MPS 257 Fall 2016-17 — Quiz 1 - Solution

Problem 1 (15 Points)

Let $\Sigma = \{0,1\}$

a. (10) Let $L = \{1, 01, 100\}$. Specify, by circling the appropriate answer, whether each of the following is true or false. Then, in each case justify your answer.

i. $110001110011 \in L^*$. (True) False

$110001110011 = 1 \circ 100 \circ 01 \circ 1 \circ 100 \circ 1 \circ 1$

So it is the concatenation of 7 strings all coming from L. So it is in L^*

ii. $11000011001 \in L^*$. True (False)

There is no way to get 4 successive 0's by concatenating strings of L. The max is 3 successive 0's by concatenating 100 with 01…

b. (5) Let L and N be the two finite languages $L = \{\varepsilon, 0, 10\}$ and $N = \{1, 01, 001, 101, 1001\}$

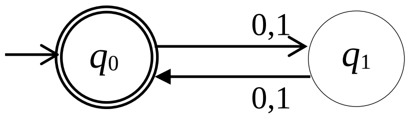
Find a language M such that $L \circ M = N$.

$M = \{1, 01\}$

Problem 2 (20 Points)

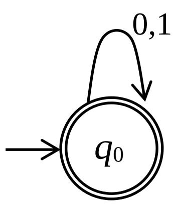
Let $\Sigma = \{0,1\}$, and consider the language $A = \{w \in \Sigma^* : w \text{ is of even length}\}$

a. (8) Give a simple DFA with as small a number of states as possible that recognizes A.

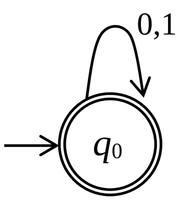


b. (5) Argue that it is impossible to have a DFA that recognizes A, that will have a smaller number of states than the DFA you have proposed in (a).

If there is a DFA that has a smaller number of states, then it must have one state. But there are only 2 DFA's that have one state:



and



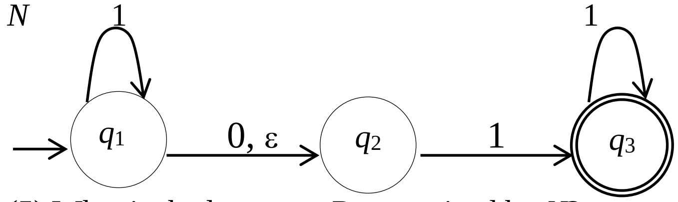
which recognize Σ^* and ϕ respectively, and not A.

c. (7) Give a simple regular expression for A.

$(\Sigma\Sigma)^*$ or $((0 \cup 1)(0 \cup 1))^*$

Problem 3 (20 Points)

Let $\Sigma = \{0,1\}$, and consider the following NFA N:



a. (5) What is the language B recognized by N?

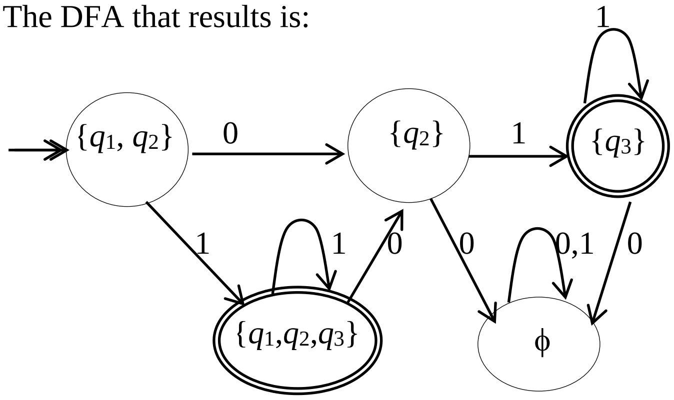
$B = \{w \in \Sigma^* \mid w$ has at most one 0 and ends with a 1$\}$

b. (5) Give a simple regular expression for the language B of N.

$1^* (0 \cup \varepsilon) 1^*\ 1$

c. (10) Give the construction of the DFA that is equivalent to the NFA N above, as in Theorem 1.39. You can include only the states reachable from the start state.

The DFA that results is:



Problem 4 (20 Points)

Let $\Sigma = \{0,1\}$, and consider the language $D = \{uu^R \mid u \in \Sigma^*\}$.

a. (10) Prove that D is not regular using the Pumping Lemma.

Assume that D is regular, and let $p>0$ be the pumping length. Let $s = 0^p 110^p$.

Then $s \in D$, and $|s| = 2p+2 > p$. So by the Pumping Lemma, s can be subdivided into three pieces , $s = xyz$ such that

1. $xy^i z \in D$, for all $i = 0, 1, 2, \ldots$.
2. $y \neq \varepsilon$
3. $|xy| \leq p$

By 3, it follows that $y = 0^k$ where $0 \leq k < p$.

By 2, $k > 0$. So $0 < k < p$.

In 1, let $i = 2$: So $xyyz = 0^{p+k}110^p$, which is not in D, since $k>0$. Contradiction.

So, our assumption that D is regular is false, and hence D is not regular.

b. (10) Using (b) and closure properties of regular languages, show that $PAL=\{w = w^R \mid w \in \Sigma^* \}$ the set of palindromes is not regular. (Hint: Find an appropriate language X such that $D = PAL \cap X$.)

Let $X = A$, the set of even length strings of Problem 1 above. A is regular.

Observe that $D = PAL \cap A$. (Palindromes and of even length)

Assume PAL is regular. A is regular, then by closure properties their intersection will be regular. But their intersection is D which is not regular as shown in part (a) above. Contradiction. So our assumption is false, and hence PAL is not regular.

Problem 5 (15 Points)

Let $\Sigma = \{0,1\}$, and consider the following language:

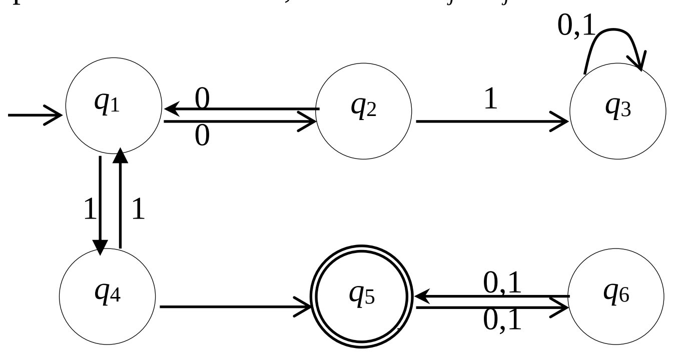
$A = \{w \in \Sigma^* \mid w = a_1b_1a_2b_2 \ldots a_nb_n,\ n > 0$, and as binary numbers $a_1a_2 \ldots a_n > b_1b_2 \ldots b_n\}$

E.g. $001011 \in A$, since 011 > 001, and $0110 \notin A$, since 01 is not > 10.

Show that A is regular by giving the state diagram of a *deterministic finite automata (DFA),* with as small a number of states as possible, that recognizes A. Give a brief explanation in English of your DFA (What is the main idea).

(Hint: For two binary numbers of the same length, how do you check which is the larger of the two?)

$a_1a_2 \ldots a_n > b_1b_2 \ldots b_n$ iff $a_j = 1$, and $b_j = 0$, where j is the smallest integer where $a_j \neq b_j$. i.e. it is the first position from the left, where the $a_j \neq b_j$



q_5 and q_6 : to make sure the accepted string is of even length.

Problem 6 (10 Points)

Let $\Sigma = \{0,1, \equiv, \times\}$, and consider

$L = \{ u \equiv 10 \times v \mid u, v \in \{0,1\}^*$, and u is twice v as binary numbers $\}$

In fact, $u = v0$. So $1010 \equiv 10 \times 101 \in L$ since 1010 is twice 101 (in decimal 10 is twice 5); $1010 \equiv 10 \times 1$ is not in L since 1010 is not twice 1.

Show that L is not regular using the pumping lemma by completing the following, (and choosing an appropriate s.)

Assume, that L is regular and let $p > 0$ be the pumping length.

Let s = $1^p0 \equiv 10 \times 1^p$. Then $s \in L$ and $|s| = 2p+5 > p$. So by the Pumping Lemma, we should be able to find x, y, z such that $s = xyz$, and

1. $xy^i z \in L$, for all $i = 0, 1, 2, \ldots$.
2. $y \neq \varepsilon$
3. $|xy| \leq p$

By 3, it follows that $y = 1^k$ where $0 \leq k < p$.
By 2, $k > 0$. So $0 < k < p$.
In 1, let $i = 2$: So $xyyz = 1^{p+k}0 \equiv 10 \times 1^p$, which is not in L, since $u = 1^{p+k}0$, $k > 0$, and $v = 1^p$, and u is not twice v. Contradiction.

So, our assumption that L is regular is false, and hence L is not regular.