CMPS 257

Time: 2 Hrs | **Final Exam** | **Spring 03-04**

Family Name	
First Name	
Id. Num	
Section	**1 (9:00 TR)** **2 (11:00 TR)**

PART/Problem	Grade	Your Grade
I	20	
II	30	
III. 1	10	
III. 2	10	
III. 3	10	
III. 4	15	
III. 5	10	
III. 6	15	
III. 7	10	
III. 8	20	
Total	**150**	

Instructions

Write your names, Id number, and circle your section in the space provided above.

THIS IS A CLOSED BOOK, CLOSED NOTES (OPEN MIND ?) Exam !!

Your answers must be presented on the question sheet itself. Use the provided space for your answers. If the space is not enough, then most probably you are writing more than what is expected. In case you need more space, put an "explicit pointer" to where your answer is written or continued (e.g. draw an arrow, or say back of page or both !!!...) For scratch work, you may use the back of the sheet, and the last blank page. **DO NOT CUT OFF ANY PAGE !!**

There are 13 pages altogether. Try to solve as much as you can in the given time (120 minutes).

Good Luck

Part I.](20 Points) True or False questions. Circle T or F [Not Both]
You might want to comment briefly, if you are not sure about your answer. But in case your comment is not relevant, and you have the right answer, you might loose the points for that part

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1. If a language is regular, then there are infinitely many regular expressions that represent it.	T	F
2. The complement of a decidable language is necessarily decidable	T	F
3. Context free languages are closed under complementation	T	F
4. Any subset of a decidable language is necessarily decidable	T	F
5. Decidable languages are closed under intersection.	T	F
6. There is an uncountable number of languages that are not Turing-recognizable	T	F
7. Every string generated by a context free grammar has exactly one left most derivation..	T	F
8. Push down automata cannot be simulated by Turing machines	T	F
9. A non-deterministic finite automaton might have an infinite computation on a string w.	T	F
10. For Turing machines, if the halting problem is decidable, then the acceptance problem will be decidable	T	F

Part II. (30 Points) Multiple choice problems, 10 of them

The multiple-choice problems, are intended to have **only one** correct answer. If you discover otherwise, do indicate that in writing and circle all what you think is a correct answer.

1. Which of the following best expresses the pumping theorem for context free languages ?

a. A language L is context free if there is a pumping length p such that for any string $w \in L$ of length greater than or equal to p, w can be subdivided as $w = uvxyz$ in such a way that either v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

b. Suppose L is a CFL. Then for some $p > 0$, the pumping length for L , any string $w \in L$ of length greater than or equal to p, every subdivision of $w = uvxyz$, with either v or y is nonempty, then $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

c. Suppose L is a CFL. Then for some $p > 0$, the pumping length for L , any string $w \in L$ of length greater than or equal to p, there is a subdivision of $w = uvxyz$, such that either v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

d. Suppose L is a CFL. Then for every number $p > 0$, and for any string $w \in L$ of length greater than or equal to p, there is a subdivision of $w = uvxyz$, such that either v or y is nonempty, $|vxy| \leq p$, and $uv^n xy^n z$ is in L for every $n \geq 0$.

2. The language $\{w \mid w$ has an equal number of occurrences of 01 and 10 $\}$is:

a. not regular, because any finite automaton has only a finite amount of memory, and so cannot keep count of the number of 01 and/or 10's, for a string with arbitrary length

b. equal to the set of strings that start and end with the same symbol and hence is regular

c. finite

d. not context free

3. Let A = {<M>: M is a Turing machine, and M satisfies a property P}. The complement of A is

a. {<M> | M is a Turing machine, and M does not satisfy property P }

b. {<M> | M is a Turing machine, and M loops on *some input* }

c. {<M> | M is a Turing machine, and M accepts all input }

d. {u | u is not a valid coding of a Turing machine} $\cup$ {<M> | is a Turing machine, and M does not satisfy property P }

4. The Church-Turing thesis

a. says that all problems can be solved by algorithms

b. presents the Turing machine as a formal definition for the intuitive notion of an algorithm

c. says that some problems cannot be solved by algorithms

d. was proven to be wrong in the 1970's

5. By a solvable problem (or decidable problem) is meant

a. The problem has a mathematical solution

b. There is an algorithm to solve the problem

c. There is an algorithm to solve the problem and the algorithm is efficient

d. The problem is in NP.

6. If A is NP – complete , then membership in A can be
a. verified quickly and decided quickly
b. verified quickly and decided slowly
c. verified slowly and decided quickly
d. verified slowly and decided slowly

7. The extended Tm model for which the one-tape simulating Turing machine is significantly slower (exponential) running time is the
a. the k-tape Turing machine
b. the non-deterministic Turing machine
c. The two-way infinite tape Turing machine
d. The enumerator

8. The language that expresses the emptiness problem for Turing machines E_{TM} is:
a. {<M, w> | M is a Turing machine that does not halt on w}
b. {<M, w> | M is a Turing machine that rejects w}
c. {<M> | M is a Turing machine that halts on ε}
d. {<M> | M is a Turing machine, and $L(M) = \phi$ }

9. The *universal* Turing machine is

a. the ultimate Turing machine, that simulates any variant (multi-tape, enumerators, etc…)of the Turing machine model
b. a decider for A_{TM}
c. capable of simulating any Turing machine on any input
d. a decider for A_{LBA}

10. The acceptance problem for linear bounded automata, A_{LBA} is decidable essentially because
a. an LBA has a finite amount of memory, so it runs out of memory in finite time
b. for a given LBA B and a string w, there is a finite number of possible configurations for the computation of B on w
c. a linear bounded automaton B must halt on any w
d. an LBA has the mechanism to guess whether it will halt on w or not

PART III. Problems.

Problem 1.(10 Points)

Give an example for each of the following (No proof is required)

a. A language that is context free but not regular

$a^n b^n$

b. A language that is decidable but not context free

$a^n b^n c^n$

c. A language that is Turing-recognizable but not decidable

Halt TM

d. A language that is not Turing-recognizable

EQ TM

e. A language that is in P ?

Problem 2.(10 Points)

Let $\Sigma = \{0,1\}$, and consider the language $L = \{w \in \Sigma^* \mid$ the i^{th} symbol in w is 0, whenever i is odd $\}$

a. Show that L is regular by giving the state diagram of a DFA that recognizes L.

b. Give a simple regular expression for describing L.

Problem 3.(10 Points)
Consider the language $L = \{0^m 1^n \mid m \le n \le 2m\}$

a. Show that L is context free by giving a *PDA* that recognizes L.

b. Give a simple *CFG* for L.

Problem 4.(15 Points)

Let $\Sigma = \{0,1\}$, and consider the language $L = \{ u\,u \mid u \in \Sigma^* \}$

a. List in lexicographic order the first 8 strings in L .

b. Show that L is not regular by using the pumping lemma for regular languages.

c. Consider the following incorrect "proof" that attempts to show that L above is regular:

Let $A = \{ u \mid u \in \Sigma^* \}$.

1. A is obviously regular.
2. Now, $L = A\,A$
3. But regular languages are closed under concatenation.
4. Therefore, L is regular.

Which step in the "proof" is false, and why ?

d. If $\Sigma = \{0\}$, would the language $L = \{ u\,u \mid u \in \Sigma^* \}$ be not regular ? Why ?

Problem 5 (10 Points)

Consider the acceptance problem for Turing machines represented by the language:

$A_{TM} = \{ <M, w> \mid M$ is a Turing machine that accepts string $w\}$

a. Show that A_{TM} is Turing-recognizable.

b. In showing that A_{TM} is undecidable, we introduced a Turing machine D as follows:

D = "On input <M>, where M is a Turing machine:
1. Run H on input <M, <____> >
2. If H *accepts,* reject; if H reject, *accept.*"

What is H supposed to be, and what is the missing second component in Step 1 of D?

c. Then a contradiction is arrived at by running D on some input. What is the input, and what is the contradiction ?

Problem 6.(15 Points)

Suppose that L is recognized by a DFA M with n states. Prove each of the following:

a. L is nonempty if and only if M accepts a string of length less than n. (Hint: Take a string in L of minimal length, and show that its length must be less than n.)

b. L is infinite if and only if M accepts some string w where $n \leq |w| < 2n$; i.e. the length of w is between n and $2n$.

c. Using (b) of the previous problem or otherwise, show that $INFINITE_{FA}$ is decidable, where $INFINITE_{FA}$ = { <*F*> | *F* is a *finite automaton (not necessarily deterministic*, and *L*(*F*) is infinite }

Problem 7.(10 Points)

Let $INFINITE_{TM}$ = { <*M*> | *M* is a Turing machine and $L(M)$ is infinite }

a. For a given Turing machine *M*, and a string *w*, consider the Turing machine M_1 :

   M_1 = "On input *x*:

   1. If $x = 01$ then, *accept.*
   2. If *x* is not 01, run *M* on input *w*
   3. If *M* accepts, then *accept* ; if *M* rejects, then *reject*."

   What is $L(M_1)$? (Hint: Consider two cases.)

b. Show that $INFINITE_{TM}$ is undecidable. (Hint You may want to make use of M_1 above)

Problem 8.(20 Points)

a. Show that the class of Turing-recognizable languages is closed under union.

b. Consider the function $f(w) = w^R$. Give a high level description of an efficient TWO-TAPE Turing machine that computes f.

M = "On input w:

1. ______________________________

c. What is the running time for your machine in (b) ? Why ? Be as precise as possible.