Problem I :

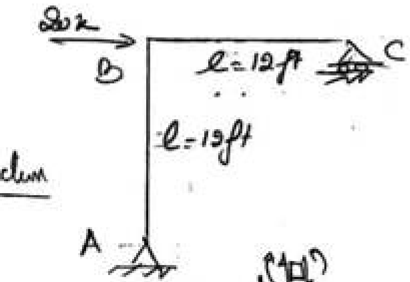
Frames

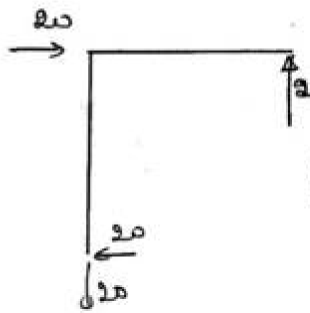
δ_{max} BC = ?

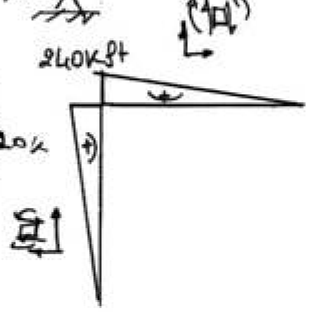
$E = 30 \times 10^3$ ksi

$I = 1000 \text{ in}^4$

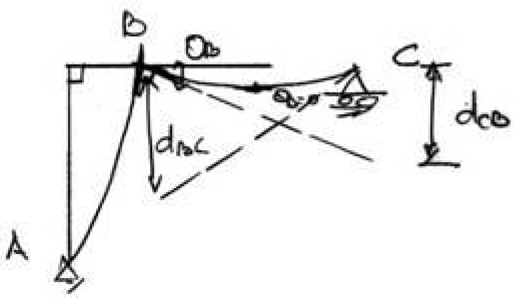
Neglect axial deflection







δ_{max} (BC)



$d_{CB} = \frac{1}{2} \times \frac{240}{EI} \times 12 \times \frac{2}{3} \times 12 = \frac{11520}{EI}$ (Point above tangent)

$\theta_B = \frac{d_{CB}}{12} = \frac{960}{EI}$ (↻)

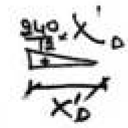
2nd (better)

$d_{BC} = \frac{1}{2} \times \frac{240}{EI} \times 12 \times \frac{1}{3} \times 12 = \frac{5760}{EI}$

$\theta_C = \frac{d_{BC}}{12} = \frac{480}{EI}$ (↺)

Check $\theta_C - \theta_B = \frac{1}{2} \times \frac{240}{EI} \times 12 = \frac{1440}{EI}$

Max @ D : X'_D from C :



$\theta_C - \theta_D = 0 \Rightarrow \frac{1}{2} \times \frac{240}{12} X'_D \times X'_D = +\frac{480}{EI}$

$\Rightarrow X'_D = 6.93$ ft

$\delta_D = |\Delta_D| - |d_{DC}| = \frac{480 \times 6.9}{EI} - \frac{1}{2}\frac{240}{12} \times \frac{6.93 \times 6.93}{EI} \times \frac{1}{3} \times 6.93 = 2209.6/EI$

$\delta_{max} = \frac{2209.6}{EI}$

$EI = 3 \times 10^4 \text{ ksi}$
$= 208333 \text{ k ft}^2$

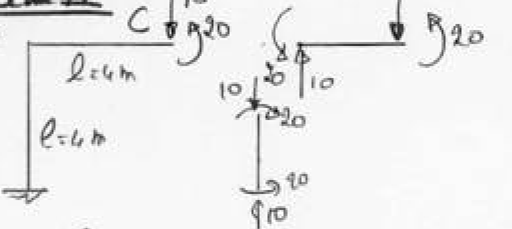
$\delta_{max} = 0.01$ [illegible] in.

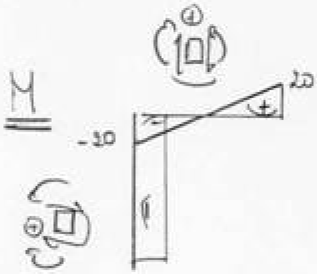
Problem II

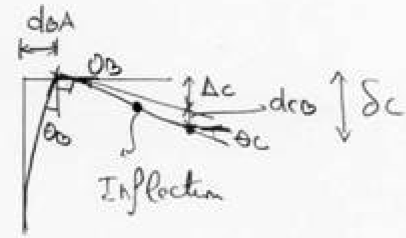
Frames

$EI = 104166.7\ kNm^2$

$\delta_C = ?$







$d_{BA} = -\frac{20}{EI} \times 4 \times \frac{4}{2} = -1.536 \times 10^{-3}\ m \quad \Rightarrow |\delta_B| = 1.536 \times 10^{-3}\ m \rightarrow$

$\theta_B - \cancel{\theta_A}^{\,0} = -\frac{1}{EI} \times 20 \times 4 = -7.68 \times 10^{-4}\ rad$

$\theta_C - \theta_B = 0 \Rightarrow \theta_B = \theta_C = -7.68 \times 10^{-4}\ rad$

$d_{CB} = +\frac{1}{EI}\left[-\frac{1}{2}(20) \times (2) \times \left(2 + \frac{2}{3} \times 2\right) + \frac{1}{2}(20)(2)\left(\frac{1}{3} \times 2\right)\right]$

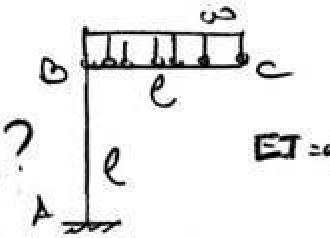
$= (-)5.12 \times 10^{-4}\ m$ OK

$|\Delta_C| = |\theta_B| \times 4 = 3.072 \times 10^{-3}\ m$

$|\delta_C| = |\Delta_C| + |d_{CB}| = 3.584 \times 10^{-3}$ (down)

Problem III

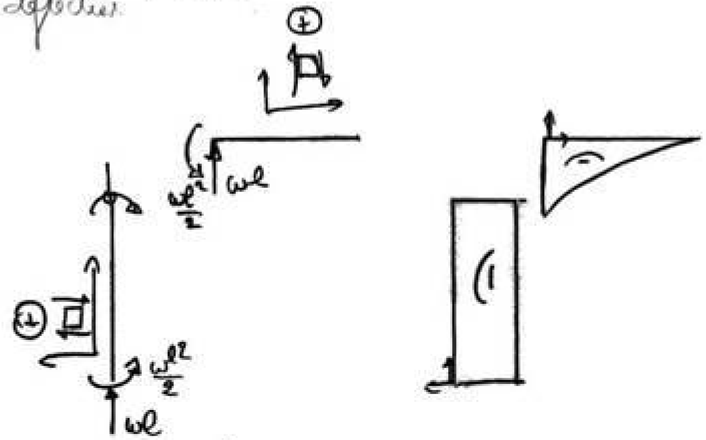
Frames

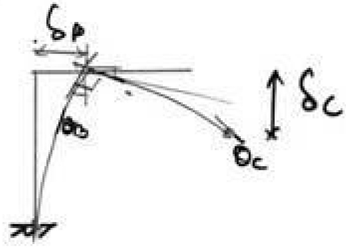


$\delta_C, \theta_C, \delta_B$?

EI = cst

Neglect axial deflection





Same

$(\rightarrow)$

$\theta_B - \cancel{\theta_A} = -\frac{w\ell^2}{2EI}\ell = -\frac{w\ell^3}{2EI} \Rightarrow \theta_B = \frac{w\ell^3}{2EI}$

$\theta_C = \frac{2}{3}\frac{w\ell^3}{EI}$

$\theta_C - \theta_B = -\frac{1}{3}\frac{w\ell^2}{2EI}\ell = -\frac{w\ell^3}{6EI} \Rightarrow \theta_C = -\frac{w\ell^3}{6EI} - \frac{w\ell^3}{2EI} = -\frac{2}{3}$

$\delta_C = \frac{5}{8}\frac{w\ell^4}{EI}$

$\Rightarrow \theta_C = \frac{2}{3}\frac{w\ell^3}{EI}$

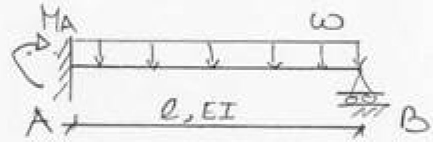
$\Delta_C = |\theta_B| \times \ell = \frac{w\ell^4}{2EI}$

$d_{CB} = -\frac{1}{3}\frac{w\ell^2}{2EI} \times \ell \times \frac{3}{4}\ell = -\frac{w\ell^4}{8EI}$ (below)

$\delta_C = |\Delta_C| + |d_{CB}| = \frac{5w\ell^4}{8EI} \downarrow$

Statically Indeterminate Beam

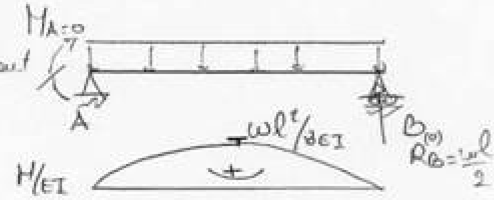
Problem I



Solve for Reactions for θ_B

Solve Using Primary

Primary: (0) M_A is redundant



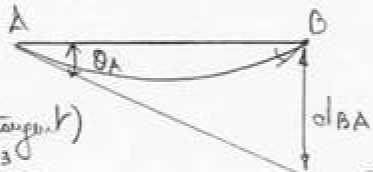
$\Theta_A^{(0)} = ?$

$\theta_A = \frac{d_{BA}}{\ell}$

where $d_{BA} = \frac{2}{3} \cdot \frac{\omega\ell^2}{8EI} \cdot \ell \cdot \frac{\ell}{2}$

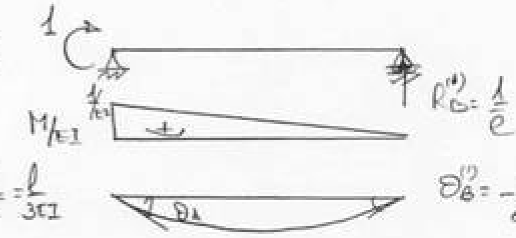
$\Rightarrow d_{BA} = + \frac{\omega\ell^4}{24EI}$ (point above tangent)

$\Rightarrow \theta_A = \frac{\omega\ell^3}{24EI} \Rightarrow \Theta_A^{(0)} = + \frac{\omega\ell^3}{24EI}$



$\theta_B^{(0)} = - \frac{\omega\ell^3}{24EI}$

Secondary: (1) $M_A = 1$



$\Theta_A^{(1)} = ?$

$\theta_A = \frac{d_{BA}}{\ell} = \frac{1 \cdot \ell}{2EI} \cdot \frac{2}{3}\frac{\ell}{\ell} = \frac{\ell}{3EI}$

$\theta_B^{(1)} = - \frac{\ell}{6EI}$

$\Theta_A^{(1)} = + \frac{\ell}{3EI}$

Superposition: $\theta_A = \theta_A^{(0)} + M_A\, \Theta_A^{(1)} \Rightarrow M_A = - \frac{\Theta_A^{(0)}}{\Theta_A^{(1)}} = - \frac{\omega\ell^2}{8}$

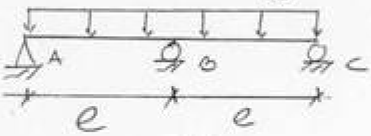
$R_B = R_B^{(0)} + M_A \cdot R_B^{(1)} = \frac{\omega\ell}{2} + \left(-\frac{\omega\ell^2}{8}\right) \cdot \frac{1}{\ell} = \frac{3}{8}\omega\ell \uparrow$

$R_A = \frac{\omega\ell}{2} + \left(-\frac{\omega\ell^2}{8}\right) \cdot \left(-\frac{1}{\ell}\right) = \frac{5}{8}\omega\ell \uparrow$

$\theta_B = -\frac{\omega\ell^3}{24EI} + \left(-\frac{\omega\ell^2}{8}\right)\left(-\frac{\ell}{6EI}\right) = - \frac{\omega\ell^3}{48EI} \Rightarrow \theta_B = \frac{\omega\ell^3}{48EI}$

Statically Indeterminate Beam

Problem II



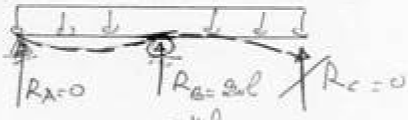
$EI = cst$

Solve for Reactions.

SOLUTIONS: Use Primary Structure (a) $R_C = 0$ or (b) $R_B = 0$

(a) Primary: R_C redundant

$\delta_C^{(0)} = ?$



Shear

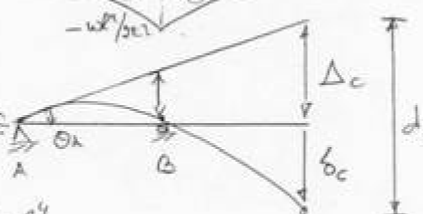
M/EI

$\theta_A = \dfrac{|d_{BA}|}{l}$

$d_{BA} = -\dfrac{\omega l^2}{2EI} \cdot \dfrac{l}{3} \cdot \dfrac{l}{4}$

$d_{BA} = -\dfrac{\omega l^4}{24EI}$ (below tangent)

Deflection



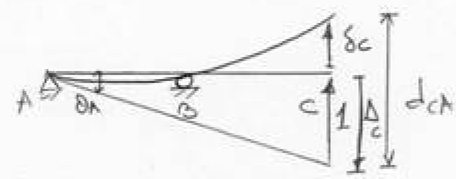
$\theta_A = \dfrac{\omega l^3}{24EI} \Rightarrow \Delta_C = \dfrac{\omega l^3}{24EI} \times 2l = \dfrac{\omega l^4}{12EI}$

$d_{CA} = -\dfrac{\omega l^2}{2EI} \times \dfrac{1}{3} \times 2l \times l = -\dfrac{\omega l^4}{3EI}$ (below tangent)

$|\delta_C| = |d_{CA}| - |\Delta_C| = \dfrac{\omega l^4}{4EI} \downarrow$

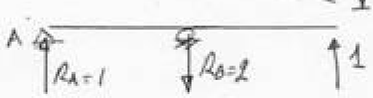
$\Rightarrow \delta_C^{(0)} = -\dfrac{\omega l^4}{4EI} \uparrow$

Secondary: $R_C = 1 \uparrow$



$\theta_A = \dfrac{|d_{BA}|}{l}$

$d_{BA} = \dfrac{l}{EI} \times \dfrac{l}{2} \times \dfrac{l}{3} = \dfrac{l^3}{6EI}$ (above tangent)



$\theta_A = \dfrac{l^2}{6EI} \Rightarrow \Delta_C = \dfrac{l^2}{6EI} \times 2l = \dfrac{l^3}{3EI}$

$d_{CA} = \dfrac{l}{EI} \times \dfrac{2l}{2} \times l = \dfrac{l^3}{EI}$ (above tangent)

M/EI

$|\delta_C| = |d_{CA}| - |\Delta_C| = \dfrac{2}{3}\dfrac{l^3}{EI} \uparrow$

$\Rightarrow \delta_C^{(1)} = +\dfrac{2l^3}{3EI}$

Indeterminate

Superposition $\delta_c = \delta_c^{(0)} + R_c\, \delta_c^{(1)} = 0$

$$R_c = \frac{-w\ell^4/4EI}{2\ell^3/3EI} = +\frac{3}{8} w\ell = 0.375 w\ell \uparrow$$

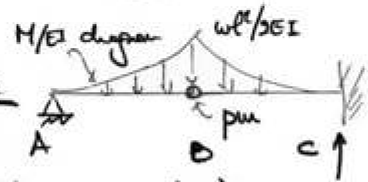
$R_A = 0 + (0.375w\ell) \times 1 = 0.375 w\ell \uparrow$ (sym +

$R_B = 2w\ell + (0.375w\ell) \cdot (-2) = .25 w\ell \uparrow$

Note: $\delta_c^{(0)}$ + $\delta_c^{(1)}$ have been obtained by the moment area theorems

Conjugate beam (optional)

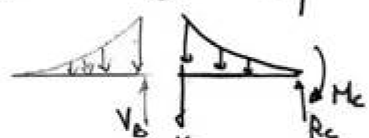
Primary (0) Conjugate



$\delta_c \equiv M_c$

$\Sigma M_A = 0 \Rightarrow V_B \cdot \ell = \frac{w\ell^2}{2EI} \cdot \frac{\ell}{3} \times \frac{3\ell}{4}$

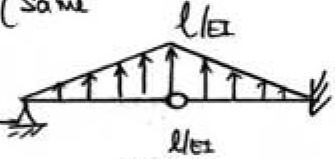
$\Rightarrow V_B = \frac{w\ell^3}{8EI}$



$\Rightarrow M_c = \frac{w\ell^4}{4EI}$ $\ominus$ $\Rightarrow$ δ_c is down $\downarrow$

$\Rightarrow \delta_c^{(0)} = -\frac{w\ell^4}{4EI}$ (Same

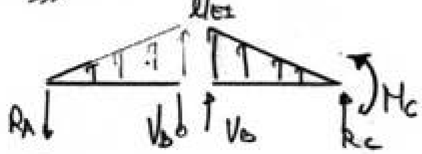
Secondary (0) Conjugate



$\delta_c \equiv M_c$

$\Sigma M_A = 0 \Rightarrow V_B \ell = \frac{\ell}{EI} \frac{\ell}{2} \times \frac{2\ell}{3}$

$\Rightarrow V_B = \frac{\ell^2}{3EI}$



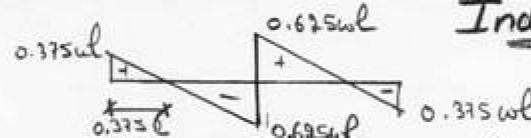
$\Sigma M_c = 0 \Rightarrow M_c = V_B \ell + \frac{\ell}{EI} \cdot \frac{\ell}{2} \times \frac{2}{3}\ell$

$\Rightarrow M_c = \frac{2}{3}\frac{\ell^3}{EI}$ (+) $\Rightarrow \delta_c$ is up $\uparrow$

$\Rightarrow \delta_c^{(1)} = \frac{2}{3}\frac{\ell^3}{EI}$ (Same)

Indeterminate

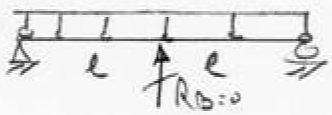
V diagram:



M diagram:

(b) Primary structure

Primary structure:
R_B redundant

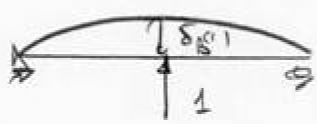


Moment ...

Compute $\delta_B^{(0)}$ .....

$$\delta_B^{(0)} = \frac{5}{384}\left(\frac{w(2l)^4}{EI}\right) = -\frac{5}{24}\frac{wl^4}{EI} \qquad \text{(down)}$$

Secondary:



Moment ..

Compute $\delta_B^{(1)}$

$$\delta_B^{(1)} = \frac{1 \times (2l)^3}{48EI} = \frac{l^3}{6EI} \qquad \text{(up)}$$

$$\delta_B = \delta_B^{(0)} + R_B\,\delta_B^{(1)} = 0 \quad \Rightarrow \quad R_B = -\frac{\delta_B^{(0)}}{\delta_B^{(1)}}$$

$\Rightarrow R_B = 1.125wl$ (↑)

Proceed ... Same V+M.

Deflected shape: