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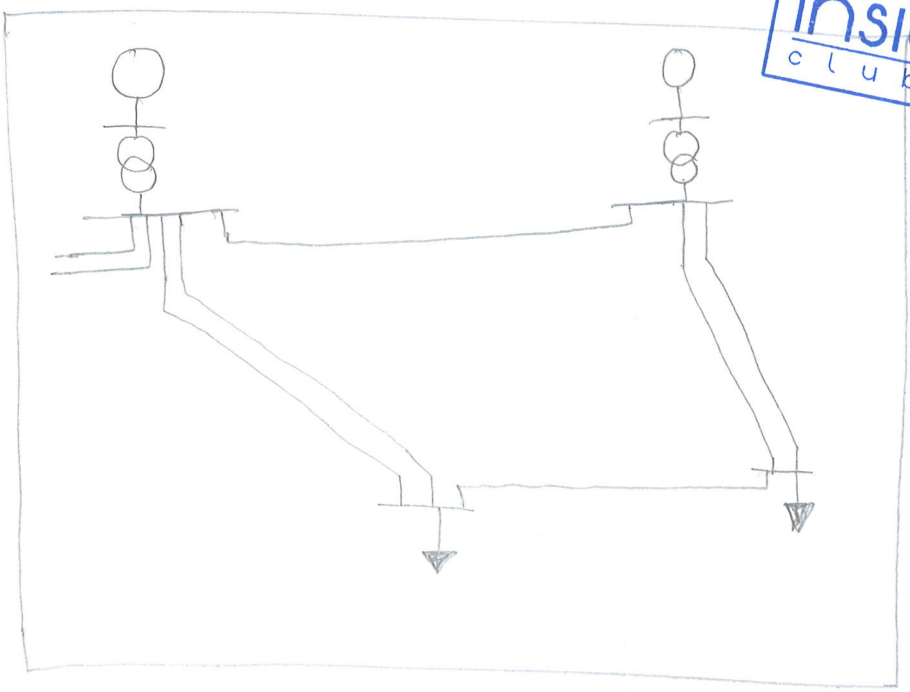
Chapter 6: Power Flows

- Calculation tool to assess the operation of a power system that is being built or operated.



- The power flow problem specifies
  - system parameters (resistance, reactance, susceptance of TL, transformers).
  - Loads (demands of active & reactive power at node).
  - The active power produced by generators, except for one generator. (reference or slack generator).
    - ↳ it cannot be fixed because it compensates for the power losses in the system.

- Outputs:
  - Voltages and phase angles
  - Line Flows
  - Generator Reactive power supply (or absorption)

- pf initialization
  - planning: used to measure (access) the adequacy of a plan
  - Operation: used to assess the effect of possible contingency on network
  - Methods of Solution: - Gauss Seidel (older method)
    - Newton-Raphson (more modern method), plus its variations

6.1: Solution to linear Equations

Example 6.4:
- $10x_1 + 5x_2 = 6$
- $2x_1 + 9x_2 = 3$

works for diagonally dominant matrices

$$x_{ii} > \sum_{j \in N_i} x_{ij} \quad \text{(sufficient condition for convergence)}$$

N_i : set of nodes adjacent to i.

$x_1 = 0$, and $x_2 = 0$ (initialization)

iterations: $x_1 = \frac{1}{10}(6 - 5x_2) = 0.6$

$x_2 = \frac{1}{9}(3 - 2x_1) = 0.2$

Iterations	0	1	2	-----	6
x_1	0	0.6	0.5	------	0.4875
x_2	0	0.2	0.22	------	0.2250

Example 6.5: Divergence of Gauss-Seidel

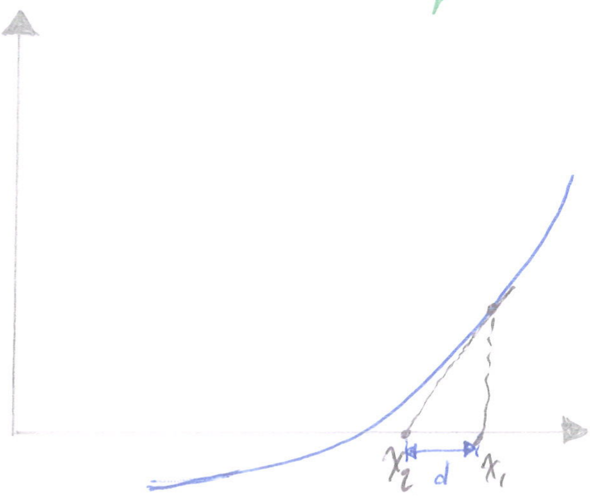
$$\begin{bmatrix} 5 & 10 \\ 9 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$x_1 = \frac{1}{5}(6 - 10x_2)$$

$$x_2 = \frac{1}{2}(3 - 9x_1)$$

insight club

6.2: Iterative Solution to Non-linear Equations using Newton-Raphson Method



$$x_2 = x_1 - \underbrace{[f'(x_1)]^{-1} \cdot f(x_1)}_{d}$$

$$f(x) = f(x_0) + f'(x_0)\cdot(x - x_0) + \frac{f''(x_0)}{2}\cdot(x - x_0)^2 + \ldots$$

$$x_1 = V;$$

$$x_{r+1} = x_r \cdot [f'(x_r)]^{-1} \cdot f(x_r)$$

- Sufficient conditions for convergence is the convexity of $f(x)$ in the neighborhood of solution.

$$f(x) = 0$$

$$x_{r+1} = x_r - [f'(x_r)]^{-1} \cdot f(x_r)$$

$$f'(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & & & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

Jacobian Matrix

Ex 6.7

$$x_1 + x_2 = 15$$
$$x_1 \cdot x_2 = 50$$

$$f_1(x_1, x_2) = x_1 + x_2 - 15 = 0$$
$$f_2(x_1, x_2) = x_1 \cdot x_2 - 50 = 0$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_{r+1} = x_r - [f'(x_r)]^{-1} \cdot f(x_r)$$

$$J = f'(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ x_2 & x_1 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 4 \\ 9 \end{bmatrix};$$

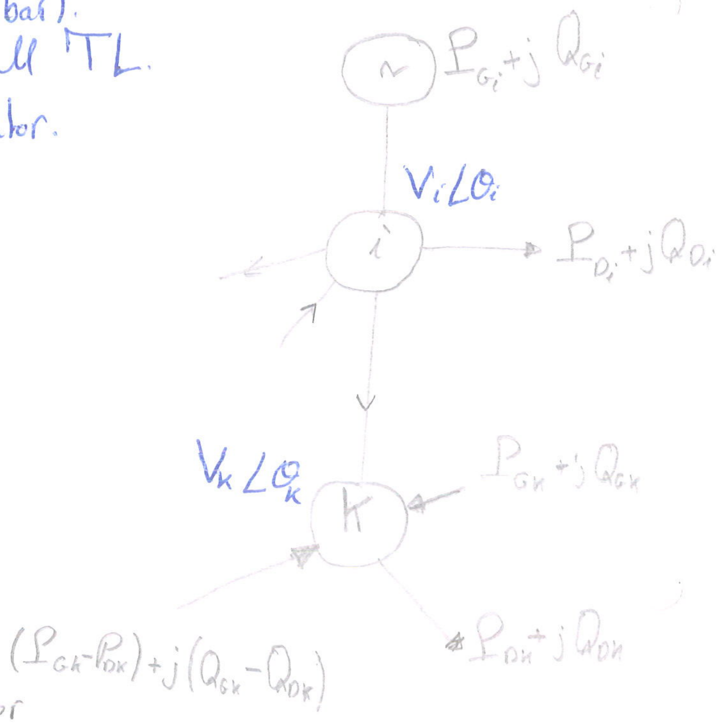
i	$x_1(i)$	$x_2(i)$
0	4	9
1	5.2	9.8
2	4.99	10.0087
3	4.99998	10.00002
4	5.0 ...	10.0 ...

6.4: The Power Flow Problem

* Starting point is the single line diagram
* Input data:
  - Active and reactive demands are specified
  - Transmission lines & transformer data. (reactances, resistors,
  - Parameters of Shunt reactors, capacitors or other series elements.
* Specify active power of all generators except the slack (reference) and voltage magnitude of all generator busses including the slack.

