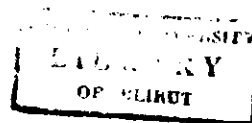


Math 219
Final Exam



January 23, 2004

Time: 90 minutes

All vector spaces are assumed to be finitely generated.

1. Let $T : U \rightarrow V$ and $S : V \rightarrow W$ be linear transformations of vector spaces. Prove that the composed map $ST : U \rightarrow W$ is a linear transformation.
2. Let $T : V \rightarrow W$ be a linear transformation of vector spaces. Prove that the map T is injective if and only if $\ker T = 0$.
3. Let $T : V \rightarrow W$ be an *injective* linear transformation of vector spaces and let $\{a_1, \dots, a_n\}$, $n \geq 1$, be a linearly independent subset of V . Prove that $\{T(a_1), \dots, T(a_n)\}$ is a linearly independent subset of W .
4. Let $T : V \rightarrow V$ be an injective linear transformation from the vector space V to itself. Prove that T is an isomorphism. (Hint: use the result in problem 3 or recall the formula $\dim V = \dim \ker T + \dim \operatorname{im} T$ and exercise 2).
5. Let V_1 and V_2 be subspaces of a vector space V such that $V_1 + V_2 = V$ and $V_1 \cap V_2 = 0$. Prove that $\dim V = \dim V_1 + \dim V_2$.
6. Let V be the vector space of all polynomials over \mathbf{R} of degree $< n$, $n \geq 1$ and let $T : V \rightarrow V$ be the operation of forming derivatives. Write down the $n \times n$ matrix A of T relative to the ordered basis $\{1, x, \dots, x^{n-1}\}$.