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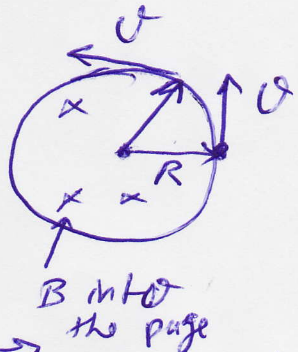
Hwork 3 / solutions

3.1 electron of kinetic energy $K_e = 45\,keV$ in circular orbit, under a magnetic force. Magnetic field is $B = 0.325\,T$

(a) Using Newton's second law

$$m\frac{v^2}{R} = e v B$$

$\Rightarrow$ Radius: $R = \frac{m}{eB} = 2.2\,nm$



$\vec{F}_m = q\,\vec{v} \times \vec{B}$

$\vec{v}$ 90° to $\vec{B}$

$\Rightarrow \sin 90^\circ = 1$

(b) Period

$$T = \frac{2\pi R}{v}$$

$$\frac{1}{2}mv^2 = K_e \Rightarrow v = \sqrt{\frac{2K_e}{m}}$$

or $$T = \frac{2\pi R}{\sqrt{2K_e/m}} = 1.1 \times 10^{-10}\,s$$

3.2

$$I(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{(e^{\frac{hc}{\lambda kT}} - 1)}$$

We want to have:

$$I_\lambda\, d\lambda = I(f)\, df$$

[Mapping of the wavelength into frequency]

or $$I(f) = I(\lambda)\left|\frac{d\lambda}{df}\right|$$

using $c = \lambda f$

$$\Rightarrow \left|\frac{d\lambda}{df}\right| = \frac{c}{f^2}$$

Then

$$I(f) = \frac{2hf^3}{c^2}\frac{1}{(e^{\frac{hf}{kT}} - 1)}$$

Figures

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3.3 we use Wien's law :

$$\lambda_{max} \cdot T = 0.0029 \text{ m} \cdot \text{k}$$

$$\Rightarrow \quad \lambda_{max} = \frac{2.9 \times 10^{-3} \text{ m} \cdot \text{k}}{2.7 \text{ K}} = 1.07 \text{ mm}$$

(b) $c = \lambda_{max} f \Rightarrow f = \frac{c}{\lambda_{max}} = 2.8 \times 10^{11} \text{ Hz}$

3.4 using $\bar{E} = \mathcal{E} f(\mathcal{E}) = \frac{\mathcal{E}}{(e^{\mathcal{E}/kT} - 1)}$

And $\mathcal{E} = \frac{hc}{\lambda}$

we get $\bar{E} = \frac{hc/\lambda}{(e^{hc/\lambda kT} - 1)} = 0.95(kT)$

(b) $\bar{E} = 4.59 \times 10^{-4} (kT)$

Short wavelength far from (kT), but the long wavelength is very close.
kT is what is called equipartition thermal energy

3.5

$$\lambda_{max} \cdot T_1 = 2.9 \times 10^{-3} \text{ m} \cdot \text{k} \Rightarrow T_1 = 107 \text{k}$$

$$I_1 = \sigma T_1^4 \qquad I_2 = \sigma T_2^4 = 2 I_1 = 2 \sigma T_1^4$$

$$\Rightarrow T_2 = 2^{1/4} T_1 = 128 \text{k}$$

(b) $\lambda_{max} = 2.3 \times 10^{-5} \text{m}$

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3.6

$$I(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{(e^{\alpha/\lambda}-1)}, \quad \alpha \equiv \frac{hc}{kT}$$

$$\frac{dI}{d\lambda} = \frac{-10hc^2}{\lambda^6}\frac{1}{(e^{\alpha/\lambda}-1)} - \frac{2hc^2}{\lambda^5}\frac{1}{(e^{\alpha/\lambda}-1)^2}\left(-\frac{\alpha}{\lambda^2}\right)$$

put $\frac{dI}{d\lambda}=0$ necessary condition for maximum.

Then
$$5(e^{\frac{\alpha}{\lambda}}-1) = \frac{\alpha}{\lambda}e^{\frac{\alpha}{\lambda}}$$

Define $x \equiv \frac{\alpha}{\lambda}$, then
$$5e^x - 5 = xe^x$$

Or $\boxed{x = 5 - 5e^{-x}}$ this is an implicit equation: x on both sides

This is solved by iteration
Let us write $g(x) = 5 - 5e^{-x}$
Then $x = g(x)$

	x	$g(x)$
guess	2.0	4.323323
new x	4.323323	4.986744
	4.986744	4.965861
	4.965861	4.965140
	4.965140	4.965115

You see it converging!

