Math 219
Final Exam

January 23, 2004

Time: 90 minutes

All vector spaces are assumed to be finitely generated.

1. Let T : $U \to V$ and S : $V \to W$ be linear transformations of vector spaces. Prove that the composed map ST : $U \to W$ is a linear transformation.
2. Let T : $V \to W$ be a linear transformation of vector spaces. Prove that the map T is injective if and only if *ker* T = *0*.
3. Let T : $V \to W$ be an ***injective*** linear transformation of vector spaces and let $\{a_1, \ldots, a_n\}$, $n \geq 1$, be a linearly independent subset of V. Prove that $\{T(a_1), \ldots, T(a_n)\}$ is a linearly independent subset of W
4. Let T : $V \to V$ be an injective linear transformation from the vector space V to itself. Prove that T is an isomorphism. (Hint: use the result in problem 3 or recall the formula dim V = dim *ker* T + dim *im* T and exercise 2).
5. Let V_1 and V_2 be subspaces of a vector space V such that $V_1 + V_2 = V$ and $V_1 \cap V_2 = 0$. Prove that dim V = dim V_1 + dim V_2.
6. Let V be the vector space of all polynomials over **R** of degree $< n$, $n \geq 1$ and let T : $V \to V$ be the operation of forming derivatives. Write down the $n \times n$ matrix A of T relative to the ordered basis $\{1, x, \ldots, x^{n-1}\}$.