CMPS 257

Time: 2 Hrs . **Final Exam** **Fall 05-06**

Family Name	
First Name	
Id. Num	
Section	**1** **2** **(9:30 TR)** **(11:00 TR)**

PART/Problem	Grade	Your Grade
I	20	
II	30	
III. 1	10	
III. 2	20	
III. 3	10	
III. 4	10	
III. 5	20	
III. 6	15	
III. 7	15	
Total	**150**	

Instructions.
Write your names, Id number, and circle your section in the space provided above.

THIS IS A CLOSED BOOK, CLOSED NOTES

Your answers must be presented on the question sheet itself. Use the provided space for your answers. If the space is not enough, then most probably you are writing more than

answer is written or continued (e.g.
scratch work, you may use the back of the sheet, and the last blank page. **DO NOT CUT OFF ANY PAGE !!**

There are 13 pages altogether. Try to solve as much as you can in the given time.

Part I. (20 Points) True or False questions. Circle T or F [Not Both]
You might want to comment briefly, if you are not sure about your answer. But in case your comment is not relevant, and you have the right answer, you might loose the points for that part
.

1. $(0 \cup 1)^* 0 (0 \cup 1)^* = 1^* 0 (0 \cup 1)^*$	T	F
2. If a language is regular, then it is also Turing-recognizable	T	F
3. Any subset of a Turing recognizable language is Turing recognizable	T	F
4. The complement of any Turing-recognizable language is Turing-recognizable	T	F
5. The union of a regular language and a context free language can never be regular	T	F
6. Let G be any context free grammar. Then, every string generated by G has exactly one left most derivation	T	F
7. A Turing machine can be simulated by a push down automaton	T	F
8. A non-deterministic finite automaton can be simulated by a deterministic finite automaton	T	F
9. If a regular expression does not involve the * operator, then the language it represents must be finite.	T	F
10. If $HALT_{TM}$ is decidable then A_{TM} will be decidable	T	F

Part II. (30 Points) Multiple choice problems, 10 of them

The following multiple-choice problems, are intended to have **only one** correct answer. If you discover otherwise, do indicate that in writing and circle all what you think is a correct answer.

1. Which of the following best expresses the pumping theorem for context free languages ?

a. A language L is context free if there is a pumping length p such that for any string $w \in L$ of length greater than or equal to p, w can be subdivided as $w = uvxyz$ in such a way that either v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

b. Suppose L is a CFL. Then for some $p > 0$, the pumping length for L , any string $w \in L$ of length greater than or equal to p, every subdivision of $w = uvxyz$, with v or y nonempty, then $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

c. Suppose L is a CFL. Then for some $p > 0$, the pumping length for L , any string $w \in L$ of length greater than or equal to p, there is a subdivision of $w = uvxyz$, such that v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

d. Suppose L is a CFL. Then for every number $p > 0$, and for any string $w \in L$ of length greater than or equal to p, there is a subdivision of $w = uvxyz$, such that v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

2. The language $\{w : w$ in $\{0,1\}^*$, w has an equal number of occurrences of 01 and 10$\}$ is:

a. regular

b. equal to $\{(01)^n(10)^n \mid n \geq 0\}$

c. finite

d. context free but not regular

3. The complement of $E_{DFA} = \{\langle M\rangle \mid M$ is a DFA, and L(M) is empty$\}$ is

a. $\{\langle M\rangle \mid M$ is a DFA, and L(M) is not empty $\}$

b. $\{\langle M\rangle \mid M$ is a DFA, and M loops on *some input* $\}$

c. $\{\langle M\rangle \mid M$ is a DFA, and L(M) $= \Sigma^*\}$

d. $\{u \mid u$ is not a valid coding of a DFA$\} \cup \{\langle M\rangle \mid$ is a DFA, and L(M) is not empty$\}$

4. An *NP* problem typically is a problem whose solution is

a. difficult to find and difficult to verify

b. difficult to define

c. difficult to find but easy to verify

d. easy to prove and easy to verify

5. The Turing-Church thesis

a. says that all problems can be solved by algorithms

b. presents the Turing machine as a formal definition for the intuitive notion of an algorithm

c.

d.

