diagonalizable.
d. −3 is an eigenvalue of A.

[4 points]

8. If A is a 3×3 matrix such that |A| =4. Then | A (adj A)| is equal to:

a. 4
b. 16
c. 64
d. none of the above

[4 points]

9. The value of the number c for which the matrix

$$A=\begin{pmatrix} 0 & c & 2 \\ 0 & 5 & -3 \\ 2 & -2 & 3 \end{pmatrix}.$$

is <u>not</u> invertible is:

a. c = 2
b. c = -10
c. c = -10/3
d. none of the above

[4 points]

10. Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be the linear transformation defined by

$$T(x,y,z)=(x+2y,\ y,\ x,\ x-y)$$

Then dim (Im T) is :

a. 3
b. 2
c. 4
d. none of the above.

[4 points]

11. Let S be the space of all skew-symmetric 4×4 matrices ($A^T = -A$). Then dim S is equal to:

a. 10
b. 12
c. 6
d. none of the above.

[4 points]

12. If T: V→ W is a linear transformation such that the image of every linearly independent subset of V is linearly independent in W, then which one of the following statements is true?

a. T is onto.
b. T is one -to -one.
c. T(**A**)=**0** for all vectors **A** in V.
d. none of the above.

[4 points]

13. The subspace of R^4 spanned by (1,-2,3,5), (0,3,1,-5), (1,1,4,0), and (0,0,0,1) has dimension:

a. 2
b. 3
c. 4
d. none of the above.

[4 points]

14. The value(s) of k for which the system

$$\begin{aligned} x+y+z&=k \\ x+2y+2z&=3 \\ y+kz&=4 \end{aligned}$$

has <u>no</u> solution are:

a. $k = 1$
b. $k = 0$
c. $k = \pm 4$
d. none of the above.

[4 points]

15. Let V and W be finite dimensional vector spaces, and $T: V \to W$ is a linear transformation. Then, which one of the following statements is true?

a. T is one-to-one $\Leftrightarrow$ T is onto.
b. dim (Ker T) + dim (Im T) = dim W
c. If dim V= dim W, then T is an isomorphism of V onto W.
d. none of the above.

[4 points]

16. Let A be an <u>invertible</u> $n \times n$ matrix over R. Which one of the following statements is <u>FALSE?</u>

a. Rank A= n.
b. A^t is invertible.
c. The number of nonzero rows in the row echelon form of A is n.
d. The rows of A are linearly dependent in R^n.

[4 points]