CMPS 257

Time: 2 Hrs | **Final Exam** | **Spring 04-05**

Family Name	
First Name	
Id. Num	
Section	**1** (9:30 TR) **2** (11:00 TR)

PART/Problem	Grade	Your Grade
1	15	
2	10	
3	15	
4	10	
5	15	
6	20	
7	25	
8	25	
9	15	
Total	**150**	

Instructions.
Write your names, Id number, and circle your section in the space provided above.

THIS IS A CLOSED BOOK, CLOSED NOTES (OPEN MIND ?) Exam !!

Your answers must be presented on the question sheet itself. Use the provided space for your answers. If the space is not enough, then most probably you are writing more than what is expected. In case you need more space, put an "explicit pointer" to where your answer is written or continued (e.g. draw an arrow, or say back of page or both !!!...) For scratch work, you may use the back of the sheet, and the last blank page.

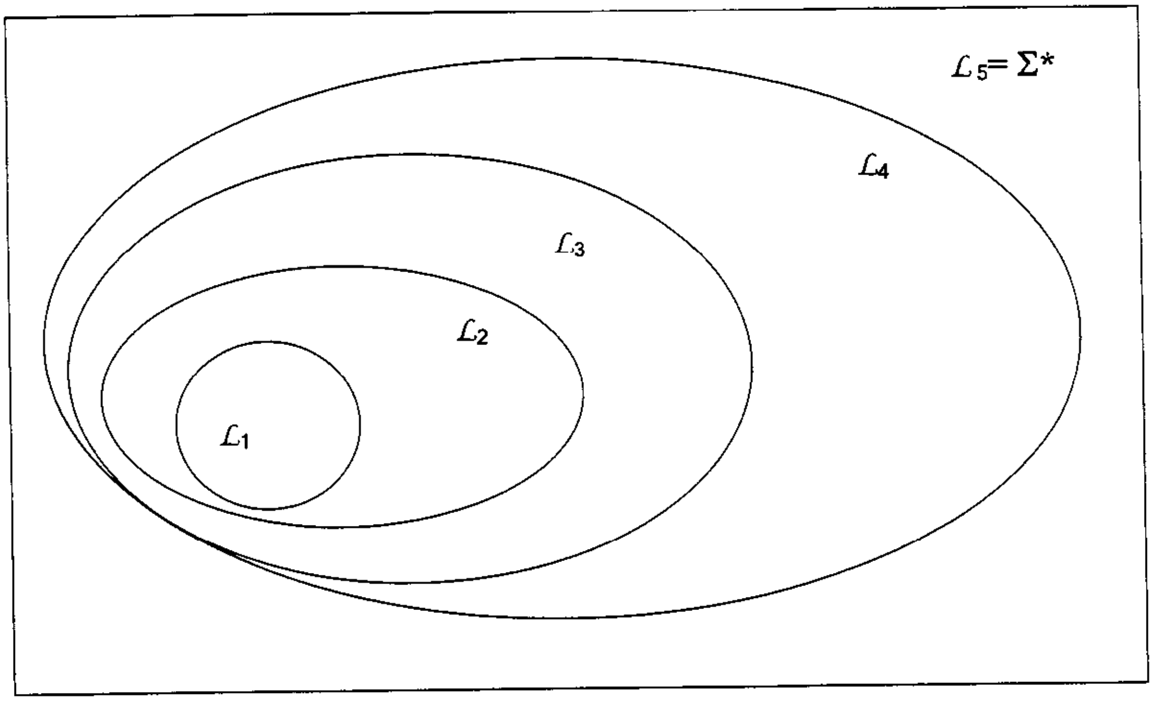
DO NOT CUT OFF ANY PAGE !!

There are 14 pages altogether. Try to solve as much as you can in the given time.

GOOD LUCK

Problem 1.(15 Points)

Consider the following diagram, which is supposed to express the relationship among classes of languages, that we have learned about this semester. Each of these classes is associated with a computation model of computing. In addition, containment is proper; i.e. there are languages that are in the outer class of languages and not in the inner class.



Complete the following table.

	Class	**AssociatedComputing Model**	**Language in $\mathcal{L}_i$ but not in $\mathcal{L}_{i-1}$**
$\mathcal{L}_1$			
$\mathcal{L}_2$			
$\mathcal{L}_3$			
$\mathcal{L}_4$			
$\mathcal{L}_5$			

Problem 2 (10 Points)

a. What are the practical implications (on compiler construction) of the fact that *the acceptance problem for context free grammars, A_{CFG} is decidable.*

b. What are the practical implications (on possibility of having a super compiler for discovering logical errors in programs) of the fact that Halting problem, $HALT_{TM}$ *is undecidable ?*

Problem 3.(15 Points)

Let $\Sigma = \{0,1\}$, and consider the language $B = \{ w \in \Sigma^* \mid w \text{ has exactly three 0's} \}$

a. Give the state diagram of a DFA M that recognizes B, using as small a number of states as possible.

b. Based on your answer in (a) above, what is the pumping length for B ? Why ?

c. Give as simple a regular expression as possible representing the language B.

