Final Exam **Phys. 212 (Intro. Mod. Physics)** **24 June, 1996**

Part A [10 marks per question]

1)- Label as true or false these statements involving the atomic quantum numbers:

(a) One of these subshells cannot exist: $n = 2, l = 0$; $n = 4, l = 3$; $n = 1, l = 1$.
(b) The number of values of m_l that are allowed depends on n and on l.
(c) The smallest value of n that can go with a given l is $l + 1$.
(d) the quantum number s can take the values of +/- 1/2

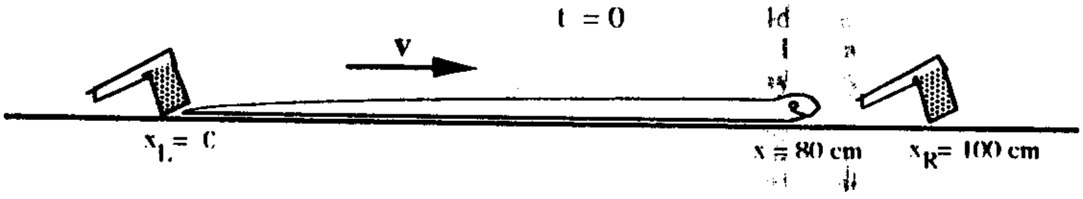
2)- Angular momentum is a vector and you might expect that it would take three quantum numbers to describe it, corresponding to the three space components of a vector. Instead, only two quantum numbers characterize the angular momentum in an atom. Explain why.

3)- Is there a change in the total energy of a hydrogen atom with n = 1, when it is placed in a magnetic field B? Justify your answer thoroughly.

4)-. Suppose that the distance from a given star to the earth is about 400 light-years and that the lifetime of a person is 70 years; at what speed must he travel to reach the star while he is still alive? (Your answer should be calculated to at least 3 decimal points.)

Part B [20 marks per problem]

1)- A relativistic snake of proper length 100 cm is moving at speed v = 0.6c to the right across a table. A mischievious boy, wishing to tease the snake, holds two hatchets 100 cm apart and plans to bounce them simultaneously on the table so that the left hatchet lands immediately behind the snake's tail. The boy argues as follows: "The snake is moving with $\beta = 0.6$. Therefore its length is contracted by a factor of $\gamma = 5/4$, and its length (as measured in my rest frame) is 80 cm. This implies that the right hatchet will fall 20 cm in front of the snake, and the snake will be unharmed." (The boy's view of the experiment is shown below.) On the other hand, the snake argues thus: "The hatchets are approaching me with $\beta = 0.6$, and the distance between them is contracted to 80 cm. Since I am 100 cm long, I shall be cut in pieces when they fall." Use the Lorentz transformation to resolve this paradox.



[As seen in the boy's frame]

2)- An electron and a proton, each with a kinetic energy $E_k = 6$ MeV, move perpendicular to a uniform magnetic field of 1 tesla.
(a) Find the ratio of the radius of curvature of the electron to that of the proton.
(b) Assume that the kinetic energy of the proton is 5 GeV. At what kinetic energy will the electron acquire the same momuntum as that of the proton.

3)- A beam of <u>thermal neutrons</u> is incident on a crystal in which the spacing between Bragg planes is $d = 0.304$ nm. An intense first-order Bragg diffraction is observed for a scattering angle θ.
(a) Determine the wavelength of the thermal neutrons
(b) Find the scattering angle θ.
(Hint: Assume that the neutrons behave like the molecules of an ideal gas at Room-Temperature)

4)- A muon (a negatively charged particle with a mass $m_\mu = 206\ m_e$) is captured by a lead nucleus ($Z_{Pb} = 82$). The resulting system will behave like a hydrogen atom.
(a) Find the "Bohr radius" for this system.
(b) Detrmine the transition energy when the muon descends from the $n = 2$ to the $n = 1$ level in this muonic (hydrogen-like) lead ion.

5)- (I) For the hydrogen atom in the ground state, $\psi_{1s} = [2/\sqrt{(4\pi a_o^3)}]\, e^{-r/a_o}$.
Evaluate the probability of finding the electron within a sphere of radius a_o = Bohr radius.
(II) A one-dimensional harmonic oscillator wave function is: $\psi = Ax\ exp[-bx^2]$
a) Show that ψ satisfies the Shrodinger equation.
b) Find the constant b and the energy E.
c) Is this a ground state or a first exited state?

6)- (a) Find the wavelength of the light from a sodium atom emitted in the $3p \rightarrow 3s$ transition, knowing that the first exited state of the sodium atom is the $3p$ level and $\Delta E = 2.1$ eV.
(b) Because of the spin-orbit interaction, the $3p$ level is actually two levels, 2.1×10^{-3} eV apart. Determine the difference $\Delta\lambda$ between the two wavelengths produced by $3p \rightarrow 3s$ transitions in Na.
(c) Represent the $3p \rightarrow 3s$ transitions in an energy-level diagram, and write down the quantum states for n=3 (using the spin-orbit coupling scheme) for the sodium atom.

7)- The measured values for the wavelength of the three emission lines in a normal Zeeman effect are 4226.90 A°, 4226.73 A° and 4226.56 A°.
(a) Find the strength of the magnetic field B?
(b) What is the energy difference $\Delta E = Ef - Ei$ for the states involved in the transition, when B is switched off?.
(c) Is there any information that can be inferred, from the above data, about the orbital quantum numbers of the initial and the final states?

SELECTED CONVERSION FACTORS

1 sec = 1.667×10^{-2} min = 2.778×10^{-4} h = 3.169×10^{-8} yr
1 m = 39.4 in. = 3.28 ft
1 Å (angstrom) = 10^{-10} m = 10^{-4} μ (micron)
1 AU (astronomical unit) = 1.496×10^{11} m
1 light year = 9.499×10^{15} m
1 kg = 2.205 lb
1 amu (atomic mass unit) = 1.6604×10^{-27} kg = 931.48 MeV
1 J = 0.239 cal = 6.242×10^{18} eV
1 cal = 4.186 J = 2.613×10^{19} eV
1 eV (electron volt) = 1.6022×10^{-19} J
1 N = 0.225 lbf

MOST COMMON REST MASSES AND ENERGIES

NAME	SYMBOL	REST MASSES (kg)	REST MASSES (amu)	REST ENERGIES (J)	REST ENERGIES (MeV)
atomic mass unit	amu	1.6604×10^{-27}	1.000000	1.4922×10^{-10}	931.48
proton	m_p	1.6725×10^{-27}	1.007287	1.5031×10^{-10}	938.26
neutron	m_n	1.6748×10^{-27}	1.008665	1.5052×10^{-10}	939.55
electron	m_e	9.1091×10^{-31}	0.000549	0.8186×10^{-13}	0.5110
hydrogen atom	m_H	1.6734×10^{-27}	1.007829	1.5038×10^{-10}	938.72

PHYSICAL CONSTANTS

CONSTANT	SYMBOL	VALUE AND UNITS
Avogadro's number	N_A	6.0222×10^{26} (kg-mole)$^{-1}$
speed of light (vacuum)	c	2.9979×10^{8} m/sec
universal gas constant	R	8.3143 J/mole-°K
Boltzmann's constant	$k = R/N_A$	1.3806×10^{-23} J/°K
mechanical equivalent of heat	J	4.1855 J/cal
vacuum permittivity	ϵ_0	8.8541×10^{-12} F/m
vacuum permeability	μ_0	$4\pi \times 10^{-7}$ H/m
gravitational constant	G	6.673×10^{-11} N-m^2/kg^2
Planck's constant	h	6.6262×10^{-34} J-sec
Dirac's constant	$\hbar = h/2\pi$	1.0546×10^{-34} J-sec
electronic charge	e	-1.6022×10^{-19} C
electron Compton wavelength	λ_0	2.4262×10^{-12} m
gyromagnetic ratio	$e/2m_e$	8.7940×10^{10} C/kg
electron Bohr magneton	$\mu_B = e\hbar/2m_e$	9.273×10^{-24} J/T
proton Bohr magneton	$e\hbar/2m_p$	5.050×10^{-27} J/T
nuclear magneton	μ_N	5.090×10^{-27} J/T
proton magnetic moment	μ_{pm}	$2.7928\mu_N$
neutron magnetic moment	μ_{nm}	$-1.1913\mu_N$
Rydberg constant	R_∞	1.0974×10^{7} m^{-1}
Bohr radius	a_0	5.2917×10^{-11} m

