Chapter 5: Transmission Lines: Steady State Operation.

ABCD constants: Two-port network Theory.

5.1: Short and Medium Transmission Lines

. Model the transmission line by a two port network

. **Two Port Network:**

$$V_s = A \cdot V_R + B I_R$$
$$I_s = C \cdot V_R + D I_R$$

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bilateral

$AD - BC = 1$

for linear Bilateral Element

Short Line ($l \leq 80$ km ~ 50 miles)

For such a line, a series model is used

$X = x \cdot l$ — x: Ω/km, l: km

$R = r \cdot l$ — r: Ω/km, l: km

. Y is neglected for overhead lines (OHL)

. ABCD of a short line:

$$\left.\begin{aligned} V_s &= V_R + Z I_R \\ I_s &= 0 \times V_R + 1 \times I_R \end{aligned}\right\} \Rightarrow \begin{aligned} A &= 1 \\ B &= Z \\ C &= 0 \\ D &= 1 = A \end{aligned}$$

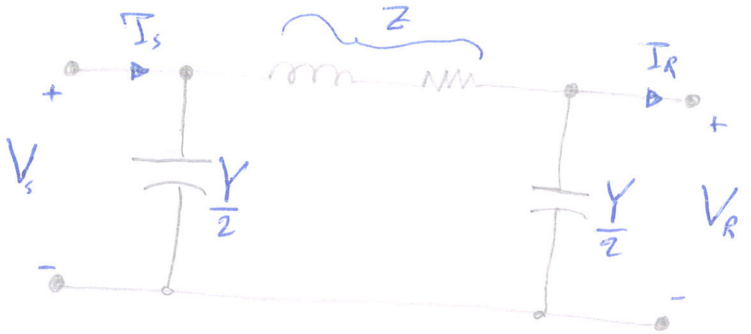
so ; $AD - BC = 1 \Longrightarrow$ this means T^{-1} exists

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}}_{T:\ \text{Transmission Matrix}} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} T^{-1} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

- $S_s = P_s + jQ_s = V_s \cdot I_s^{*}$ in per unit

- A Medium-Length-Line ($50\,km < l < 240\,km$)



π - Equivalent

$$Z = R + jX = (r + ja) \cdot l \,/\, Z_{Base}$$

$$Y = jy\,l = j\omega C L \ (* Z_{Base})$$

- ABCD of a medium Line:

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$$V_s = V_R + z \cdot \left(I_R + V_R \cdot \frac{Y}{2}\right)$$

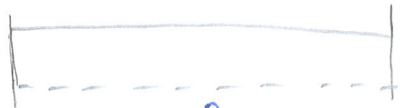
$$= \underbrace{\left(1 + \frac{zY}{2}\right)}_{A} V_R + \underbrace{z}_{B} I_R$$

$$I_s = \left(I_R + \frac{Y}{2} V_R\right) + V_s \frac{Y}{2}$$

$$= \left(I_R + \frac{Y}{2} V_R\right) + \left[\left(1 + \frac{zY}{2}\right) V_R + z I_R\right] \cdot I_R$$

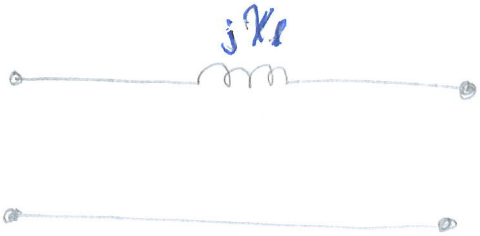
$$= \underbrace{Y\left(1 + \frac{zY}{4}\right)}_{C} V_R + \underbrace{\left(1 + \frac{zY}{2}\right)}_{D} I_R$$

$$\Longrightarrow \begin{cases} A = \left(1 + \frac{zY}{2}\right) \\ B = z \\ C = Y\left(1 + \frac{zY}{4}\right) \\ D = A = \left(1 + \frac{zY}{2}\right) \end{cases}$$



- A doubled line $\left\{ \begin{array}{l} \cdot\ Y \text{ is doubled} \\ \cdot\ Z \text{ is halved} \end{array} \right.$ because they are in parallel.

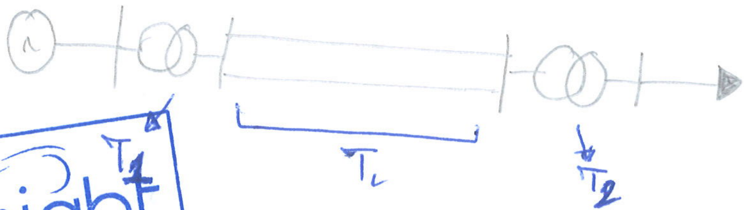
- Simple Transformer Model

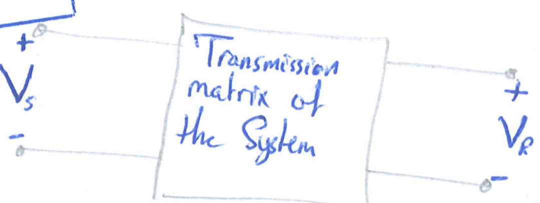


$R = 0$
$Y = 0$

$$A = D = 1$$
$$B = Z$$
$$C = 0$$

- System Matrix





$$T_S = [T_1] \cdot [T_L] \cdot [T_2]$$

- Voltage Regulation

$$VR = \frac{V_{R(no\ load)} - V_{R(full\ load)}}{V_{R(full\ load)}} \times 100$$

at no load:

$I_R = 0$

$V_s = A \cdot V_{R(no\ load)}$

$\rightarrow V_{R(no\ load)} = \frac{V_s}{A}$

Exercise: Take data from Assignment 1.

Calculate impedences in P.U.

Form a transmission matrix of the whole system ($T_1 - T_L - T_2$)

Consider the voltage on receiving end is 0.95 p.u. ($V_R = 0.95 \angle 0°$)

For the given load data, Calculate the voltage and current at sending end.

Deduce the active & reactive power delivered by generator.

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2

Example 5.1: ABCD parameters of a medium Line

345 KV, 200 Km, 2 * 795000 cmil

$z = 0.032 + j0.35 \ \Omega/Km$

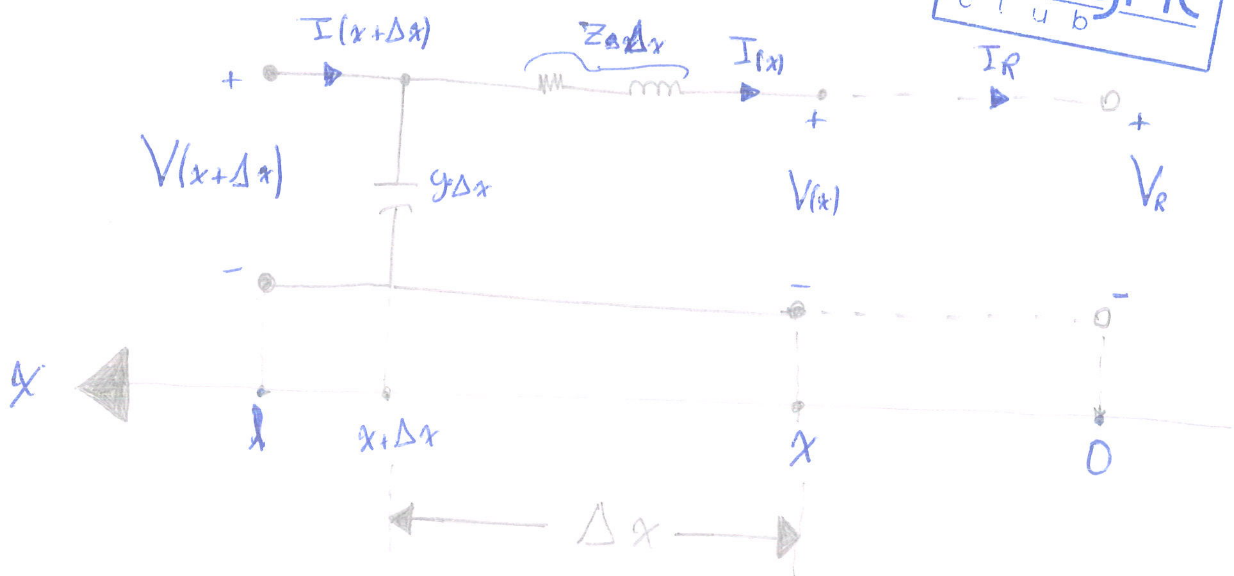
$y = j4.2 \times 10^{-6} \ S/Km$

$Z_{Base} = \frac{345^2}{100} = 1190.25 \ \Omega$

700 MW @ 0.99 pf loading ; $V_R = 0.95$

- $Z = zl = 70.29 \angle 84.8° \longrightarrow Z_{p.u.} = \frac{70.79}{1190.25} \angle 84.8°$
- $Y = yl = 840 \times 10^{-6} \angle 90° \longrightarrow Y_{pu} = 840 \times 10^{-6} * 1190.95 \angle 90°$
- $A = \left(1 + \frac{YZ}{2}\right) = D = 0.9706 \angle 0.159°$

5.2: Transmission Line Differential Equations.



- $z = r + j\omega l \ \Omega/m = r + jx \ \Omega/m$
- $y = j\omega C \ S/m$
- $V(x+\Delta x) = V(x) + z \cdot \Delta x \cdot I(x)$

$$\hookrightarrow \frac{V(x+\Delta x) - V_x}{\Delta x} = z \cdot I(x)$$

as $\Delta x \longrightarrow 0$; $\boxed{\frac{dV(x)}{dx} = z I(x)}$

KCL: $I(x+\Delta x) = I(x) + y \cdot \Delta x \cdot V(x+\Delta x)$

$$\rightarrow \frac{I(x+\Delta x) - I(x)}{\Delta x} = y \cdot V(x+\Delta x)$$

$$\boxed{\frac{d\,I(x)}{dx} = y \cdot V(x)}$$

$$\frac{d^2 V(x)}{dx^2} = z \cdot \frac{d\,I(x)}{dx} = z \cdot y \cdot V(x)$$

$$\boxed{\frac{d^2 V(x)}{dx^2} = z \cdot y \cdot V(x)}$$

Solution is an exponential function:

$$V(x) = A_1 \cdot e^{\gamma x} + A_2 \cdot e^{-\gamma x}$$

where $\gamma = \sqrt{yz}$ $\rightarrow$ propagation constant (m^{-1})

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$$\gamma = \sqrt{j0.3 \times j3.3 \times 10^{-6}} = \sqrt{j^2 \times 0.3 \times 3.3 \times 10^{-6}} = j10^{-3}\ m^{-1}$$

$$= j\beta$$

- Replace $V(x)$ in D.E. of $V(x)$.

$$I(x) = \frac{A_1 \cdot e^{\gamma x} - A_2 \cdot e^{-\gamma x}}{z/\gamma}$$

- $z/\gamma = \sqrt{\frac{z^2}{\sqrt{yz}}} = \sqrt{\frac{z}{y}} = Z_c$ characteristic Impedance

$$\approx \sqrt{\frac{j0.3}{j3.3 \times 10^{-6}}} = \sqrt{0.1 \times 10^{6}} \approx 300\ \Omega$$

KV	Z_c
150	350 Ω
220	310 Ω
275	300 Ω
500	275 Ω
750	250 Ω

if $x=0$; $V(x=0)=V_R$

$$\left.\begin{aligned} V_R &= A_1 + A_2 \\ I_R &= \frac{A_1 - A_2}{Z_c} \end{aligned}\right\} \rightarrow \quad A_1 = \frac{V_R + Z_c \cdot I_R}{2}, \quad A_2 = \frac{V_R - Z_c \cdot I_R}{2}$$

$$\bullet\ V(x) = \left(\frac{V_R + Z_c \cdot I_R}{2} \cdot e^{\gamma x}\right) + \left(\frac{V_R - Z_c \cdot I_R}{2} \cdot e^{-\gamma x}\right)$$

$$\rightarrow \boxed{V(x) = \underbrace{\left(\frac{e^{\gamma x} + e^{-\gamma x}}{2}\right)}_{\cosh(\gamma x)} \cdot V_R + \underbrace{\left(\frac{e^{\gamma x} - e^{-\gamma x}}{2}\right)}_{\sinh(\gamma x)} \cdot Z_c \cdot I_R}$$

So, $\boxed{I(x) = \frac{1}{Z_c} \cdot \sinh(\gamma x) \cdot V_R + \cosh(\gamma x) \cdot I_R}$

$$V(l) = \overbrace{\cosh(\gamma l)}^{A} \cdot V_R + \overbrace{Z_c \cdot \sinh(\gamma l)}^{B} \cdot I_R$$

$$I(l) = \underbrace{\frac{1}{Z_c} \cdot \sinh(\gamma l)}_{C} \cdot V_R + \underbrace{\cosh(\gamma l)}_{D} \cdot I_R$$

$$* \ AD - BC = \cosh^2(\gamma l) - \sinh^2(\gamma l) = 1 \ \checkmark$$

$$\cosh(\alpha l + j\beta l) = \cosh(\alpha l) \cdot \cos(\beta l) + j \sinh(\alpha l) \cdot \sin(\beta l)$$

$$\sinh(\alpha l + j\beta l) = \sinh(\alpha l) \cdot \cos(\beta l) + j \cosh(\alpha l) \cdot \sin(\beta l)$$

$$V_s = \cosh(\gamma l) \cdot V_R + Z_c \cdot \sinh(\gamma l) \cdot I_R$$

$$I_s = \frac{1}{Z_c} \cdot \sinh(\gamma l) \cdot V_R + \cosh(\gamma l) \cdot I_R$$

$$\gamma = \sqrt{yz} = \underbrace{\alpha}_{\text{loss coefficient}} + j\overbrace{\beta}^{\text{phase coefficient}}$$

(Propagation Constant)

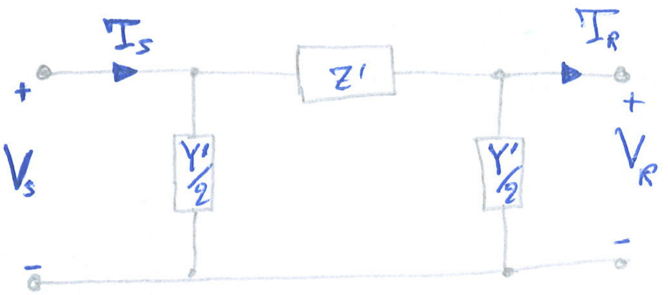
5.3: Equivalent π - Circuits.

$$Z = z \cdot l = (r + jx) \cdot l = R + jX$$

$$Y = yl = j\omega l C = jB$$

$$V_s = \left(1 + \frac{Y' \cdot Z'}{2}\right) \cdot V_R + Z' \cdot I_R$$

$$I_s = Y'\left(1 + \frac{Y' \cdot Z'}{4}\right) \cdot V_R + \left(1 + \frac{Y' \cdot Z'}{2}\right) \cdot I_R$$



- $\cosh(\gamma l) = 1 + \frac{Y'Z'}{2}$; $Z_c \cdot \sinh(\gamma l) = Z'$

- $Z' = \frac{Z * \sinh(\gamma l)}{\gamma l}$

- $\frac{Y'}{2} = \frac{Y}{2} * \frac{\tanh(\gamma l/2)}{(\gamma l/2)}$

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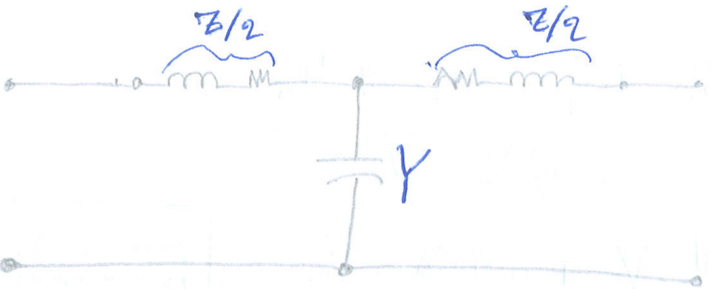
Exercise:



T- Model

- Derive the ABCD constants of a T-equivalent in terms of Y & Z

5.4: Loss-less Lines:

- $Z_c = \sqrt{\frac{Z}{y}} = \sqrt{\frac{\cancel{r}^{\,0} + j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{l \times \cancel{length}}{C \times \cancel{length}}} = \sqrt{\frac{l}{c}}$

Surge impedance

- $\gamma = \sqrt{yz} = \sqrt{(j\omega C)(j\omega L)} = j\omega \underbrace{\sqrt{LC}}_{B} = \cancel{\alpha}^{\,0} + j\beta$

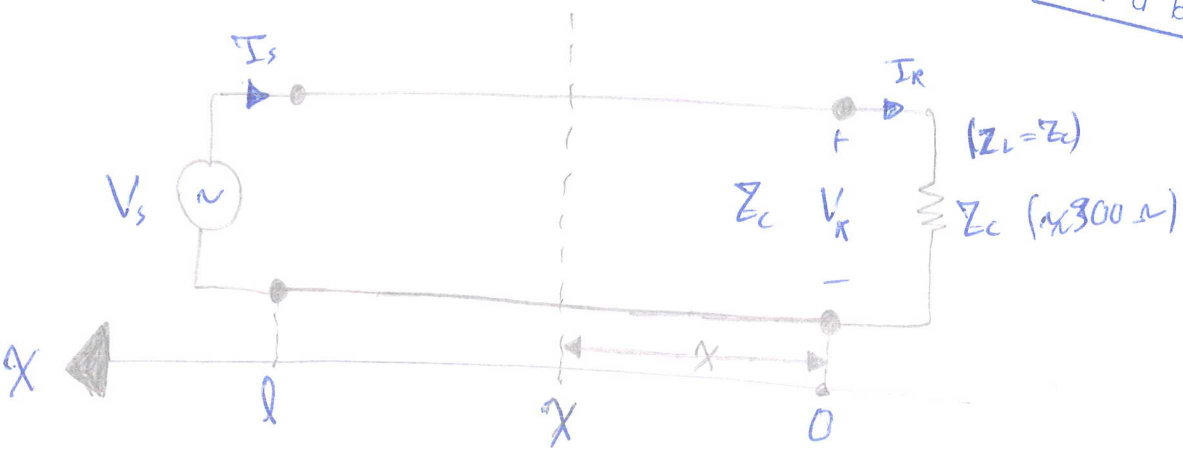
* $A = \cosh(\gamma l) = \cosh(j\beta l) = \cos(\beta l) = D$
* $B = Z_c \sinh(\gamma l) = j \cdot Z_c \cdot \sin(\beta l)$
* $C = \dfrac{\sinh(\gamma l)}{Z_c} = j\dfrac{\sin(\beta l)}{Z_c}$

. wavelength: distance to change the angle by 2π

So, $\beta\lambda = 2\pi$

$$\hookrightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{2\pi}{2\pi f\sqrt{LC}} = \frac{1}{f\sqrt{LC}}$$

$$\begin{cases} @\ f = 50\,Hz\ ;\ \lambda = 6324\ km \\ @\ f = 60\,Hz\ ;\ \lambda \approx 5000\,km \end{cases}$$



* $V(x) = \cos(\beta x) \cdot V_R + jZ_c \cdot \sin(\beta x) \cdot I_R$ $\quad \frac{V_R}{Z_c}$

$$= (\cos(\beta x) + j\sin(\beta x)) \cdot V_R = V_R \cdot e^{j\beta x}$$

* $I(x) = j\dfrac{\sin(\beta x)}{Z_c} \cdot V_R + \cos(\beta x) \cdot \dfrac{V_R}{Z_c}$

$$= \frac{V_R}{Z_c}\left[e^{j\beta x}\right]$$

* Steady-State Stability Limit,

$$P_{max} = V_s \cdot V_R \cdot P_{SIL} \cdot \frac{\sin(\delta_{max})}{\sin(\beta l)}$$

5.4

Missing +

27/10/2014

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5.5: Maximum Power Flow

- $V_R = |V_R| \angle 0°$
- $V_s = |V_s| \angle \delta$
- $S_R = V_R \cdot I_R^*$
- $I_R = \dfrac{V_s - V_R}{Z'}$

$Z' = |Z'| \angle \theta_z$

$P_R = Re(S_R)$

$= \dfrac{|V_s| \cdot |V_R|}{Z'} \cdot \cos(\theta_z - \delta) - \dfrac{|V_R|^2}{Z'} \cdot \cos(\theta_z$

(read the rest & solve example)

5.6: Line Loadability

- maximum power that can be sent down a transmission line for a specified voltage, and maximum angle.

Example 5.7: (data from Itaipu Project)

9000 MW
500 Km
60 Hz

$\delta_{max} = 35°$
$V_s = 1$
$V_R = 0.95$

- Determine the number of circuits at 345; 500 and 765 KV capable of transmission of power after one outage.

Solution: $Z_c = 297\,\Omega$ @ 345 KV
$277\,\Omega$ @ 500 KV
$266\,\Omega$ @ 765 KV

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a) At 345KV

Three gorges

$P_{SIL} = \dfrac{345^2}{297} = 401\,MW$

- $P_{max} = \dfrac{1 * 0.95 * 401 * \sin(35°)}{\sin\left(\frac{2\pi}{5000} * 500 * \frac{180}{\pi}\right)} = 401 * (0.927)$ → able to transmit 92.7% of the surge impedance

$= 372\ MW/line$

- $N_{345} = \dfrac{9000}{372} + 1 = 24.2 + 1 = 25.2$ → 26 circuits (25 circuits are not enough!)

↳ I need 13 towers in Parallel.

b/ At 500 KV

$$P_{SIL} = \frac{500^2}{277} = 903 \text{ MW}$$

$$P_{max} = 903 \times 0.927 = 837 \text{ MW/line}$$

$$N_{500} = \frac{9000}{837} + 1 = 12$$

c/ At 765KV

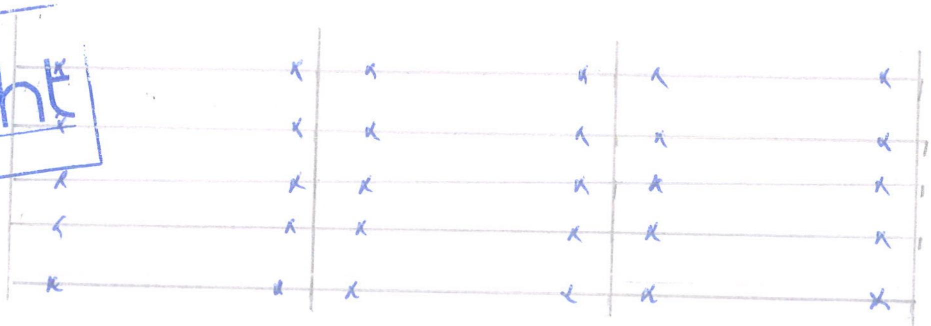
$$P_{SIL} = \frac{(765)^2}{266} = 2200 \text{ MW}$$

$$P_{max} = 2200 \times 0.927 = 2039 \text{ MW/line}$$

$$N_{765} = \frac{9000}{2039} + 1 = 4.4 + 1 \approx 6$$

Example 5.8:

Can 5 instead of 6 transmit the power if there are two intermediate substations that divide each line into 3×167 Km sections.



$$X' = 266 \cdot \sin\left(\frac{2\pi}{5000} \times 500\right) = 156.4\ \Omega$$

$$X_{eq}(-1) = \frac{1}{5}\left(\frac{X'}{3}\right) \times 2 + \frac{1}{4} \cdot \frac{X'}{3} = 33.88\ \Omega$$

$$P_{max} = \frac{V_1 \cdot V_2}{X} \cdot \sin\delta_{max}$$

$$P_{max} = \frac{765\,(765 \times 0.95)}{33.88} \times \sin(35°)$$

Voltage @receiving end

$$= 9412 \text{ MW}$$

5.7: Reactive Compensation Techniques.

- Capacitive Shunt Compensation: done at heavy loading when the system experiences low voltage.
  . Placed near load.

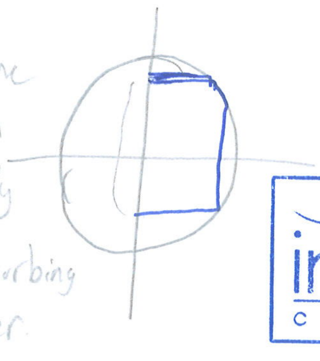
- Capacitive Series Compensation: . Placed in series with the transmission line to increase the line loadability.

  <u>Problems:</u> ① High short circuit current, have to be diverted (disconnect the capacitance, short circuit it, should be bypassed).
  ② Subsynchronous resonance with machines (generators) inductance
  ③ Series compensation can go up to 75% of X' (series inductance of T.L).

- Shunt Inductor Compensation: done at low loading when transmission system has an access of reactive power. It is usually to protect the generator from absorbing too much reactive power.

- Synchronous Condenser (or Capacitor): takes active power from the grid (not operated by prime mover) and supplies reactive power only (or consumes reactive power only).

machine operating on q-axis only supplying/absorbing reactive power



It is a synchronous machine, producing zero power (no power) to the grid (no-prime mover), gets its real power losses from the grid directly.

<u>**Problem:**</u> Costly: Cost is higher than static compensation.

<u>Advantage:</u> Controllability of the reactive power, without producing harmonics