A better way is to use Newton Raphson iteration
This works like this:
Define: $f(x) = 5e^{-x} + x - 5$
expand $f(x)$ in Taylor series
$f(x) \approx 1 + f'(x_0)(x - x_0) + \cdots$
This is a linearization
We want $f(x) = 0 \Rightarrow$

$$\boxed{x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}}$$ "Newton-Raphson recipe"

Apply this to our case: $x_n = x_{n-1} - \frac{(5e^{-x} + x - 5)}{5e^{-x}(-1) + 1}$

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$\boxed{3.6}$ (continuation)

Let us start with the same guess, $x_0 = 2.0$

n	x_{n-1}	$f(x_{n-1})$	$f'(x_{n-1})$	x_n
1	$x_0 = 2.0$	-2.3233	0.323	7.19288
2	7.19288	2.19664	0.99624	4.98798
3	4.98798	0.022077	0.965903	4.96512

we reach the same value after 3 iterations instead of 5

since $x = \frac{hc}{\lambda kT} \Rightarrow \lambda_{max} \cdot T = 2.9 \times 10^{-3}\ m \cdot k$

$\boxed{3.7}$ Blue Supergiant

$T_{surface} = 30000\ k$, Luminosity $L = 10^5 L_{sun}$

$P_{sun} = 3.86 \times 10^{26}\ W$ (Power of the Sun)

(a) $\lambda_{max} \cdot T = 2.9 \times 10^{-3}$

$\Rightarrow \lambda_{max} = 97\ nm$ Ultraviolet

(b) $P = \sigma A T^4$ $\qquad \sigma = 5.67 \times 10^{-8} \frac{W}{m^2 k^4}$

$A = 4\pi R^2$

$\Rightarrow R = 8.2 \times 10^9\ m \approx 12\ R_{sun}$

(c) NO: since a large amount of light is not emitted in the visible range

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3.8

$$I(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{\left(e^{\frac{hc}{\lambda kT}}-1\right)}$$

using result of problem 3.2

$$\int_0^\infty I(\lambda)\,d\lambda = \int_\infty^0 I(f)\,df\left(-\frac{c}{f^2}\right)$$

$$= \int_0^\infty \frac{2\pi h f^3}{c^2\left(e^{hf/kT}-1\right)}\,df$$

$$= \frac{2\pi (kT)^4}{c^2 h^3}\int_0^\infty \frac{x^3}{(e^x-1)}\,dx$$

$(x \equiv hf/kT)$

$$= \frac{2\pi\,(kT)^4}{c^2h^3}\,\frac{1}{240}(2\pi)^4 = \frac{2\pi^5k^4T^4}{15c^2h^3}$$

$$= \sigma T^4$$

3.9

(a) $\lambda_{th} = \frac{hc}{\Phi}$ from $hf = \Phi + K_{max}$ and $K_{max} = 0$

$\lambda_{th} = 653\ nm$

frequency $f_{th} = \frac{c}{\lambda_{th}} = \frac{\Phi}{h} = 4.59 \times 10^{4}\ Hz$

(b) $V_0 = \frac{1}{e}\left(\frac{hc}{\lambda} - \Phi\right) = 2.23\,eV$

(C) $1.20\,V$

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3.10

(a) $\lambda = 550\,nm$

$$E = Nhf, \quad f = \frac{c}{\lambda}$$

power $$P = \frac{E}{t} = \frac{Nhf}{t} \Rightarrow N = \frac{(0.05)(100\,W)(550\times10^{-9}\,m)}{(6.63\times10^{-34}\,J\cdot s)(3\times10^{8}\,m/s)}$$

$$N = 1.38\times10^{19}\ 1/s$$

(b) $$Flux = \frac{1.38\times10^{19}/s}{4\pi(2m)^3} = 2.57\times10^{17}\ \frac{m^2}{s}$$

3.11

$$eV_0 = hf - \phi = \frac{hc}{\lambda} - \phi$$

$$\left.\begin{aligned} 0.95 &= \frac{h}{e}\frac{c}{\lambda_1} - \frac{\phi}{e} \\ 0.38 &= \frac{h}{e}\frac{c}{\lambda_2} - \frac{\phi}{e} \end{aligned}\right\} \Rightarrow \frac{\phi}{e} = 1.87\,eV$$

Solve & find $f = 4.57\times10^{14}\,Hz$ from $\frac{hf}{e} = \frac{\phi}{e}$

3.12

(a) $$\phi = \frac{hc}{\lambda} = 1.90\,eV$$

(b) $$K = \frac{hc}{\lambda} - \phi = 2.23\,eV$$

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$\boxed{3.13}$ (a)