TRIGONOMETRIC IDENTITIES

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\sin\theta/\cos\theta = \tan\theta$

$\sin^2\theta + \cos^2\theta = 1 \quad \sec^2\theta - \tan^2\theta = 1 \quad \csc^2\theta - \cot^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$

BINOMIAL THEOREM

$(1 \pm x)^n = 1 \pm \dfrac{nx}{1!} + \dfrac{n(n-1)x^2}{2!} + \cdots \quad (x^2 < 1)$

$(1 \pm x)^{-n} = 1 \mp \dfrac{nx}{1!} + \dfrac{n(n+1)x^2}{2!} + \cdots \quad (x^2 < 1)$

EXPONENTIAL EXPANSION

$e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$

LOGARITHMIC EXPANSION

$\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots \quad (|x| < 1)$

TRIGONOMETRIC EXPANSIONS (θ in radians)

$\sin\theta = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} - \cdots$

$\cos\theta = 1 - \dfrac{\theta^2}{2!} + \dfrac{\theta^4}{4!} - \cdots$

$\tan\theta = \theta + \dfrac{\theta^3}{3} + \dfrac{2\theta^5}{15} + \cdots$

DERIVATIVES AND INTEGRALS

In what follows, the letters *u* and *v* stand for any functions of *x*, and *a* and *m* are constants. To each of the indefinite integrals should be added an arbitrary constant of integration. The *Handbook of Chemistry and Physics* (CRC Press Inc.) gives a more extensive tabulation.

1. $\dfrac{dx}{dx} = 1$
2. $\dfrac{d}{dx}(au) = a\dfrac{du}{dx}$
3. $\dfrac{d}{dx}(u + v) = \dfrac{du}{dx} + \dfrac{dv}{dx}$
4. $\dfrac{d}{dx}x^m = mx^{m-1}$
5. $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$
6. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
7. $\dfrac{d}{dx}e^x = e^x$
8. $\dfrac{d}{dx}\sin x = \cos x$
9. $\dfrac{d}{dx}\cos x = -\sin x$
10. $\dfrac{d}{dx}\tan x = \sec^2 x$
11. $\dfrac{d}{dx}\cot x = -\csc^2 x$
12. $\dfrac{d}{dx}\sec x = \tan x\sec x$
13. $\dfrac{d}{dx}\csc x = -\cot x\csc x$
14. $\dfrac{d}{dx}e^u = e^u\dfrac{du}{dx}$
15. $\dfrac{d}{dx}\sin u = \cos u\dfrac{du}{dx}$
16. $\dfrac{d}{dx}\cos u = -\sin u\dfrac{du}{dx}$

1. $\int dx = x$
2. $\int au\,dx = a\int u\,dx$
3. $\int (u + v)\,dx = \int u\,dx + \int v\,dx$
4. $\int x^m\,dx = \dfrac{x^{m+1}}{m+1} \quad (m \neq -1)$
5. $\int \dfrac{dx}{x} = \ln|x|$
6. $\int u\dfrac{dv}{dx}\,dx = uv - \int v\dfrac{du}{dx}\,dx$
7. $\int e^x\,dx = e^x$
8. $\int \sin x\,dx = -\cos x$
9. $\int \cos x\,dx = \sin x$
10. $\int \tan x\,dx = \ln|\sec x|$
11. $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$
12. $\int e^{-ax}\,dx = -\dfrac{1}{a}e^{-ax}$
13. $\int xe^{-ax}\,dx = -\dfrac{1}{a^2}(ax + 1)e^{-ax}$
14. $\int x^2e^{-ax}\,dx = -\dfrac{1}{a^3}(a^2x^2 + 2ax + 2)e^{-ax}$
15. $\int_0^\infty x^ne^{-ax}\,dx = \dfrac{n!}{a^{n+1}}$
16. $\int_0^\infty x^{2n}e^{-ax^2}\,dx = \dfrac{1\cdot3\cdot5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\dfrac{\pi}{a}}$