6. By an unsolvable problem (or undecidable problem) is meant

a. the problem does not have a mathematical solution

b. there is no algorithm to solve the problem

c. there is an algorithm to solve the problem but the algorithm is not efficient

d. The problem has no logical solution

7. The halting problem indicates that

a. every Turing machine must halt
b. there cannot be a general purpose algorithm for discovering infinite loops in programs
c. Turing decidable languages are also Turing recognizable
d. a Turing machine might loop on its input

8. We have shown that a nondeterministic Turing machine, N, can be simulated by a standard Turing machine M. The runtime of M is exponential in terms of the runtime of N, basically because

a. M uses a breadth first search instead of depth first search
b. M might loop on the input while N will not
c. the number of nodes in the tree representing the computation of N on a string w is exponential in terms of the height of the tree
d. the height of the tree representing the computation of N on a string w is exponential in terms of the number of leaves

9. The *universal* Turing machine is

a. the ultimate Turing machine, that halts on all input
b. a decider for A_{TM}
c. capable of simulating any Turing machine on any input
d. a recognizer for any language

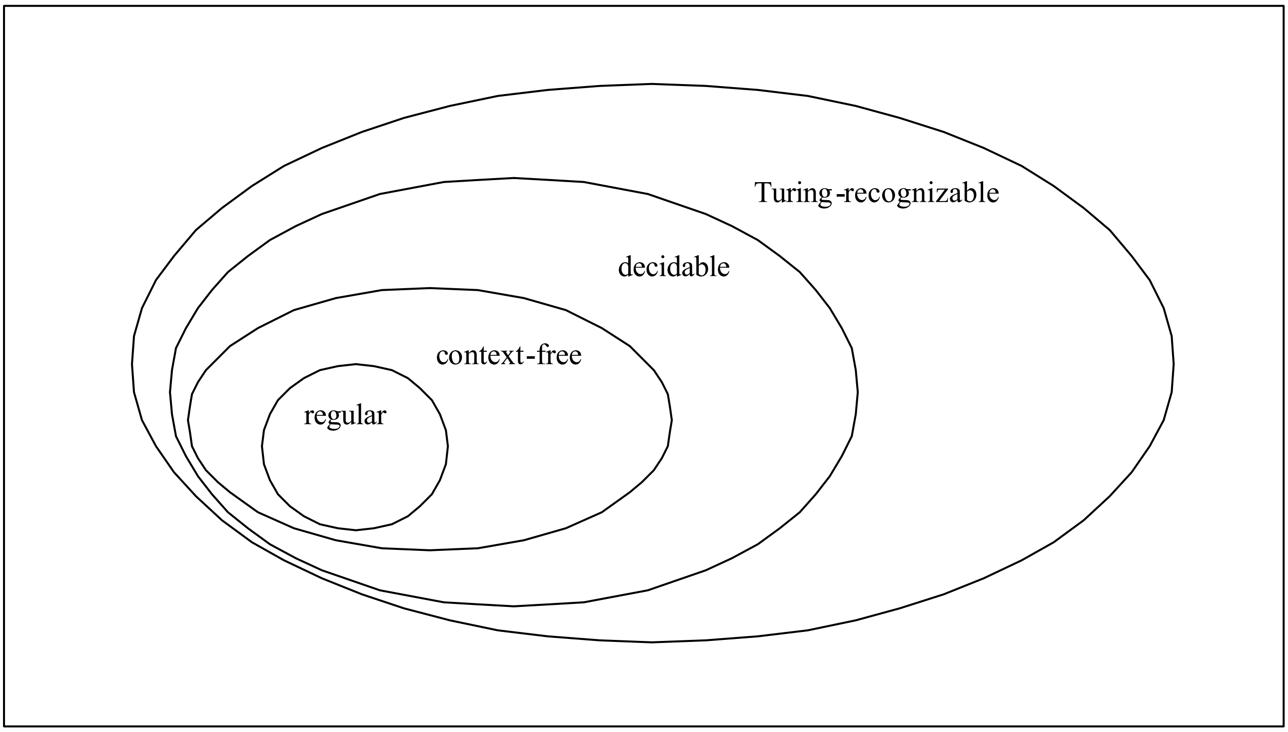
10. In showing that for a given Turing machine M we can find an enumerator that enumerates the language of M, we did not use the following approach:

Let s_1, s_2, s_3 Σ^*. Then,

E = Ignore the input.
1. Repeat the following for i
2. Run M on s_i
3. If it accepts, print s_i

a. an enumerator cannot ignore its input
b. M may loop on some s_i for some i in line 2
c. E should not have an infinite computation
d. the enumerator cannot check on line 3 whether M accepts or not

PART III. Problems.
Problem 1.(15 Points)
The diagram below represents the relationship among classes of languages, where the outermost rectangle represents the class of all languages.



On this diagram, place the symbols representing the languages as below in their correct position

$A = \{ a^n b^n : n \geq 0 \}$
$B = \Sigma^*$
$C = \{ a^n b^n c^n : n \geq 0 \}$
$D = A_{TM}$
$E =$ the complement of A_{DFA}
$F = EQ_{TM}$
$G =$ the complement of A_{TM}
$H = HAMPATH = \{ \langle G,u,v \rangle \mid G$ is a directed graph with a Hamiltonian path from u to $v\}$

Problem 2.(20 Points)

Let $\Sigma = \{0,1\}$, and consider the following languages:

$E = \{w : w \in \Sigma^* \text{ , and } |w| \textit{ is even } \}$, the set of even length strings

$EP = \{ww^R : w \in \Sigma^* \}$, the set of even length palindromes, and

$P = \{w = w^R : w \in \Sigma^* \}$, the set of palindromes.

a. Show that E is regular by giving the state diagram of a *DFA* that recognizes L.

b. Give a regular expression for E

c. Show that EP is not regular using the pumping lemma for regular languages

d. Using closure properties, and the results of (a) and (c) above, argue that P is not regular

Problem 3.(10 Points)

For this problem *EP* stands for the same language (even length palindromes) as in the previous problem.

a. Show that *EP* is a context free language by giving a context free grammar that generates it

b. Show that the complement of *EP* is also context free by giving a *PDA* that recognizes it. Give a brief explanation of the main idea

