American University of Beirut
CMPS 257
Theory of Computation
Spring 2002-2003

Final Exam

Date: June 7th, 2003. 8:00 – 10:00am.
Instructor: Jihad Boulos

Name: ....

ID #: ....

Section: ....

This is an open-book, open-note exam. Your exam should have 11 pages, and there are 10 questions totaling 100 points. You may use your notes (i.e., class handouts) and the main course textbook. You are **NOT** allowed to use any external notes. Your answers should be concise, and when possible should be a list of important points rather than prose. Solve as much problems as you can. I advise you to spend time on understanding the problem and budget your time for solving each problem, or else you will be wasting a lot of time on one problem and will run out of time for other problems.

Wordy and/or irrelevant answers will reduce your score for that problem. Your answers should be the summary of work done on scratch paper that you do not hand in. If I could not read your writing, I will just give a ZERO without bothering myself trying to understand what you are writing. The space allocated for answers should be sufficient for your answers. If not, use the back of papers.

Exercise 1 (7.5 points):

Circle for each statement (TRUE) for true, (FALSE) for false, or (UNKNOWN) for unknown to researchers.

1. Recursively Enumerable (RE) languages are closed under union and under intersection.

   TRUE FALSE UNKNOWN

2. Deterministic and non-deterministic finite automata accept exactly the same class of languages.

   TRUE FALSE UNKNOWN

3. Regular languages are closed under union, complementation, and intersection.

   TRUE FALSE UNKNOWN

4. Context-free languages are closed under union and under intersection.

   TRUE FALSE UNKNOWN

5. The problem of determining if a string over {a, b} is described by a given regular expression over that alphabet is decidable.

   TRUE FALSE UNKNOWN

6. The problem of determining if a string over {a, b} is generated by a given context-free grammar over that alphabet is decidable.

   TRUE FALSE UNKNOWN

7. $\{ww \mid w \text{ in } \{0, 1\}^*\}$ is not regular but it is context-free.

   TRUE FALSE UNKNOWN

8. If a language A is a subset of a language B, and A is non-recursive, then B is also non-recursive.

   TRUE FALSE UNKNOWN

9. Every regular language can be recognized by a deterministic pushdown automaton.

   TRUE FALSE UNKNOWN

10. A non-deterministic TM accepts its input if every possible computation on the input leads to an accepting state.

TRUE FALSE UNKNOWN

11. Every finite set is regular.

TRUE FALSE UNKNOWN

12. Every context-free grammar can be made *unambiguous*.

TRUE FALSE UNKNOWN

13. The computational path of a TM on an input w either halts accepting, halts rejecting, or enters a never halting loop of repeating configurations.

TRUE FALSE UNKNOWN

14. Let A and B be two sets. If there exists a surjection $f: A \rightarrow B$ and an injection $g: A \rightarrow B$ between A and B, then there exists a bijection $h: A \rightarrow B$.

TRUE FALSE UNKNOWN

15. For all finite sets A, the complement of A is a context-free language.

TRUE FALSE UNKNOWN

Exercise 2 (5 points):

This question explores the relations among the various classes of languages.

a. Indicate ALL the regular languages below (by circling the item number to its left).

   i. All strings over $\{0, 1\}$ that have equally many 0's and 1's.
   ii. All strings over $\{0, 1\}$ that have an even number of 0's and an even number of 1's.
   iii. All strings over $\{0, 1\}$ that start with a 0 and do *not* contain fifty 1's in a row.

b. Indicate ALL the languages below that ARE context-free but ARE NOT regular (by circling the item to its left.)

   i. $\{a^n b^n c^n \mid n \geq 0\}$
   ii. $\{a^n b^n c^k \mid n, m, k \geq 0\}$

iii. $\{a^n b^m \mid 0 \le n \le m \le 2n\}$

c. Indicate ALL the languages below that ARE *recursive* but NOT *context-free* (by circling the item to its left.)

i. $\{ww^R \mid w \text{ in } \{a, b\}^*\}$, where w^R is the string w written in reverse.
ii. $\{wcw \mid w \text{ in } \{a, b\}^*\}$
iii. $\{a^{2n} b^n c^{2n} \mid n \ge 0\}$

d. Indicate ALL the languages below that ARE *recursively enumerable (RE)* but NOT *recursive* (by circling the item to its left.)

