Economics 227: Intermediate Macroeconomics
Assignment 2 Answers

1. In the long-run model, suppose that a stock market crash reduces consumers' wealth. Use graphs to explain the effect on investment and the interest rate.

   The stock market crash will reduce autonomous consumption, thereby increasing national savings. The interest rate will fall and level of investment will rise. (In a graph of the market for loanable funds, the supply of loanable funds shifts to the right.)

2. Suppose that production in an economy is given by the Cobb-Douglas function:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$$

   (a) Suppose the real wage is 1 unit in 2011, 2 units in 2012, and 2.4 units in 2013. If labor earns $\frac{1}{2}$ of total income, then what's average labor productivity?

$$\frac{W}{P} = (1-\alpha) AK^{\alpha}L^{-\alpha} = (1-\alpha)\frac{Y}{L}$$

   So average labor productivity is twice the real wage in all three years.

   (b) Is it possible for this growth in the real wage to coincide with a constant capital-labor ratio? Why or why not?

   From part (a) above, we have $\frac{W}{P} = (1-\alpha) A \left(\frac{K}{L}\right)^{\alpha}$. So if the real wage grows while $\frac{K}{L}$ is constant, the level of technology (indexed by A) must be rising.

   (c) Now suppose the real wage is some number $\frac{W}{P}$. If a computer virus destroys 15% of Lebanon's capital stock (and doesn't change the number of workers), what is the percentage change in $\frac{W}{P}$?

   Let $\frac{W}{P}$ and $\left(\frac{W}{P}\right)'$ denote the real wage before and after the computer virus destroys 15% of Lebanon's capital stock. We have

$$\frac{\left(\frac{W}{P}\right)'}{\frac{W}{P}} = 0.85^{\alpha} = \sqrt{0.85} = 0.922$$

   So the real wage falls about 7.8%.

   (d) If this computer virus destroys 15% of Lebanon's capital stock and doesn't change the number of workers, what is the percentage change in the real interest rate / rental return on capital r?

   If r and r' denote the real interest rate before and after the computer virus destroys 15% of Lebanon's capital stock, then since $r = \alpha A \left(\frac{L}{K}\right)^{1-\alpha}$ and $r' = \alpha A \left(\frac{L}{0.85K}\right)^{1-\alpha}$, we have

$$\frac{r'}{r} = \left(\frac{1}{0.85}\right)^{1-\alpha} = 1.085$$

   So the interest rate rises 8.5%.

(e) Now suppose a new virus destroys a further 15% of Lebanon's capital. Will the magnitude of the change in the real wage be larger, smaller, or the same as it was in part (d)?

The real wage will rise another 9.22%: the magnitude of the change in the real wage will be the same as it was before. To verify this, note that if $\left(\frac{W}{P}\right)''$ is the real wage after this second virus hits, then $\left(\frac{W}{P}\right)'' = (1-\alpha)\,A\left(\frac{0.85^2 K}{L}\right)^{\alpha}$.

3. Suppose production in Lebanon is given by the Cobb-Douglas function $Y = AK^{\alpha}L^{1-\alpha}$ We estimate Lebanon's GDP is \$40 billion, that there are 4 million workers, and total wages are about 64% of GDP. Finally, the interest rate / rental return on capital is $r = 10\%$. Then what's the estimated value of Lebanon's capital stock?

Euler's Theorem gives $Y - wL - rK = 0$. If total wages are 64% of GDP, then $Y - wL = 0.36Y = \$14.4B$. So therefore

$$K = \frac{\$14.4B}{0.1} = \$144B$$

4. Consider the CES production function

$$Y = A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}}$$

(a) Show that the CES production function is CRS.

Given $F(K,L) = A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}}$

$$F(zK, zL) = A\left[\alpha (zK)^{\xi} + \beta (zL)^{\xi}\right]^{\frac{1}{\xi}} = zF(K,L)$$

(b) If production is CES, what are the real wage and rental return on capital?

$$\begin{aligned}\frac{W}{P} &= A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}-1}\beta L^{\xi-1}\\ \frac{R}{P} &= A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}-1}\alpha K^{\xi-1}\end{aligned}$$

(c) Show that Euler's Theorem holds.

Euler's Theorem says that there are no profits if production is CRS. To verify Euler's Theorem, we need to show that

$$F(K,L) = \frac{W}{P}L + \frac{R}{P}K$$

We have

$$\begin{aligned}\frac{W}{P}L + \frac{R}{P}K &= A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}-1}\beta L^{\xi} + A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}-1}\alpha K^{\xi}\\ &= A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}-1}\left(\beta L^{\xi} + \alpha K^{\xi}\right)\\ &= A\left[\alpha K^{\xi} + \beta L^{\xi}\right]^{\frac{1}{\xi}} = F(K,L)\end{aligned}$$

So Euler's Theorem holds.