Threshold frequency $f_{th} = 1.25 \times 10^{15}$ Hz

$\Rightarrow \quad \phi = h f_{th} = 4.8\,eV$

(b) $eV_0 + \phi = hf \Rightarrow V_0 = \frac{hf}{e} - \frac{\phi}{e}$

slopes of V_0 : $3.8 \times 10^{-15} = \frac{h}{e} \Rightarrow h = 6.1 \times 10^{-34}$ J.s

(c) No photoelectrons emitted

(d) For a differ.t metal, f_{th} and ϕ will be different. The slope will be the same. The graph will be shifted to the right depending on f_{th}.

$\boxed{3.14}$

$\lambda = 652\,nm$, $\Delta t = 20\,ms$, $P = 0.60\,W$

(a) $P = \frac{\text{Energy}}{\text{time}} \Rightarrow \text{energy} = P \cdot t$

$= 7.5 \times 10^{16}\,eV = 1.2 \times 10^{-2}\,J$

(b) $E = \frac{hc}{\lambda} = 3.05 \times 10^{-19}\,J = 1.91\,eV$

(c) $N = \frac{1.2 \times 10^{-2}\,J}{3.05 \times 10^{-19}\,J} = 3.93 \times 10^{16}$ photons

$\boxed{3.15}$

$\lambda_{th} = 272\,nm$

$f = 1.45 \times 10^{15}$ Hz

$hf = \phi + K_{max} \Rightarrow K_{max} = hf - \phi$

But $\phi = \frac{hc}{\lambda_{th}} = 1.1 \times 10^{15}$ Hz

$\Rightarrow \quad K_{max} = 1.45\,eV$

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3.16

$$P = \frac{h}{\lambda} = \frac{hc}{\lambda c}$$

(a) $\lambda = 400\ nm \Rightarrow P = 1.66 \times 10^{-27}\ \frac{kg.m}{s}$

(b) $\lambda = 0.1\ nm \Rightarrow P = 6.63 \times 10^{-24}\ kg.m/s$

3.17

$\Theta = 110^0$

$$\lambda - \lambda_0 = \frac{h}{mc}(1 - \cos\Theta) = 3.26 \times 10^{-12}\ m$$

$$\lambda_0 = \frac{hc}{E_1} = 2.43 \times 10^{-12}\ m \quad , \quad E_1 = 0.511\ MeV$$

$$\lambda = \lambda_0 + 3.26 \times 10^{-12}\ m = 5.69 \times 10^{-12}\ m$$

Energy of scattered photon

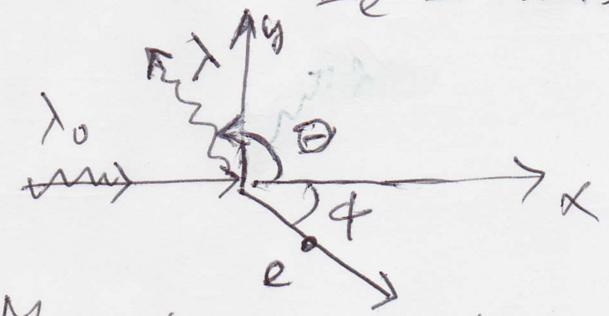
$$E = \frac{hc}{\lambda} = \frac{1240\ eV.nm}{5.69 \times 10^{-3}\ nm} = 0.218\ MeV$$

Energy of recoiled electron:

$$E_e = \frac{hc}{\lambda_0} - \frac{hc}{\lambda} = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)$$

or $$E_e = (1240)\,eV.nm\left(\frac{1}{2.43\,10^{-3}} - \frac{1}{5.69 \times 10^{-3}}\right)\frac{1}{nm}$$

or $$E_e = 0.293\ MeV$$



Momentum conservation

$$E_0 = \frac{hc}{\lambda_0} = P_0 c \Rightarrow P_0 = \frac{h}{\lambda_0}$$

$$\frac{h}{\lambda}\cos 20^0 - P_e \sin\phi = 0 \qquad y\text{-direction}$$

or $$\frac{h}{\lambda}\cos 20^0 = P_e \sin\phi = \sqrt{E_e^2 - (m_e c^2)^2}\ \sin\phi$$

$$\Rightarrow \sin\phi = \frac{h \cos 20^0}{\lambda \sqrt{E_e^2 - (m_e c^2)^2}} = \frac{(6.63 \times 10^{-34}\ J.s)\ 0.94}{5.69 \times 10^{-12}\ m \sqrt{(0.293 + 0.511)^2 - (0.511)^2}\ *}$$