i. $\{\langle M, w\rangle \mid$ TM M accepts input $w\}$
ii. $\{M \mid$ TM M is such that $L(M) \neq \Phi\}$
iii. $\{\langle M_1, M_2\rangle \mid$ TMs M_1 and M_2 are such that $L(M_1) = L(M_2)\}$

e. Indicate ALL the languages below that ARE NOT *recursively enumerable (RE)* (by circling the item to its left.)

i. $\{\langle M, w\rangle \mid$ TM M halts on input $w\}$
ii. $\{M \mid$ TM M is such that $L(M) \neq \Phi\}$
iii. $\{\langle M_1, M_2\rangle \mid$ TMs M_1 and M_2 are such that $L(M_1) \cap L(M_2) = \Phi\}$

Exercise 3 (6 points):

Construct a grammar G over {a, b, c} such that $L(G) = \{a^n b^m c^i \mid 0 \le n + m \le i\}$.

ANSWER:

Exercise 4 (16 points):

a. Give a pushdown automaton that accepts the following language:

$$L = \{ wa^n \in \{a, b\}^* \mid |w| = n \}$$

ANSWER:

b. Give a regular expression for the language L of all strings $w \in \{0, 1\}^*$ such that string w begins or ends with 00 or 11. For example, 00101 and 011 are in L, but 010 is not in L.

ANSWER:

c. Transform the following regular expression into an equivalent NFA using the procedure studied in class. (You may omit some of the redundant ε transitions, but your automaton should have a structure that closely resembles the regular expression.)

$$((a \cup ba)(ab)^*ba)^*$$

ANSWER:

d. Give a context free grammar for the language of L:

$$L = \{ \ a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i \neq j \text{ or } i = 2k \text{ (or both)} \}$$

ANSWER:

Exercise 5 (6 points):

Is it possible to m-reduce A_{TM} to EQ_{CFG}? (2 points)

Yes: ---------------
No: ---------------

Explain your answer bellow: (4 points)

ANSWER:

Exercise 6 (10 points):

State whether the language $L \subseteq \{a, b, c, d\}^*$, defined by

$$L = \{a^m b^n c^m d^n \mid n, m \in \mathrm{N}\}$$

is context-free or not and prove your statement.

ANSWER:

Exercise 7 (12 points):

Let $\text{INF}_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ is infinite}\}$.

Suppose that you want to show that $A_{TM} \leq_m INF_{TM}$ using a reduction f that maps $\langle M, w \rangle$ to $\langle M_1 \rangle$.

a. Fill in the blanks in the following two statements in a way that states what you have to do to make the reduction works. (In both cases you will be writing down something about the behavior of the Turing machine M_1.)

- If M accepts w, then

- If M does not accept w, then

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b. Give the definition of the desired Turing machine M_1, given M and w.

ANSWER:

c. What conclusion about INF_{TM} follows from the fact that $A_{TM} \leq_m INF_{TM}$? (You can get credit for this part without doing the previous two parts.)

ANSWER:

Exercise 8 (10 points):

For each of the following languages L, circle D if L is decidable, circle R if L is recognizable but undecidable, circle N if L is not recognizable.

D R N $L_1 = \{\langle M \rangle \mid L(M) \text{ is finite}\}$

D R N $L_2 = \{\langle M \rangle \mid L(M) \in (P - NP)\}$

D R N $L_3 = \{\langle M \rangle \mid L(M) \text{ is recognized by at least one TM}\}$

D R N $L_4 = \{\langle M \rangle \mid L(M) \text{ is recognized by at least one polynomial-time NTM}\}$

D R N L_5 a language recognizable by at least one polynomial-time NTM.

Exercise 9 (12 points):

Your last CMPS-257 problem:

Let $L = \{\langle M \rangle \mid M$ prints 257 on some input$\}$.

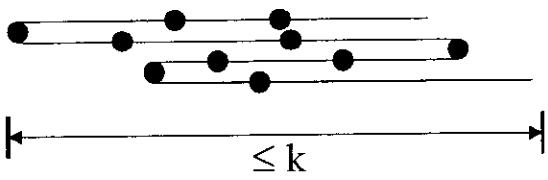
Prove that:

a. L is recognizable (i.e., give a (very simple) TM that would recognize it).
b. L is not decidable.

ANSWER:

Exercise 10 (16 points):

(The *Carpenter's Rule Problem*). Prove that the following problem is NP-complete: given a sequence of rigid rods of various integral lengths connected end-to-end by hinges, can it be folded so that its overall length is at most k?



You may assume the following problems are NP-complete: knapsack, subset-sum, partition, CNF-SAT, clique, graph coloring.

ANSWER: