CMPS 257

Time: 2 Hrs **Final Exam** **Fall 04-05**

Family Name	
First Name	
Id. Num	
Major	

PART/Problem	Grade	Your Grade
I	10	
II	30	
III. 1	10	
III. 2	15	
III. 3	10	
III. 4	15	
III. 5	15	
III. 6	25	
III. 7	10	
III. 8	10	
III. 9	10	
Total	**160**	

Instructions.
Write your names, Id number, and circle your section in the space provided above.

THIS IS A CLOSED BOOK, CLOSED NOTES (OPEN MIND ?) Exam !!

Your answers must be presented on the question sheet itself. Use the provided space for your answers. If the space is not enough, then most probably you are writing more than what is expected. In case you need more space, put an "explicit pointer" to where your answer is written or continued (e.g. draw an arrow, or say back of page or both !!!...) For scratch work, you may use the back of the sheet, and the last blank page. **DO NOT CUT OFF ANY PAGE !!**
There are 14 pages altogether. Try to solve as much as you can in the given time.

Part I.](10 Points) True or False questions. Circle T or F [Not Both]
You might want to comment briefly, if you are not sure about your answer. But in case your comment is not relevant, and you have the right answer, you might loose the points for that part

1. If a language is regular, then there are only a finite number of DFA's that recognize it. infinit	T	F
2. Any subset of a Turing recognizable language is necessarily Turing recognizable Σ^*	T	F
3. The complement of a decidable language is decidable	T	F
4. There is an uncountable number of languages that are not Turing-recognizable	T	F
5. Push down automata can be simulated by Turing machines	T	F

Part II. (30 Points) Multiple choice problems, 10 of them
The multiple-choice problems, are intended to have **only one** correct answer. If you discover otherwise, do indicate that in writing and circle all what you think is a correct answer.

1. Which of the following best expresses the pumping theorem for context free languages ?

a. A language L is context free if there is a pumping length p such that for any string $w \in L$ of length greater than or equal to p, w can be subdivided as $w = uvxyz$ in such a way that either v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

b. Suppose L is a CFL. Then for some $p > 0$, the pumping length for L , any string $w \in L$ of length greater than or equal to p, every subdivision of $w = uvxyz$, with v or y nonempty, then $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

c. Suppose L is a CFL. Then for some $p > 0$, the pumping length for L , any string $w \in L$ of length greater than or equal to p, there is a subdivision of $w = uvxyz$, such that v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

d. Suppose L is a CFL. Then for every number $p > 0$, and for any string $w \in L$ of length greater than or equal to p, there is a subdivision of $w = uvxyz$, such that v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

2. The language $\{w \mid w$ has an equal number of occurrences of 0's and 1's $\}$ is:

a. not regular
b. equal to $\{0^n 1^n \mid n \geq 0\}$ but is CF
c. finite
d. not context free

3. The complement of $E_{TM} = \{<M> \mid M \text{ is a Turing machine, and } M \text{ has an empty language}\}$ is

a. $\{<M> \mid M$ is a Turing machine, and $L(M)$ is not empty $\}$
b. $\{<M> \mid M$ is a Turing machine, and M loops on *some input* $\}$
c. $\{<M> \mid M$ is a Turing machine, and M accepts all input $\}$
d. $\{u \mid u$ is not a valid coding of a Turing machine$\} \cup \{<M> \mid$ is a Turing machine, and $L(M)$ is not empty$\}$

4. The Church-Turing thesis

a. says that all problems can be solved by algorithms
b. presents the Turing machine as a formal definition for the intuitive notion of an algorithm
c. was proved by Turing in the 1930's
d. was proved to be wrong in the 1970's

5. By an unsolvable problem (or undecidable problem) is meant

a. The problem has a mathematical solution
b. There is no algorithm to solve the problem
c. There is an algorithm to solve the problem but the algorithm is not efficient
d. The problem has no mathematical solution

6. The halting problem shows that

a. Every Turing machine must halt
b. There cannot be an algorithm for discovering logical errors in programs
c. Turing decidable languages are also Turing recognizable
d. A Turing machine might loop on its input

7. In showing that a nondeterministic Turing machine can be simulated by a regular Tm, the traversal of the tree of computations

a. Must be done using breadth search   Page 138
b. Must be done using depth first search
c. Depth first search or breadth first search may both be used
d. will not end since there will definitely be an infinite path in the tree

8. The language that expresses the equivalence problem for Turing machines EQ_{TM} is:

a. $\{<M, w> \mid M$ is a Turing machine that does not halt on $w\}$
b. $\{<M, N> \mid M$ and N are Turing machines, and $L(M) = L(N)$ $\}$
c. $\{<M, N> \mid M$ and N are DFA's, and $L(M) = L(N)$ $\}$
d. $\{<M, N> \mid M$ and N are Turing machines, and $L(M) = L(N) = \phi\}$

9. The *universal* Turing machine is

a. the ultimate Turing machine, that simulates any variant (multi-tape, enumerators, etc…)of the Turing machine model
b. a decider for A_{TM}
c. capable of simulating any Turing machine on any input
d. a decider for E_{TM}

10. The acceptance problem for context free grammars, A_{CFG} was shown to be decidable essentially because

a. every grammar has an equivalent unambiguous grammar
b. the length of a derivation of a string w in a context free grammar in Chomsky normal form is of length bounded by $2|w|$ -1.
c. there is an equivalent PDA for any CFG
d. it is reducible to the acceptance problem for DFA's (A_{DFA})

PART III. 9 Problems.

Problem 1.(10 Points)

Give an example for each of the following (No proof is required)

a. A language that is context free but not regular

$0^n 1^n$ not DFA (not reg) yes PDA (yes CF)

b. A language that is decidable but not context free

$0^n 1^n 2^n$

c. A language that is Turing-recognizable but not decidable

$HAlt_{AM}$

d. A language that is not Turing-recognizable

$\overline{HAlt_{AM}}$

e. A language that is not context free, but its complement is context free

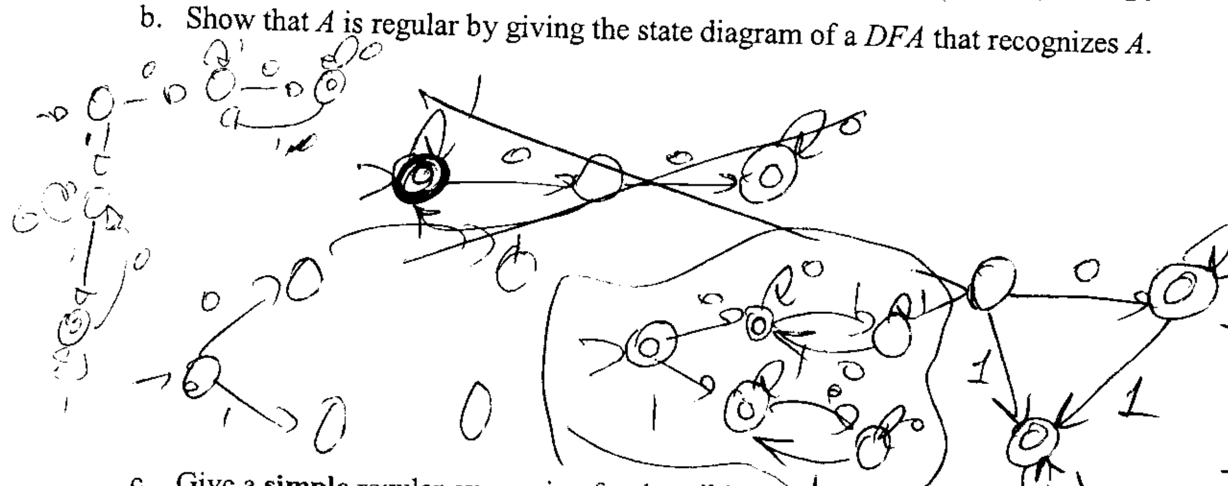
Problem 2.(15 Points)

Let $\Sigma = \{0,1\}$, and consider $A = \{w \in \Sigma^* \mid$ the first and last symbol in w are the same$\}$

a. List the first 6 elements of A in lexicographic order.