Output Result

- Voltage magnitudes and phase angles at all nodes (fundamental results)
- Active & reactive power produced by the slack generator. (at the slack busbar).
- Active and reactive flows in all TL.
- reactive power produced by generator.



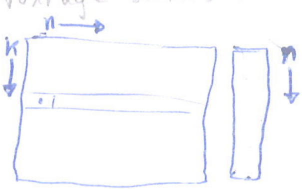
* Alternative Method of Modelling a Generator
  - → Specify active power and reactive power (case I)
  - → Specify voltage magnitudes (case II)

* Nodal Equations:

$$I = Y \cdot V$$

* matrix
* All voltage sources are converted to current sources

$$I_k = \sum_{n=1}^{N} Y_{kn} \cdot V_n =$$



Complex Power delivered to network

$$S_K = P_K + jQ_K = V_K \cdot I_K^* = V_K \cdot \left[\sum_{n=1}^{N} Y_{Kn}^* \cdot V_n^* \right] \quad (1)$$

power flow going into network

Express in terms of real and imaginary terms:

$$V_K \triangleq |V_K| \angle \delta_K$$

$$V_n \triangleq |V_n| \angle \delta_n = |V_n| \cdot (\cos\delta_n + j\sin\delta_n)$$

$$Y_{Kn} = G_{Kn} + jB_{Kn}$$

$$P_K = V_K \cdot \sum_{n=1}^{N} V_n \left[G_{Kn} \cdot \cos(\delta_K - \delta_n) + B_{Kn} \cdot \sin(\delta_K - \delta_n) \right]$$

$$Q_K = V_K \sum_{n=1}^{N} V_n \cdot \left[G_{Kn} \cdot \sin(\delta_K - \delta_n) + B_{Kn} \cdot \cos(\delta_K - \delta_n) \right]$$

Nodal Power Equation

$$P_{GK} - P_{DK} = P_K \Rightarrow$$

$$\Delta P_K = P_K - \underbrace{P_{GK} + P_{DK}}_{\text{specified injection}} = 0$$

active power mismatch at node K

power going into network

$$\Delta Q_K = Q_K - Q_{GK} + Q_{DK} = 0$$

reactive power mismatch

Power Flow Equations

$$P_K + jQ_K = V_K \cdot \left(\sum_{n=1}^{N} Y_{Kn}^* \cdot V_n^* \right) \quad (1)$$

$$S_K + S_D - S_{GK} = 0 \quad (2)$$

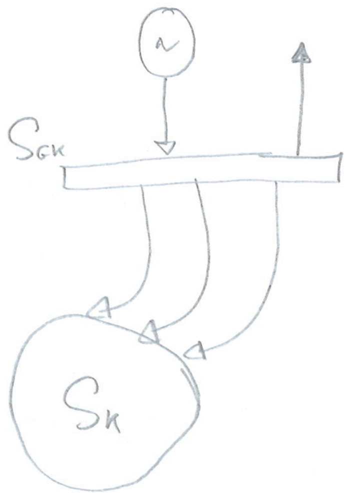
$K: 2 \to N$
except for $K=1$ (slack node)

$$V_K = |V_K| \cdot e^{j\delta_K}$$

$$Y_{Kn} = G_{Kn} + jB_{Kn}$$

$$P_K = V_K \cdot \sum V_n \left(G_{Kn} \cdot \cos(\delta_K - \delta_n) + B_{Kn} \cdot \sin(\delta_K - \delta_n) \right) \quad (3)$$

$$Q_K = V_K \cdot \sum V_n \left(G_{Kn} \cdot \sin(\delta_K - \delta_n) + B_{Kn} \cdot \sin(\delta_K - \delta_n) \right) \quad (4)$$



* Active and Reactive Power Mismatch

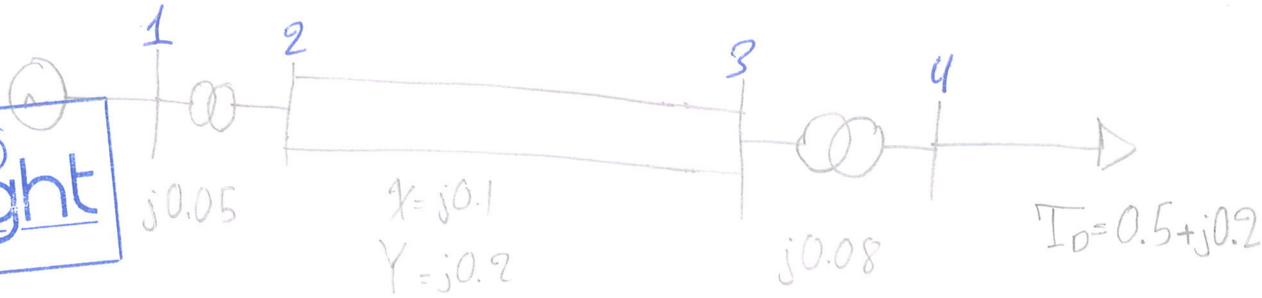
$$\Delta P_k = P_k + P_{Dk} - P_{Gk} = 0 \quad (5)$$

$$\Delta Q_k = Q_k + Q_{Dk} - Q_{Gk} = 0 \quad (6)$$

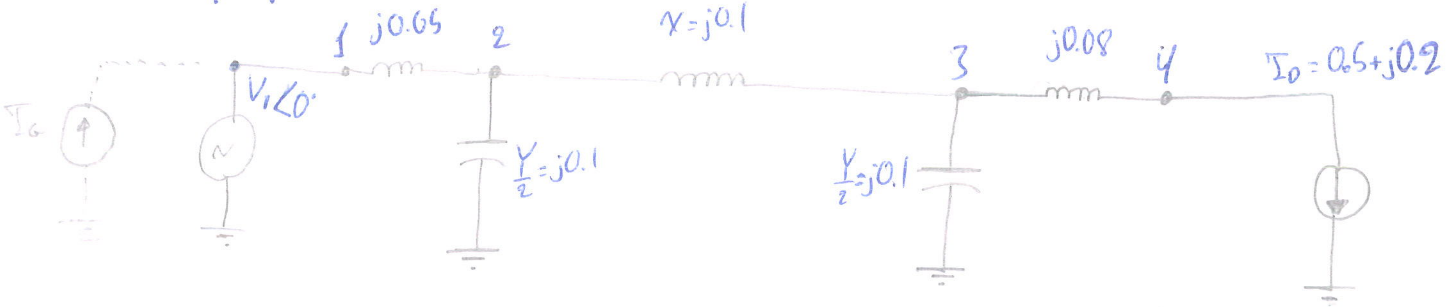
- In Gauss Seidel, we solve (2) after replacing S_k by its value given in (1)
- In N-R method, we solve (5) & (6) after replacing P_k and Q_k by their values given in (3) & (4)

2.4: Network Equations

- Nodal equations of a simple system:



$I = Y \cdot V$



1. $\dfrac{V_1 - V_2}{j0.05} = I_G$

2. $\dfrac{V_2 - V_1}{j0.05} + V_2(j0.1) + \dfrac{V_2 - V_3}{j0.1} = 0$

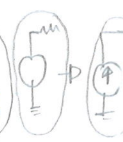
3. $\dfrac{V_3 - V_2}{j0.1} + V_3(j0.1) + \dfrac{V_3 - V_4}{j0.08} = 0$

4. $\dfrac{V_4 - V_3}{j0.08} = -(0.5 + j0.2)$

Rearrange:

equivalent to current source of G

$$+\left(\frac{1}{j0.05}+j0.1+\frac{1}{j0.1}\right)V_2+\quad -\frac{1}{j0.1}V_3\quad+\quad 0\cdot V_4\quad=\frac{V_1}{j0.05}$$



$$-\frac{1}{j0.1}V_2\quad+\left(\frac{1}{j0.1}+j0.1+\frac{1}{j0.08}\right)V_3\quad+\left(-\frac{1}{j0.08}\right)V_4\quad=0$$

$$0\cdot V_2\quad+\quad\left(-\frac{1}{j0.08}\right)V_3\quad+\quad\frac{1}{j0.08}V_4\quad=\text{[illegible]}$$