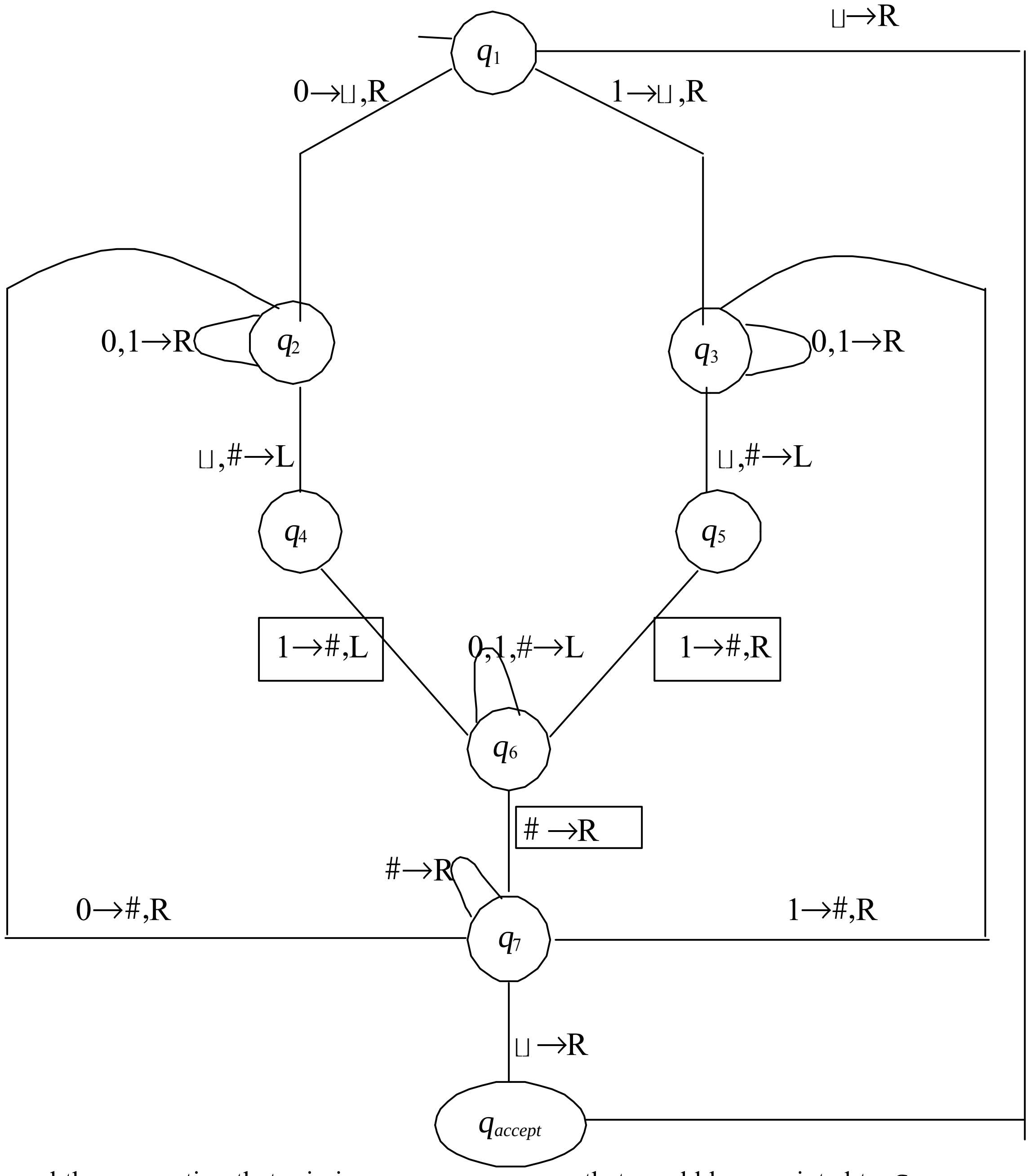
Problem 4.(10 Points)

Consider the language $L = \{w\#w : w \in \{0,1\}^*\}$

Show that L is not context free using the pumping lemma for context free languages.

Problem 5. (15 Points)

Consider the following state diagram of a Tm M whose language is supposed to be $EP = \{ww^R \mid w \in \{0,1\}^* \}$



We have used the convention that missing arrows are arrows that would have pointed to q_{reject} .

a. (the boxes). The correct labels should be as follows:

	Arrow	Label should be
1	From q_4 to q_6	
2	From q_5 to q_6	
3	From q_6 to q_7	

b. Give a high level description of M.

M w:

1.

c. What is the time complexity of M ? Justify your answer.

d. Give a high level description of a 2-tape Turing machine that decides EP in $O(n)$ time.

Problem 6 (15 Points)

Given $<M, w>$, consider the following Turing machine,

M_1 x do:

1. If $x = 00$ then ***accept***.
2. If $x \neq 11$ then ***reject***.
3. If $x = 11$ then simulate M on w

a. What is the language that M_1 accepts? i.e. complete the following:

$$L(M_1) = \begin{cases} \qquad\qquad & \text{if } M \text{ accepts } w \\ \qquad\qquad & \text{if } M \text{ does not accept } w \end{cases}$$

b. Using (a) or otherwise, show that the language

$\{<M> \mid M$ is a Turing machine and whenever M accepts 00 it also accepts $11\}$

is not Turing-decidable.

Problem 7.(15 Points)

a. Let A and B be two languages, such that $A \leq_m B$. Show that $\overline{A} \leq_m \overline{B}$

b. Using (a) or otherwise, show that if A is Turing recognizable, and $A \leq_m \overline{A}$ then A is decidable.