$= \{ \varepsilon, 0, 1, 00, 11, 000, 010, 101, 111 \}$

b. Show that A is regular by giving the state diagram of a *DFA* that recognizes A.



c. Give a **simple** regular expression for describing A.

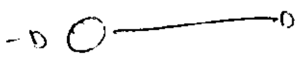
$(0(100)^*0) \cup (1(100)^*1)$

$\Sigma\Sigma\Sigma$

$\Sigma = \{0,1\}$

d. Show that $\{w \mid w$ has an equal number of occurrences of 01 and 10 $\}$ is regular.

$(10\,001)^{14}$ not   by DFA of 6



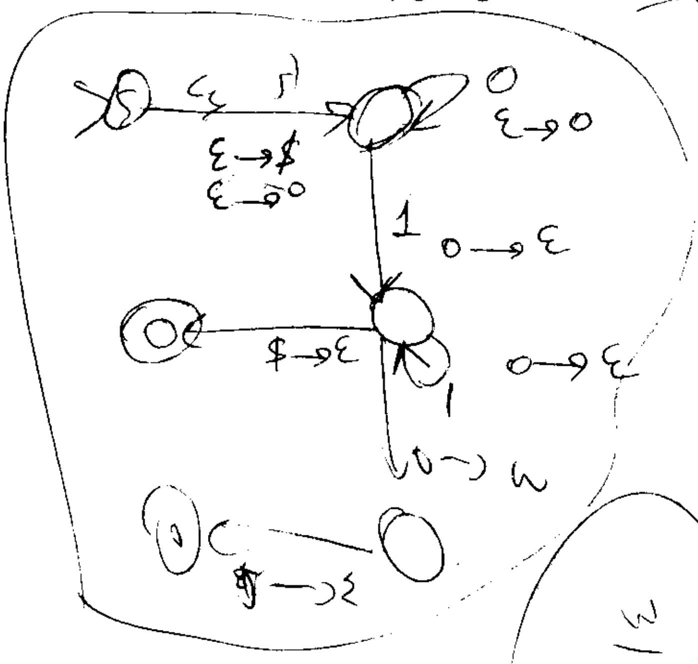
Problem 3.(10 Points)

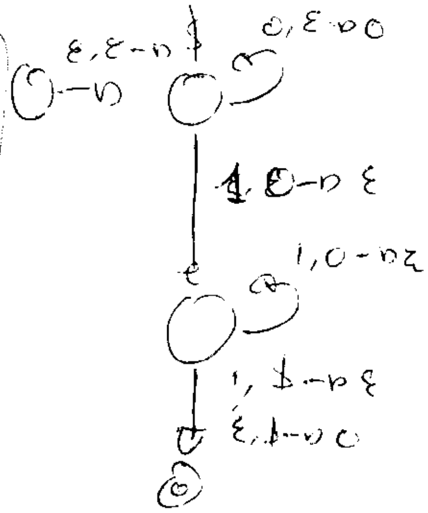
Consider the language $B = \{0^m 1^n \mid m \le n \le m+1\}$ ε, 01, 011, 0011, 00111, 000111

a. List the first 6 elements of B in lexicographic order.

$L = \{\varepsilon, 0, 1, 01, 011, 0011$

b. Show that B is context free by giving a *PDA* that recognizes B.





$S \to 0S1 \mid 1 \mid \varepsilon$

c. Give a **simple** *CFG* for B.

$S \to 0/1/$

$S \to 0S1 \mid 1 \mid \varepsilon$

Problem 4 (15 Points)
Consider the acceptance problem for Turing machines represented by the language:

$A_{TM} = \{ <M, w> \mid M \text{ is a Turing machine that accepts string } w\}$

a. Show that A_{TM} is Turing-recognizable.

Co-Tur
not decideable
So recog.

b. In showing that A_{TM} is undecidable, we introduced a Turing machine D as follows:

D = "On input <M>, where M is a Turing machine:
1. Run H on input <M, <_n_> >
2. If H *accepts,* reject; if H reject, *accept.*"

What is H supposed to be, and what is the missing second component in Step 1 of D ?

Halting learning machine that we suppose
is decidable.
step 1 → itself n

c. Then a contradiction is arrived at by running D on some input. What is the input, and what is the contradiction?

the input is D

→ it will reject if it accept
and accept if it reject
contradict.

Problem 5. (15 Points)
Consider the language $B = \{0^n1^n \mid n \geq 0\}$. Suppose that M is a Turing machine that decides B.

a. Is $<M, 001>$ in A_{TM} ? Why ?

No

b. Is $<M, 0011>$ in A_{TM} ? Why ?

Yes

c. Is $<M>$ in E_{TM} ? Why ?

not ϕ No

d. If we run the universal Turing machine U on input $<M, 001>$ will it accept, reject, or what? Why ?

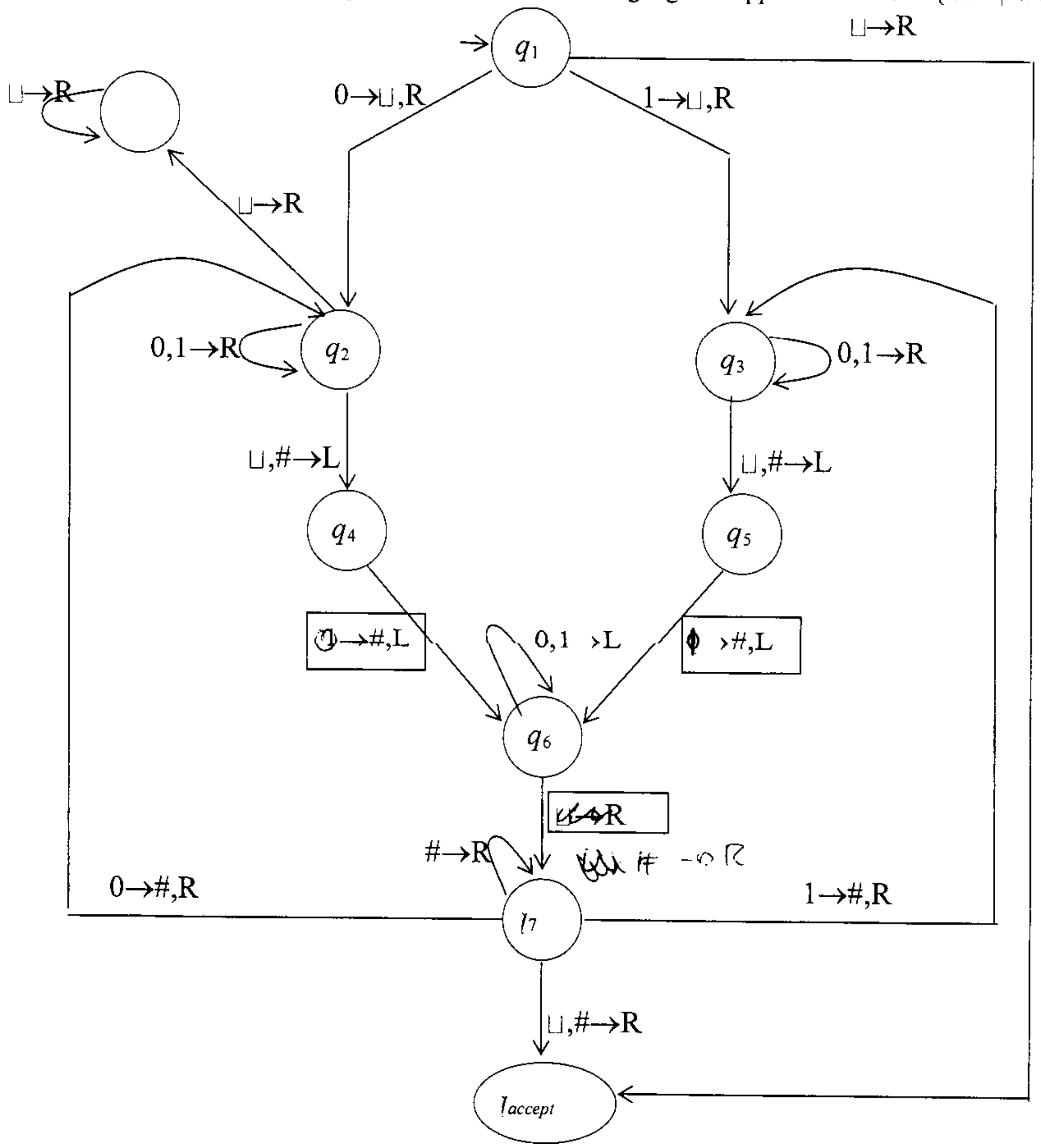
reject

e. If we run the universal Turing machine U on input $<M>$ will it accept, reject, or what? Why ?

reject → no valid string [illegible] re

Problem 6. (25 Points).

Consider the following state diagram of a Tm M whose language is supposed to be $C = \{ww^R \mid w \in \{0,1\}^*\}$



We have used the convention that missing arrows are arrows that would have pointed to q_{reject}.

a. There are some 'bugs' in the transition rules as given. Correct them in the following table:

	Arrow	Label should be
1	From q_4 to q_6	
2	From q_5 to q_6	
3	From q_6 to q_7	

b. Give a high level description of M.as corrected

M = "On input w:

1. ______________________________

____

____

____

____

____

____

____

c. If the arrows from q_4 to q_6 and q_5 to q_6 are left as given the language of the Tm would be

$\{wu^R \mid w \in \{0,1\}^*$ and u is ~~inverse of w~~ $\}$

d. Is the given Tm M after correction, a decider ? Why ?

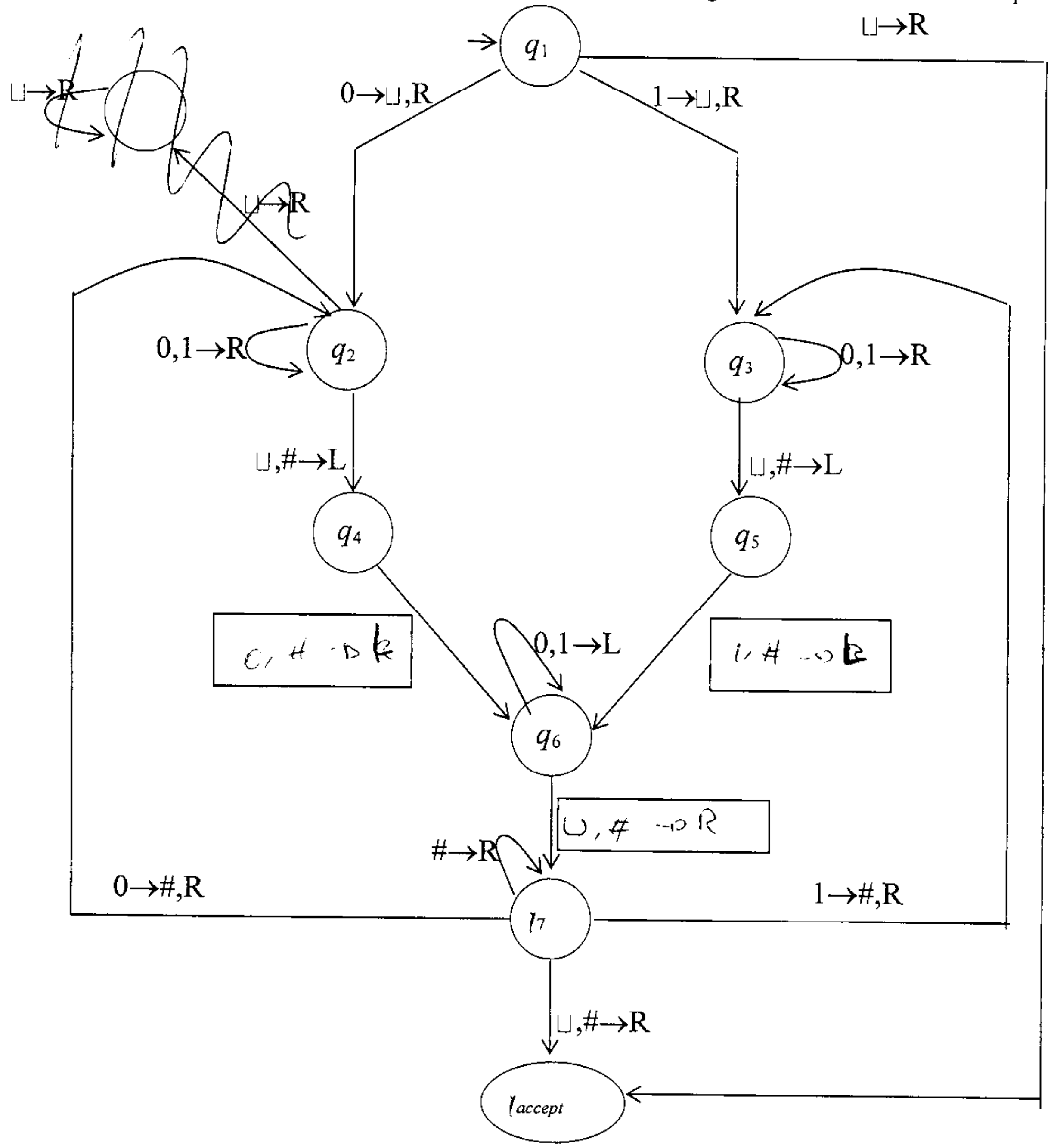
no, because of loop decide.

e. Given your answer in (d), is the language $C = \{ww^R \mid w \in \{0,1\}^*\}$ decidable ? Justify your answer.

yes, we only need to remove the loop.

ww^R decidable

f. Modify to the given Tm so that it becomes a decider for $P = \{w{=}w^R \mid w \in \{0,1\}^*\}$ i.e. set of all palindromes over $\{0,1\}$. Indicate your modifications on the diagram below. Give a brief explanation.



Problem 7.(10 Points)

a. Suppose that M and N are two DFA's, with m and n states respectively. Show that $L(M) = L(N)$ if and only if M and N accept the same set of strings of length less than or equal to mn. (Hint: In case the languages of M and N differ, consider a string t on which they differ of minimal length)

$M \cap N$ — number ≤ state

Converse is true $M \cup N$ — mn

$M \cap N = N \cup M \Rightarrow M = N$

$M \cap N$ $M \cup$

P — $\Sigma M \cap$

① $L(M) = L(N)$ → ~~they are accepted~~ by a DFA

construct a DFA Z

$\Rightarrow$ Z: $Q_Z = (Q_M \times Q_N \times F_N)$

$F_Z = F_M \times F_N$

$\delta Z : Q_Z \times \Sigma_Z \to Q_Z$

→ we have $m \times n$ states and we know by pumping that we need the DFA to accept string ≤ mn size of states

b. Use (a) above to show that EQ_{DFA} is decidable.

Method

① Use above DFA Z we try emptiness of lemma

to construct a DFA for both on both.

② run input on Z.

if it accept accept

Problem 8 (10 Points)
Given <M, w>, consider the following Turing machine,

M_1 = "On input x do:
1. If $x = 01$ then ***accept***.
2. If $x \neq 10$ then ***reject***.
3. If $x = 10$ then simulate M on w."

01 ∪ 10 if m accep

a. What is the language that M_1 accepts? i.e. complete the following:

$$L(M_1) = \begin{cases} 01 & \text{if } M \text{ does not accept } w \\ \{01 \cup 10\} & \text{if } M \text{ accepts } w \end{cases}$$

W =

b. Using (a) or otherwise, show that the language {<M> | M is a Turing machine and whenever M accepts w it also accepts w^R} is not Turing-decidable.

if this T-dec ⟹ A_{TM} is de

But, by Contrad.

(1) construct M_1:

(2) run W <M_1>

(3) if it accepts accept, else reject

but contradict...

Problem 9.(10 Points)

a. Show that the class of Turing-recognizable languages is closed under union.

do 2 TM
sim both at a time
it.

b. Show that if A is Turing-recognizable, then its reversal, A^R is also Turing-recognizable.

A rec. => Their exist a TM
that recognize A call it T

Construct T'
on input x
- compute x^R
- simulate x^R on T

T' recognize A^R

① construct T' such that
T' begins opposite of T
T' accepts opposite of T
run T' on input.