Problem 4.(10 Points)

For any DFA $M = (K, \Sigma, \delta, s, F)$ define the following sequence $\{ Q_i \}$ of subsets of K recursively:

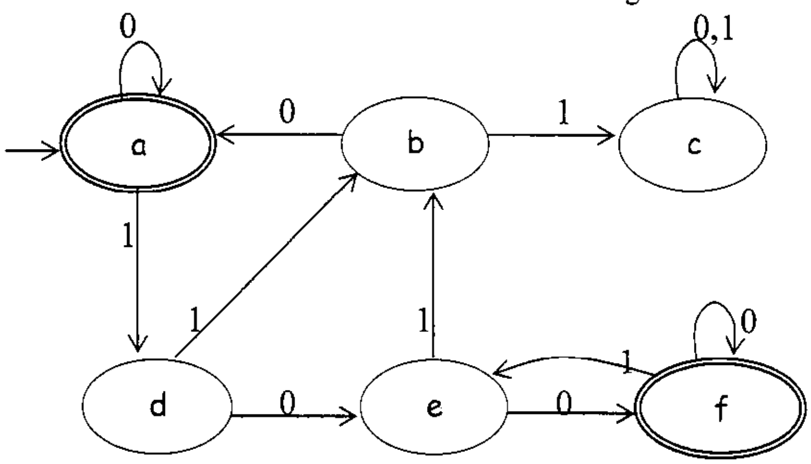
$Q_0 = F$

$Q_{i+1} = \{ q \in Q \mid \text{ there is } x, \text{ s.t. } \delta(q, x) \in Q_i \} \qquad i = 0, 1, 2, \ldots$

i.e. $q \in Q_j$ if from q an accept state might be reached in j steps.

a. Can the sequence of sets Q_i be all distinct ? Why ?

b. Assume here that M is the DFA whose state diagram is as follows:



What are the sets Q_0, Q_1, Q_2 ?

Problem 5.(15 Points)

For any language A, let $A_{\frac{1}{2}}$ be the set of all first halves of strings in A; that is

$$A_{\frac{1}{2}} = \{ u \mid \text{for some } v, |u| = |v| \text{ and } uv \in A \}$$

E.g. $A = \{010, 10, 101, 0011, 1010\}$ then $A_{\frac{1}{2}} = \{1, 00, 10\}$ (Contributions from 10, 0011, and 1010; odd length strings of A do not contribute to $A_{\frac{1}{2}}$.

a. **(Difficult)** Show that if A is regular, then $A_{\frac{1}{2}}$ is also regular. (Hint: Make use of the sets Q_j as introduced in the previous problem.)

b. What is $A_{\frac{1}{2}}$ if A is the set of palindromes; i.e. = $\{w = w^R \mid w \in \{0,1\}^*\}$? Why ?

c. Is the converse of (a) true? (i.e. If $A_{\frac{1}{2}}$ is regular, then A is also regular.) Why ?

Problem 6.(20 Points)

Let $G=(V, \Sigma, R, S)$ be a CFG. A variable $A \in V$ is said to be **useful** if there is at least one $w \in L(G)$ such that:

$$S \Rightarrow \ldots \Rightarrow x\,A\,y \Rightarrow .. \Rightarrow w$$

where $x, y \in (V \cup \Sigma)^*$. In words a variable is useful if it occurs in at least one derivation. A variable that is not useful is called **useless**. A production rule is **useless** if it involves any useless variable. Thus, a variable is useless because either there is no way of getting a terminal string from it or because there is no way it may be reached in a derivation starting at *S (or both)*.

Consider the Grammar

$S \rightarrow aA \mid aBB$
$A \rightarrow aaA \mid \varepsilon$
$B \rightarrow bB \mid bbC$
$C \rightarrow B$

a. Are there useless variables in the grammar as given ? What are they and why ?

b. Based on your answer in the previous part, give an equivalent grammar to the given one, with no useless variables.

c. What is the language generated by the given grammar.

d. Suggest a general algorithm for removing useless variables and their associated production rules, to convert a given grammar to an equivalent grammar without useless variables.

Problem 7 (25 Points)

Consider the following implementation level description of a Turing machine M_1

M_1 = " On input string w:

1. Scan the tape and mark the first 0 which has not been marked. If no unmarked 0 is found, go to stage 4. Otherwise move the head back to the front of the tape.
2. Scan the tape and mark the first 1 which has not been marked. If no unmarked 1 is found, ***reject***
3. Move the head back to the front of the tape and go to stage 1.
4. Move the head back to the front of the tape. Scan the tape to see if any unmarked 1s remain. If none are found then ***accept***; else ***reject*** "

a. Argue that M_1 is a decider

b. What is the language, A_1 decided by M_1? Give a brief explanation

c. Based on (b), what is the smallest k for which $A_1 \in$ TIME(n^k). Why ?

d. Give a state diagram that represents the machine M_1.

e. Suppose we were to decide A_1 using a two-tape Turing machine. How can you improve on the complexity obtained in (c) ?

Problem 8. (25 Points)

Consider the languages

$$A = \{a^n b^n c^m \mid n, m \geq 0\}$$
$$B = \{a^n b^n c^n \mid n \geq 0\}$$

a. Show that A is context free by giving the state diagram of a PDA that recognizes A

b. Give a CFG that generates A

c. Show that B is not context free by using the pumping theorem for CFLs.

d. Consider

CFL_{TM} = { <*M*> | *M* is a Turing machine, & $L(M)$ is context free }

Is < M_1> (of Problem 7 above) in CFL_{TM} ? Why ?

Is the empty string ε in CFL_{TM} ? Why ?

e. Given a Turing machine *M* and a string *w*, consider the Turing machine

M_2 = " On input *x*

1. If *x* is ***not*** of the form $a^n b^n c^m$ then ***reject***
2. If $x = a^n b^n c^n$ for some *n*, then ***accept***
3. Else run *M* on *w*. If *M* accepts then ***accept*** If *M* rejects then ***reject***

Complete the following and explain your answer:

$L(M_2) = \begin{cases} \quad & \text{if } M \text{ accepts } w \\ \quad & \text{if } M \text{ does not accept } w \end{cases}$

Problem 10 (15 Points)

a. Using the previous problem above or otherwise, show that CFL_{TM} is undecidable.

b. What is the mapping reduction f underlying your argument above.