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Mathematics 219 Final Examination (Time: 2 hrs.)

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1. (40 %) Prove (concisely) or disprove (by a counter example) any 10 of the following 12 parts (in an inner product space V)

(a) $4(u,v) = \|u+v\|^2 - \|u-v\|^2$
(b) Non-zero orthogonal vectors are linearly independent.
(c) Two $m \times n$ matrices have the same null space iff they have the same row space.

(d) Similar $n \times n$ matrices have the same row space.
(e) Similar matrices have the same eigenvalues.
(f) If a set of 4 vectors (say in R^4) have the property that each 3 of them are linearly independent, then this set is linearly independent.

(g) $f(A \oplus B) = f(A) \oplus f(B)$ (for every linear transformation f & A and B are subspaces of V)
(h) For an $m \times n$ matrix A, A & $RRE(A)$ have the same null space & row space.
(i) For an $m \times n$ matrix A, A & $RRE(A)$ have the same column space.

(j) A 4×4 matrix with 4 distinct eigenvalues is diagonlizable.
(k) There exists a square matrix with $\lambda = 0$ as an eigenvalue .
(l) For any symmetric $n \times n$ matrix A, A and A^7 have the same null space. (Hint: Prove it)

2. (8 %) Find the least squares (*best possible*) solutions of the system $\{x+y+z=0 \ \& \ x+y+z=8\}$.

3. (8 %) (a) What do we know about arbitrary symmetric matrices regarding eigenvalues & diagonalization ?
(b) Apply the Cauchy-Schwarz inequality on C[a, b]= The inner product space of continous functions on the interval [a, b] (with the well-known way of "dotting" functions).

4. (7 %) Let T: V→W be a linear transformation of vector spaces. State the rank-nullity theorem for T. Then use it to show that if T is injective and dimV=dimW=n, then T must be onto.

5. (7 %) Let $T: V \to W$ be a given injective linear transformation of vector spaces (say dim V=4, dim W=6) Find a linear transformation $S: W \to V$ such that SoT=I (identity on V). (Hint: Apply the Linear Extension Theorem with good choices of bases)

6. (13 %) Let $A = \begin{bmatrix} 3 & 1 & 0 \\ 6 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(i) Find the eigen values of A and a basis for each eigen space of A.
(ii) Show that A is diagonalizable and find the exact relation between A , P and D. (Do **not** calculate P^{-1}).

7. (12 %) Let $A = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 2 & 1 & 4 & 11 \\ 0 & 3 & 6 & 3 \end{bmatrix}$ & $RRE(A) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(i) Find a basis for each of the row space, column space, and the null space of A. **Justify**
(ii) Does the system $AX = B$ have a solution for every B in R^3 ? **Explain.**

8. (5 %) Prove or disprve (by a counter example):
similar $n \times n$ matrices have the same row space & the same column space.