$*\ 1.69 \times 10^{-13}\ J$ to change from MeV $\rightarrow$ J

$$\sin\phi = 0.330 \Rightarrow \boxed{\phi = 19.3^0}$$

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3.18

x-rays produced at 18 kV
They are Compton-scattered thru an angle of 45°
(a) What are the original & scattered wavelengths?
+(b)

$$\lambda - \lambda_0 = \frac{h}{mc}(1-\cos\theta)$$

$$\Rightarrow \lambda = \lambda_0 + \frac{h}{mc}(1-\cos\theta) \quad (1)$$

And: $eV = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{eV} = 0.0691\ \text{nm}$ original

Then $\lambda \underset{(1)}{=} 0.0698\ \text{nm}$ (scattered)

(c) $E = \frac{hc}{\lambda} = 17.8\ \text{keV}$

3.19

$$eV_0 = hf - \phi = \frac{hc}{\lambda} - \phi$$

$$e(0.52\,V) = \frac{hc}{450\,nm} - \phi \quad (1)$$

$$e(1.90\,V) = \frac{hc}{300\,nm} - \phi \quad (2)$$

solve to get $\frac{\phi}{e} = 2.24\ eV$ from eather one

(b) $\frac{hc}{e(300\,nm)} = 1.90 + 2.24 \Rightarrow h = 6.63\times10^{-34}\ J\cdot s$

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$\boxed{3.20}$

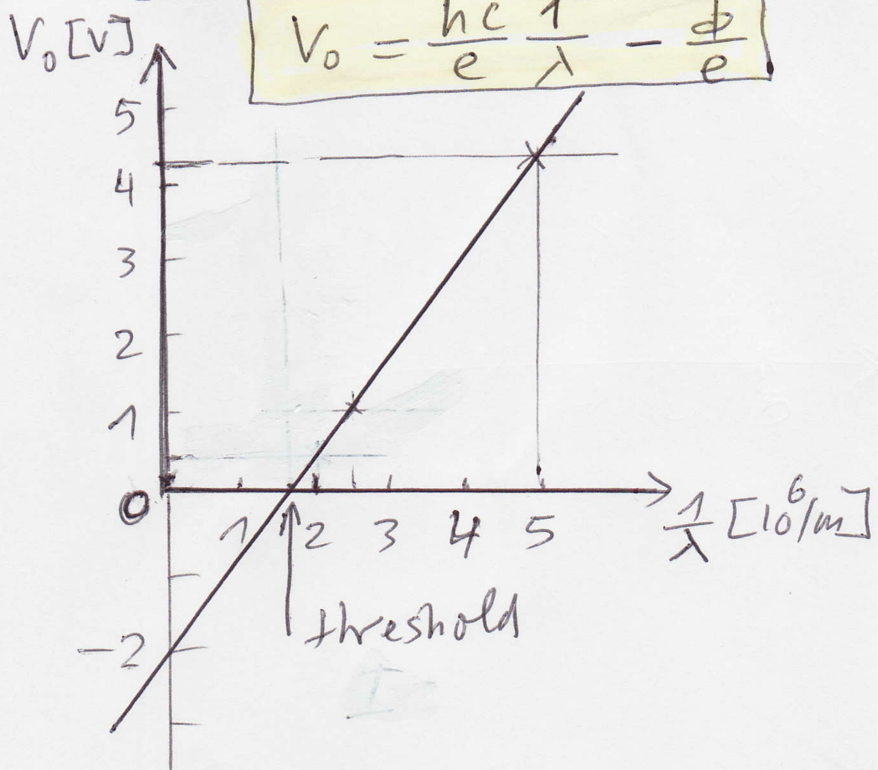
Table

V_0 [V]	4.20	2.06	1.05	0.41	0.03
$\frac{1}{\lambda}$ [10^6/m]	5.0	3.3	2.5	2.0	1.7

Stopping potential (see also problem 3.13)

$$\boxed{V_0 = \frac{hc}{e}\frac{1}{\lambda} - \frac{\phi}{e}}$$

we expect a straight line in a graph V_0 versus $\frac{1}{\lambda}$



Intercept of the $\frac{1}{\lambda}$ - axis at $\frac{1}{\lambda_{th}} = 1.65 \times 10^6 \frac{1}{m}$

Then $f_{th} = \frac{c}{\lambda_{th}} = 4.95 \times 10^{14}$ Hz

(c) Slope $= \frac{h}{e}$

You get $\frac{h}{e} = 4.19 \times 10^{-15}$ eV / Hz