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Spring 2000-01	MATHEMATICS 219	NAME ------------------
Time : 2 Hours.	FINAL EXAMINATION	ID# ----------------
Prof. H. Abu-Khuzam		JUNE 20 ,2001

1. Let $T: P_2(R) \to P_3(R)$ be the linear transformation defined by

$$T(p(x)) = x^2 p'(x) + 3p(x)$$

a. Find the matrix of T relative to the standard ordered bases of $P_2(R)$ and $P_3(R)$.

[6 points]

b. Find a basis for Ker T. Find dim (Im T).

[6 points]

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2. Let $A=\begin{pmatrix} 5 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

a. Find the eigenvalues and a basis for each eigenspace of A.

[8 points]

b. Deduce that there is a basis of R^3 consisting of eigenvectors of A and find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP=D$. (Do not verify)

[4 points]

2c. Use part (b) above to find an orthonormal basis of R^3 consisting of eigenvectors of A.

[3 points]

3. Let A and B be n×n matrices such that AB is invertible. Prove that A must be invertible. (Hint: Use Determinants)

[7 points]

4. Show that an orthogonal set of nonzero vectors in an inner product space V is linearly independent.

[8 points]

5. Let T: V→V be a linear operator on a finite dimensional vector space V. Show that

$$(\text{Ker } T) \cap (\text{Im } T) = \{0\} \Rightarrow V = (\text{Ker } T) + (\text{Im } T)$$

[7 points]

6. Use the row echelon form of the augmented matrix of the following system

$$x + 2y + z = 3$$
$$x + 3y - z = 4$$
$$x + 2y + (a^2 - 8)z = a.$$

to find the value(s) of a for which the system has

a. no solution

b a unique solution

c. infinitely many solutions.

[10 points]

7. Let T: $V \to W$ be a linear transformation. Show that If $A_1, A_2, \ldots, A_n$ are vectors in V, such that $T(A_1), T(A_2), \ldots, T(A_n)$ are linearly independent in W, then $A_1, A_2, \ldots, A_n$ are linearly independent in V.

[8 points]

8. Let V be a vector space of dimension 3, and let U be a subspace of V of dimension 2. Show that there exists a linear transformation T: $V \to V$ such that $T(A)=0$ for all $A \in U$, and T is <u>not</u> the zero map on V.

[6 points].

9. Answer **TRUE or FALSE only:** **[3 points for each correct answer , 0 for no answer, and –1 for each wrong answer].**

a. ----- Let A be a 4×4 matrix such that (rank A)=3, then $|A| = 0$

b. ----- The subspace of $\mathbf{R}^3$ spanned by the vectors (1,0 ,-1), (2,1, -3), and (5,2,-7) has dimension 3.

c. ----- Similar matrices have the same deteminant.

d. ----- Any set of 2 vectors in $\mathbf{R}^3$ can be extended to a basis of $\mathbf{R}^3$.

e. ----- If N is a square matrix such that $N^2=\mathbf{0}$, then I-N is invertible

f. ----- The space of all symmetric 3×3 matrices has dimension 5.

g. ----- If A, B, and C are n×n matrices such that AB=AC, then B=C.

h. ----- Let W be a subspace of a vector space V, then the set $U= \{A \in V : A \notin W\}$ is a subspace V.

i ----- Let A be an n×n matrix such that $AA^T=I$, where I is the identity n×n matrix , then $|A| = 1$.

[27 points]