$$\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\ \hline
1 & \times & \times & \times & \times \\
2 & \times & \frac{1}{j0.05}+j0.1+\frac{1}{j0.1} & -\frac{1}{j0.1} & 0 \\
3 & \times & -\frac{1}{j0.1} & \frac{1}{j0.1}+j0.1+\frac{1}{j0.08} & -\frac{1}{j0.08} \\
4 & \times & 0 & -\frac{1}{j0.08} & \frac{1}{j0.08}
\end{array}
\cdot
\begin{array}{|c|}\hline V1 \\ \hline V2 \\ V3 \\ V4 \\ \hline\end{array}
=
\begin{array}{|c|}\hline \times \\ \hline \frac{V_1}{j0.05} \\ 0 \\ -(0.5+j0.2) \\ \hline\end{array}$$

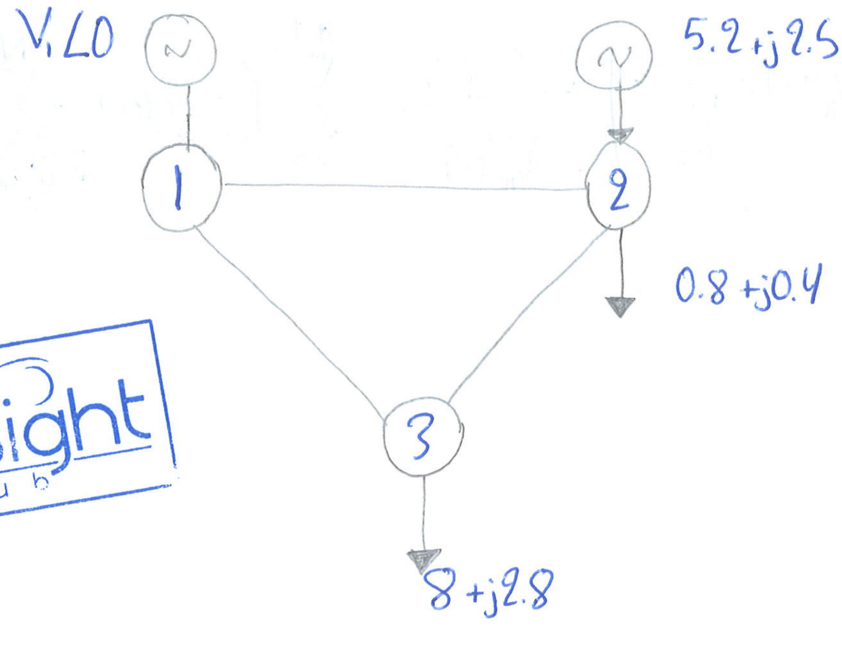
$$Y \cdot V = I$$

Exercise: Write the nodal Equation of the following Network:

2-3: $Z_s = 0.01 + j0.1$;

1-3: $Z_s = 0.01 + j0.1$;

1-2: $Z_s = 0.02 + j0.2$.



Summary:

Nodal Admittance Matrix:

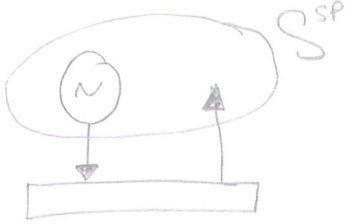
- Diagonal Element: Y_{ii} = the sum of all admittance connected to node i.

- off-Diagonal Element: Y_{ij} = The <u>NEGATIVE</u> of the admittance connecting nodes i & j

  : $Y_{ij}=0$ when there is no connection between i & j

2.5: Power Flow Solution by Gauss Seidel:

$$\underbrace{V_k \cdot \left(\sum_{n=1}^{N} Y_{kn}^{*} \cdot V_n^{*}\right)}_{S_k} + \underbrace{S_{Dk} - S_{Gk}}_{-S_k^{SP}} = 0$$



$$\frac{1}{V_k^{*}} \cdot \left[V_k \cdot Y_{kk}^{*} \cdot V_k^{*} + V_k \cdot \sum_{n \neq k} Y_{kn}^{*} \cdot V_n^{*} = S_k^{SP} \right]^{*} \qquad K = 1 \ldots N$$

- take conjugates
- divide by V_n^{*}
- rearrange

$$V_k^{(r+1)} = \frac{1}{Y_{kk}} \cdot \left[\frac{S_k^{SP*}}{V_k^{*(r)}} - \sum_{n \neq k} Y_{kn} \cdot V_n^{(r)} \right] \qquad K = 2 \ldots N$$

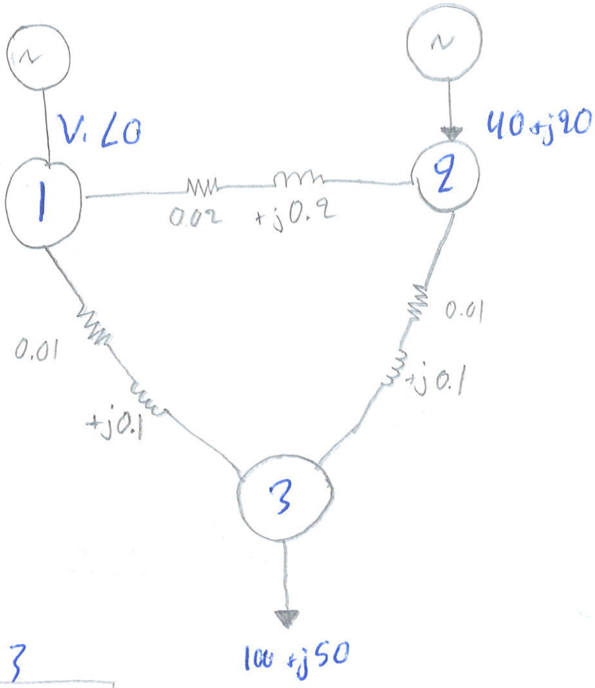
$$V_k(r+1) = \frac{1}{Y_{kk}} \cdot \left[\frac{P_k^{SP} - jQ_k^{SP}}{V_k^{*}(r)} - \sum_{n \neq k}^{k-1} Y_{kn} \cdot V_n(r+1) - \sum_{n=k+1}^{N} Y_{kn} \cdot V_n(r) \right]$$

Node 1 is a slack node!

$$Y_{11} = \frac{1}{0.01+j0.1} + \frac{1}{0.02+j0.2} = 1.485 - j14.85$$

$$Y_{12} = -\frac{1}{0.02+j0.2} = -0.495 + j4.95$$

$$Y_{22} = \frac{1}{0.02+j0.2} + \frac{1}{0.01+j0.1} = 1.485 - j14.85$$



Y

Y =	1	2	3
1	1.485 − j14.85	−0.495+j4.95	−0.9901 +j 9.901
2	−0.495 +j4.95	1.485 −j14.85	−0.9901 +j 9.901
3	−0.9901 +j9.901	−0.99 +j9.901	1.98 −j 19.8

Write the Guass - Seidel Iterative Equations

$$V_k(r+1) = \frac{1}{Y_{kk}}\left[\frac{P_k^{sp} - jQ^{sp}}{V_k^*(r)} - \sum_{n=1}^{k-1} Y_{kn} \cdot V_n(r+1) - \sum_{n=k+1}^{N} Y_{kn} \cdot V_n(r)\right]$$

$$V_2(r+1) = \frac{1}{Y_{22}}\left[\frac{0.4 - j0.2}{V_2^*(r)} - Y_{21} \cdot V_1 - Y_{23} \cdot V_3(r)\right]$$

$$V_3(r+1) = \frac{1}{Y_{33}}\left[-\frac{(1 - j0.5)}{V_3^*(r)} - Y_{31} \cdot V_1 - Y_{32} \cdot V_2(r)\right]$$

Demand

- $V_1 = 1\angle 0$
- $V_2(0) = 1\angle 0$
- $V_2(0) = 1\angle 0$

* Correction for PV node

$$V_2(r+1) = V_2(r+1) * \frac{|V_2|}{|\tilde{V}_2(r+1)|}$$

- $$\hat{Q}^{sp} = Q = \mathrm{Im}\left(V_2 \cdot \sum_{n=1}^{3} Y_{2n}^{*} \cdot V_n^{*}\right)$$

$$S_K = P_K + jQ_K = V_K \sum_{n=1}^{N} Y_{Kn}^{*} \cdot V_n^{*}$$

Exercise on using G.S for Power Flow Calculation:

- Take the network of H.W #1

- Data given

a) Form Y matrix

b) Write Iterative Equations

- 3 iterative equations
- plug in the numbers
- carry out the first iteration (& 2nd iteration)

No generation
No demand

$$V_2(r+1) = \frac{1}{Y_{22}}\left[\frac{\overset{0}{\cancel{P_2^{sp}}} - j\overset{+j0}{\cancel{Q_2^{sp}}}}{V_2^{*}(r)} - \cdots\right]$$

6.6: Power Flow using Newton-Raphson's Method

$f(x)=0$

$$x_{r+1} = x_r - [J'(x_r)]^{-1} \cdot f(x)$$

- When applied to the Power Flow Problem:

$$x = \begin{bmatrix} \delta \\ V \end{bmatrix} \begin{matrix} 2 \\ \vdots \\ N \\ 2 \\ \vdots \\ N \end{matrix} \Bigg\} 2(N-1)$$

$$f(x) = \begin{array}{c} \begin{bmatrix} \Delta P(\delta, V) \\ \\ \Delta Q(\delta, V) \end{bmatrix} \end{array} \begin{array}{c} 2 \\ \vdots \\ N \\ 2 \\ \vdots \\ N \end{array}$$

$$J = \frac{\partial f}{\partial x} = \left[\begin{array}{ccc|ccc} \frac{\partial \Delta P_2}{\partial \delta_2} & \cdots & \frac{\partial \Delta P_2}{\partial \delta_N} & \frac{\partial \Delta P_2}{\partial V_2} & \cdots & \frac{\partial \Delta P_2}{\partial V_N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \Delta P_N}{\partial \delta_2} & \cdots & \frac{\partial \Delta P_N}{\partial \delta_N} & \frac{\partial \Delta P_N}{\partial V_2} & \cdots & \frac{\partial \Delta P_N}{\partial V_N} \\ \hline \frac{\partial \Delta Q_2}{\partial \delta_2} & \cdots & \frac{\partial \Delta Q_2}{\partial \delta_N} & \frac{\partial \Delta Q_2}{\partial V_2} & \cdots & \frac{\partial \Delta Q_2}{\partial V_N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ \frac{\partial \Delta Q_N}{\partial \delta_2} & \cdots & \frac{\partial \Delta Q_N}{\partial \delta_N} & \frac{\partial \Delta Q_N}{\partial V_2} & \cdots & \frac{\partial \Delta Q_N}{\partial V_N} \end{array}\right]$$

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$P^{sp} = P_{GK} - P_{DK}$

P_K

$$* \; \Delta P_K = \underbrace{\sum_{n=1}^{N} V_K \cdot V_n \left[G_{Kn} \cdot \cos(\delta_K - \delta_n) + B_{Kn} \cdot \sin(\delta_K - \delta_n)\right]}_{P_{\text{going to Network.}}} - (P_{GK} - P_{DK}) = 0$$

$k = 2 --- N$

$$* \; \Delta Q_K = \sum_{n=1}^{N} V_K \cdot V_n \left[G_{Kn} \cdot \sin(\delta_K - \delta_n) + B_{Kn} \cdot \cos(\delta_K - \delta_n)\right] - (Q_{GK} - Q_{DK}) = 0$$

$n \neq k$ since

$\cos(\delta_K - \delta_n) = 0^{1}$

$\sin(\delta_K - \delta_n) = 0$

$$\frac{\partial \Delta P_K}{\partial \delta_K} = J_1(k,k) = \sum_{n \neq k} V_K V_n \left[-G_{Kn} \cdot \sin(\delta_K - \delta_n) + B_{Kn} \cdot \cos(\delta_K - \delta_n)\right]$$

comes from the special form in ΔP_K when $n = k$

$$\frac{\partial \Delta P_K}{\partial V_K} = J_2(k,k) = \sum_{n \neq k} V_n \left[G_{Kn} \cdot \cos(\delta_K - \delta_n) + B_{Kn} \cdot \sin(\delta_K - \delta_n)\right] + 2 V_K G_{KK}$$

$V_n^2 G_{KK}$

$$\frac{\partial \Delta Q_K}{\partial \delta_K} = J_3(k,k) = \sum_{n \neq k} V_K V_n \left[G_{Kn} \cdot \cos(\delta_K - \delta_n) + B_{Kn} \cdot \sin(\delta_K - \delta_n)\right]$$

coming from the special form

$$\frac{\partial \Delta Q_K}{\partial V_K} = J_4(k,k) = \sum_{n \neq k} V_n \left[G_{Kn} \cdot \sin(\delta_K - \delta_n) - B_{Kn} \cdot \cos(\delta_K - \delta_n)\right] - 2 V_K \cdot B_{KK}$$

when $n = k$

$(-V_K^2 \cdot B_{KK})$

$$J_1(k,n) = \frac{\partial \Delta P_k}{\partial \delta_n} = V_k V_n \left[G_{kn} \cdot \sin(\delta_k - \delta_n) - B_{kn} \cdot \cos(\delta_k - \delta_n) \right]$$

$$J_2(k,n) = \frac{\partial \Delta P_k}{\partial V_n} = V_k \left[G_{kn} \cdot \cos(\delta_k - \delta_n) + B_{kn} \cdot \sin(\delta_k - \delta_n) \right]$$

$$J_3(k,n) = \frac{\partial \Delta Q_k}{\partial \delta_n} = -V_k V_n \left[G_{kn} \cdot \cos(\delta_k - \delta_n) + B_{kn} \cdot \sin(\delta_k - \delta_n) \right]$$

$$J_4(k,n) = \frac{\partial \Delta Q_k}{\partial V_n} = V_k \left[G_{kn} \cdot \sin(\delta_k - \delta_n) - B_{kn} \cdot \cos(\delta_k - \delta_n) \right]$$

very small

N.B.: $P = \frac{V_1 V_2}{X} \cdot \sin(\delta_1 - \delta_2)$

dominant

over

$\cos\left(\text{very small}\right)$

$$\frac{\partial P}{\partial \theta} \approx \frac{V_1 V_2}{X} \cdot 1$$

$\sin\left(\text{very small}\right)$

$$\frac{\partial P}{\partial V} \approx \frac{V}{X} \cdot 0.0\ldots1$$

$$\begin{bmatrix} \theta_{r+1} \\ \hline V_{r+1} \end{bmatrix} = \begin{bmatrix} \theta_r \\ \hline V_r \end{bmatrix} - \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1}_{(\theta_r, V_r)} \cdot \begin{bmatrix} \Delta P(\theta_r, V_r) \\ \hline \Delta Q(\theta_r, V_r) \end{bmatrix}$$

has to be calculated exactly as given in expressions. Because, they represent KCL at node K

Fast Decoupled Load Flow

. Newton-Raphson Power Flow

$$\begin{bmatrix} \theta_{r+1} \\ \hline V_{r+1} \end{bmatrix} = \begin{bmatrix} \theta_r \\ \hline V_r \end{bmatrix} - \underbrace{\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P(\theta_r, V_r) \\ \hline \Delta Q(\theta_r, V_r) \end{bmatrix}}_{\begin{bmatrix} \Delta \theta_r \\ \Delta V_r \end{bmatrix}}$$

- The calculation matrix J and its inversion at every iteration is a major computational burden of the method.

- This justifies the approximations sought.

- It was noticed through the computations that J_1, J_2, J_3 & J_4 don't significantly vary. (consult example in old note, from Bergen & Kittal)

- Stott & others tried successfully to keep them constant.
- Also, noticed (conf. example in old notes) that elements of J_2 are much smaller than elements in $J1$, similarly elements in $J3$ are much smaller than elements in $J4$!

- The idea occured to neglect $J2$ & $J3$!

$$J_1(k,n) = V_k V_n \left[G_{kn} \cdot \sin(\theta_k - \theta_n) - B_{kn} \cdot \cos(\theta_k - \theta_n) \right]$$

$$J_2(k,n) = V_k \left(G_{kn} \cdot \cos(\theta_k - \theta_n) + B_{kn} \cdot \sin(\theta_k - \theta_n) \right]$$

- In a reasonable systems:

$$V_n \approx V_k \approx 1$$

- High X/R (>5)

θ_k & θ_n are small : $\sin(\theta_k - \theta_n)$ small (~ 0)

$\cos(\theta_k - \theta_n) \sim 1$