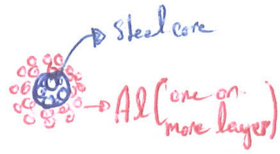
Chapter 4: Transmission Line Parameters

4.1: Transmission Line Design Consideration

Conductors

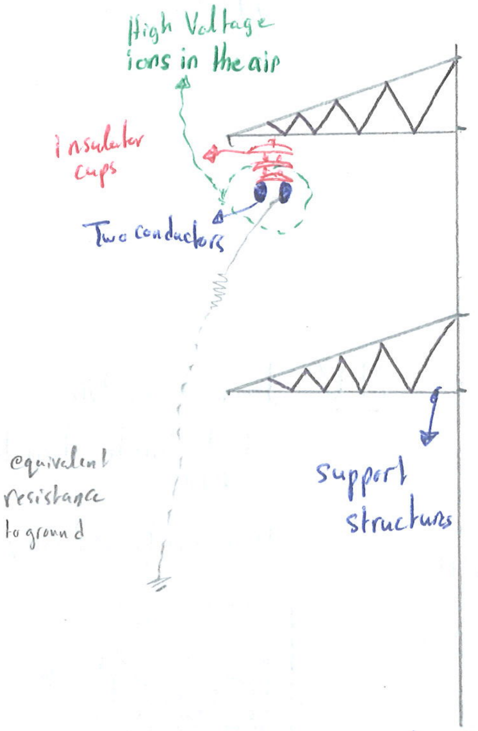
* A C S R : Most Common — Overhead
  - A → Al
  - C → Conductor
  - S → Steel
  - R → Reinforced



* A A A C — Overhead
  - A → All
  - A → Alum.
  - A → Alloy
  - C → Conductor

* A A C — Underground
  - A → All
  - A → Aluminium
  - C → Conductor

Insulators



* insulator cups: High quality glass or porselen;
* Two Conductors: to reduce surface electric field on the conductor and reduce corona losses.
* bundles / phase

220K	2
345K	3 or 4
500K	4

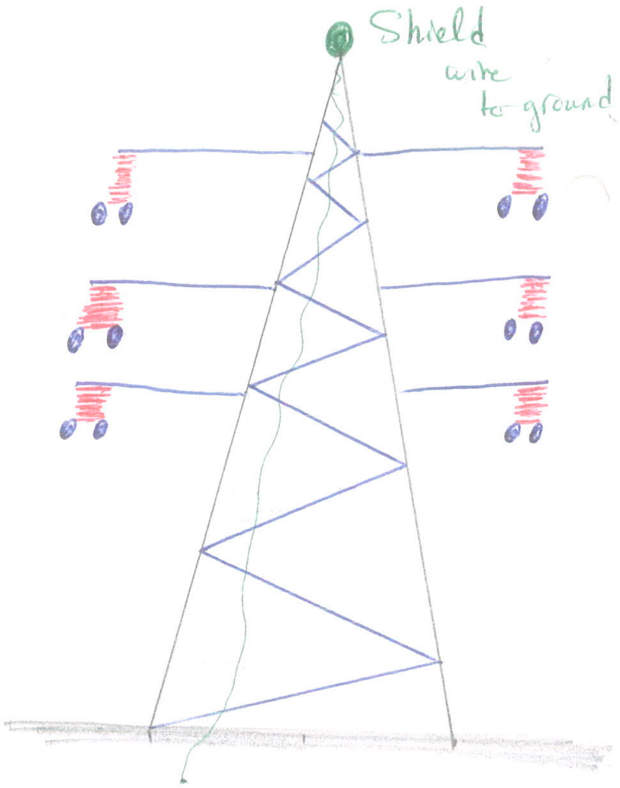
if its rain and wet

275K	4

* Support Structures : - wooden poles
  - Steel Structure

⟹ Electrical Factors

⟹ Mechanical Factors



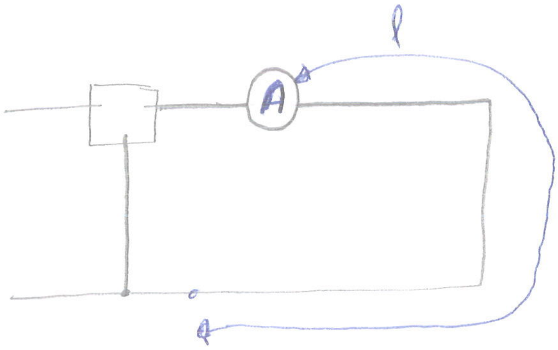
4.2: Resistance

- DC Resistance:

$$R_{dc,T} = \rho_T \cdot \frac{l}{A}$$

- AC Resistance:

$$R_{ac,T} = \frac{P_{loss}}{|I|^2}$$



Peacock - 605 000 cmils

N.B.: * 1 cmil is the area of a circle of $\frac{1}{1000}$ inch diameter

* 1000 cmils $\approx 0.507\ mm^2$

@ 25°C
- $R = 0.155\ \Omega/mile$ at 50 Hz
- $R = 0.155\ \Omega/mile$ at 60 Hz
- $R_{dc} = 0.154\ \Omega/mile$

insight club

@ 50°C - $R = 0.1755\ \Omega/mile$ at 50 Hz

Joree 2 516 000 cmils

@ 50°C - $R = 0.045\ \Omega/mile$ at 60 Hz

$$\rho_{T_2} = \rho_{T_1} \cdot \left(\frac{T_2 + T_0}{T_1 + T}\right)$$ → Aluminum Constant: 228.1

$$\times \Rightarrow R_{T_2} \approxeq R_{T_1} \cdot \left(\frac{T_2 + T}{T_1 + T}\right)$$

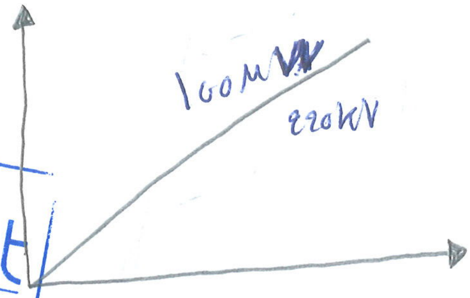
temperature

$3 \cdot I^2 \cdot R_{75^\circ} \leqslant 0.02 P$

N.B: losses should be max. 2% of rated power

Example

* If we have the load & voltage, we can get current:



$$I = \frac{100}{0.9\sqrt{3} \times 220} = 0.29\,kA$$

$$R_{75} \leqslant \frac{0.02 \times 100M}{3 \times (0.29k)^2} = 7.93$$

Assume $l = 150Km$

$$r_{75}\ (\Omega/mil) = \frac{7.93}{150k} \times 1.609 = 0.085\ \Omega/mile$$

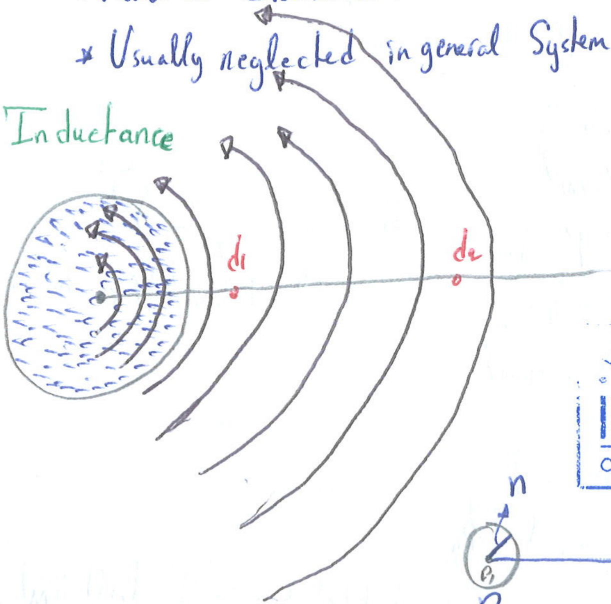
$$\hookrightarrow r_{50} = 0.085 \times \left(\frac{50 + 228.1}{75 + 228.1}\right) = 0.078\ \Omega/mile$$

↳ which is according to the table: Plover.

4.3: Conductance (self read)

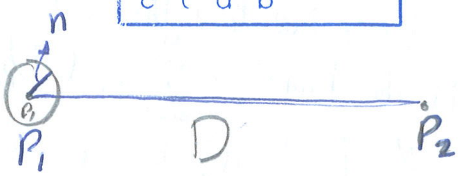
* Parallel conductance due to air ionization and corona loss
* Usually neglected in general System Studies

4.4: Inductance



* Inside Wire: $\lambda_{int} = \frac{\mu_0}{8\pi} \cdot I$

$$\lambda_{d_1 d_2} = \frac{\mu_0}{2\pi} \times I \times \ln\left(\frac{D_2}{D_1}\right)$$



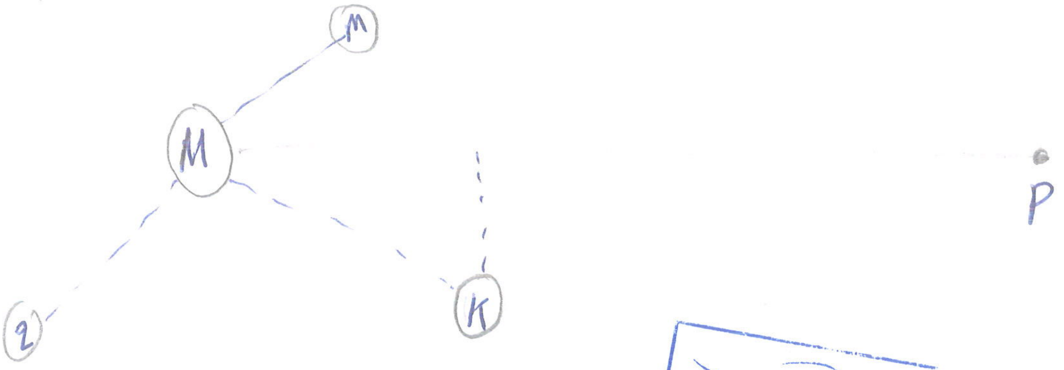
$$\lambda_p = \frac{\mu_0}{8\pi} \cdot I + \frac{\mu_0}{2\pi} I \ln\left(\frac{D}{r}\right)$$

$$= \frac{\mu_0}{2\pi} I \times \ln\left(\frac{D}{r'}\right)$$

N.B: $r' = re^{-1/4}$ geometric mean radius.

$$\sum_{m=1}^{M} I_m = 0!$$

i.e. $\rightarrow I_1 + I_2 + I_3 + \cdots + I_m = 0$ Balanced



N.B: λ_{KP_m} is the flux linkage which links conduct "K" out to point P due to current I_m.

1

* $\sum_{m=1}^{M} I_m = 0$!

$$\lambda_{KPK} = \frac{\mu_0}{2\pi} \cdot I_K \cdot \ln\left(\frac{D_{PK}}{r'_K}\right)$$

* flux linkage between conductors K & P due to I_K:

$$\lambda_{KPK} = \frac{\mu_0}{2\pi} \cdot I_K \cdot \ln\left(\frac{D_{PK}}{r'_K}\right)$$

* Flux linkage between conductor K & P due to I_m

$$\lambda_{mPm} = \frac{\mu_0}{2\pi} I_m \cdot \left(\frac{D_{Pm}}{D_{Km}}\right)$$

* Total flux linkage between K & P due to all wires up to P:

$$\lambda_{KP} = \frac{\mu_0}{2\pi} \cdot \sum_{m=1}^{M} I_m \ln\left(\frac{D_{Pm}}{D_{Km}}\right)$$ with $D_{KK} \triangleq r'_K$ when $m = k$

* let $P \rightarrow \infty$ (very far from the wires)

Add the term: $-\sum_{m=1}^{M} I_m \ln D_{Pm}$

$$\lambda_{KP} = \frac{\mu_0}{2\pi}\left(\sum I_m \cdot \ln\frac{1}{K_m} + \underbrace{\sum I_m \cdot \ln(D_{Pm}) - \sum I_m \ln(D_{PM})}_{\sum I_m \cdot \ln\left(\frac{D_{Pm}}{D_{PM}}\right)}\right)$$

* as $P \rightarrow \infty$; $D_{Pm} \rightarrow D_{PM} \rightarrow \ln\left(\frac{D_{Pm}}{D_{PM}}\right) = 0$

* as $P \rightarrow \infty$; $\lambda_{KP} \rightarrow \lambda_K$

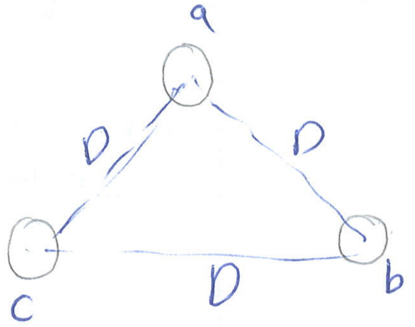
$\Rightarrow$ Total flux linkage of K up to ∞

$$\lambda_K = \frac{\mu_0}{2\pi}\left[\sum_{m=1}^{M} I_m \cdot \ln\left(\frac{1}{D_{Km}}\right)\right]$$ when $\sum I_m = 0$!

* phasors
* sinusoides

4.5: Inductance; Single phase Two-Wire line and 3Φ Three-Wire line with equal Spacing.

* 1-Φ: Two-wire Line [self-read]
* 3-Φ: Three-Wire Lines



* m = a; b; c
* $\lambda_a = \frac{\mu_0}{2\pi} * \left[I_a \ln\left(\frac{1}{r_a}\right) + I_b \cdot \ln\left(\frac{1}{D}\right) + I_c \cdot \ln\left(\frac{1}{D}\right) \right]$

But $I_a + I_b + I_c = 0$

$$= \frac{\mu_0}{2\pi} * \left[(I_b + I_c) \cdot \ln\left(\frac{1}{D}\right) + I_a \cdot \ln\left(\frac{1}{r_a}\right) \right]$$

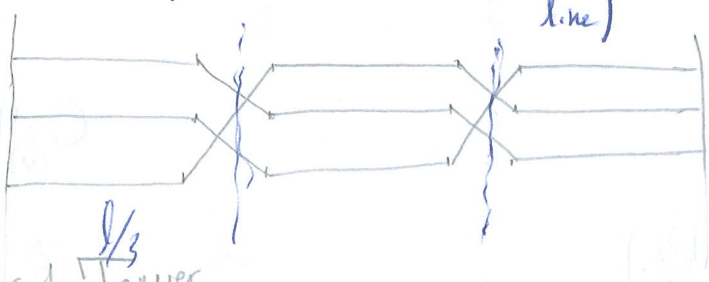
$$= \frac{\mu_0}{2\pi} * \left[-I_a \ln\left(\frac{1}{D}\right) + I_a \ln\left(\frac{1}{r_a}\right) \right]$$

$$= \frac{\mu_0}{2\pi} * I_a * \ln\left(\frac{D}{r_a'}\right)$$

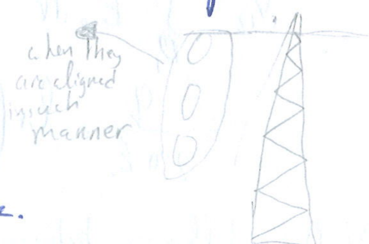
* Generalization of Flux Linkage Formula

$$L_a = \frac{\lambda_a}{I_a} = \frac{\mu_0}{2\pi} * \ln\left(\frac{D}{r'}\right)$$

* For long lines; phases are transposed (2 times over the length of Transmission line)



→ To Balance Inductance between phases

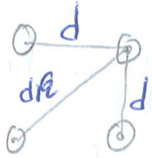


* For a Transposed Tower

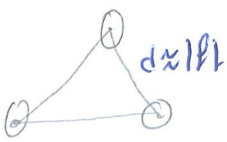
or when we have more than one conductor per phase.

$$D = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}}$$

$$D_{SL} = \sqrt[2]{r'd}$$



$$D_{SL} = \sqrt[4]{r'd^3\sqrt{2}}$$



$$D_{SL} = \sqrt[3]{r'd^2}$$

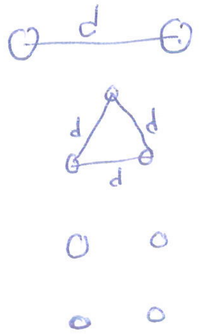
* Generalization

$D \longrightarrow D_{eq}$ for a transposed Line

$$D_{eq} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ac}}$$

$r \longrightarrow D_{sc}$

$$D_{sc} = \begin{cases} r & \text{; 1 one bundle/phase} \\ \sqrt{rd} \\ \sqrt[3]{rd^2} \\ \sqrt[4]{rd^3 \cdot \sqrt{2}} \end{cases}$$



[gmr : read from table of OHL wire]

overhead lines

gmr : geometric mean radius $\longrightarrow$ r'

standard conductors

cylindrical conductance

$$L = \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{D_{eq}}{r'}\right)$$

between phases

$$* \; L = \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{D_{eq}}{D_{sc}}\right) \quad H/m$$

for/between bundles

4.8 : Electric Field and Voltage of Solid Cylindrical Conductor

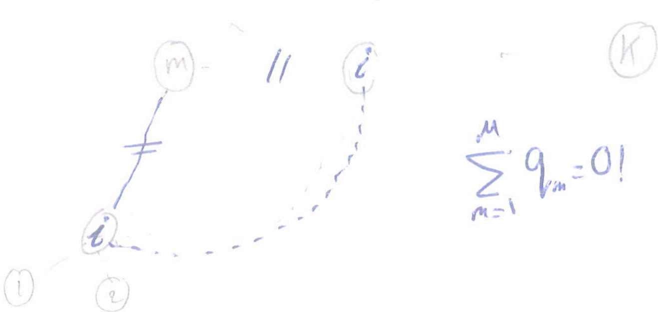
$$\oiint \vec{D} \cdot \vec{ds} = Q_{encl.}$$

$$E = \frac{q}{2\pi\varepsilon_0} \cdot \frac{1}{x} \; V/m$$

$$V_{12} = \int_{D_1}^{D_2} \vec{E} \cdot \vec{dl}$$

$$= \frac{q}{2\pi\varepsilon_0} \cdot \ln\left(\frac{D_2}{D_1}\right)$$

$C = q \cdot V$



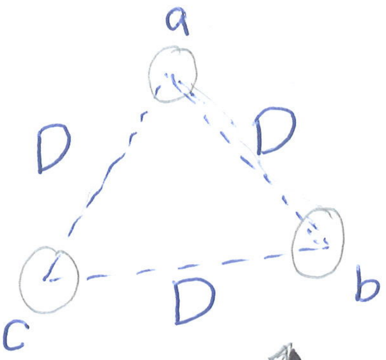
$$\sum_{m=1}^{M} q_m = 0!$$

* Voltage difference between conductor K and i due to charge on m ($V_i - V_k \triangleq V_{ki}$)

$$\therefore V_{kim} = \frac{q_m}{2\pi\varepsilon} \cdot \ln\left(\frac{D_{im}}{D_{km}}\right)$$

$$\therefore V_{ki} = \sum_{m=1}^{M} \frac{q_m}{2\pi\varepsilon} \cdot \ln\left(\frac{D_{im}}{D_{km}}\right)$$

4.9: Capacitance of a 3-ϕ; Three-Line System of Equal Spacing.



$$* \; V_{ab} = \frac{1}{2\pi\varepsilon} \cdot \left[q_a \cdot \ln\left(\frac{D_{ab}}{r}\right) + q_b \cdot \ln\left(\frac{r}{D_{ab}}\right) + q_c \cdot \ln\left(\frac{D_{bc}}{D_{ac}}\right) \right].$$

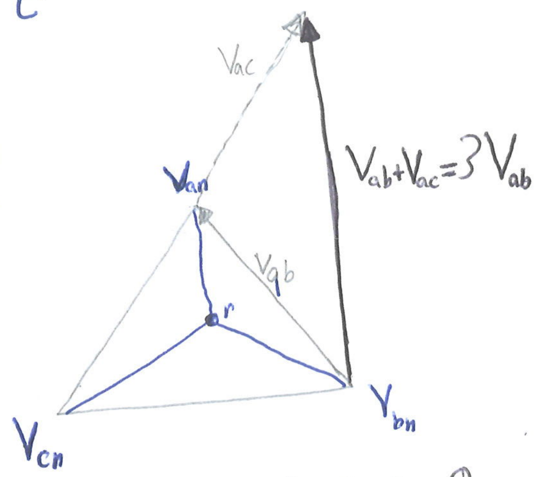
($m=a$, $m=b$, $m=c$; k l; $\frac{D_{bc}}{D_{ac}} \rightarrow 0$)

$$* \; V_{ac} = \frac{1}{2\pi\varepsilon} \cdot \left[q_a \cdot \ln\left(\frac{D}{r}\right) + q_c \cdot \ln\left(\frac{r}{D}\right) \right]$$

$$\rightarrow V_{an} = \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right) * \left[2q_a \cdot \ln\left(\frac{D}{r}\right) + \underbrace{(q_b + q_c)}_{-q_a} \ln\left(\frac{r}{D}\right) \right]$$

$$= \frac{1}{\cancel{3}}\left(\frac{1}{2\pi\varepsilon}\right) * \left[\cancel{3} q_a \cdot \ln\left(\frac{D}{r}\right) \right]$$

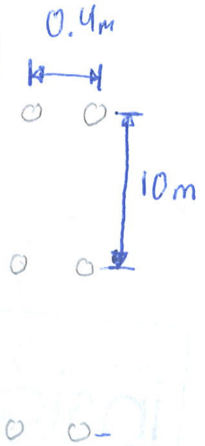
$$V_{an} = \frac{1}{2\pi\varepsilon} \cdot q_a \cdot \ln\left(\frac{D}{r}\right)$$



$$q_a + q_b + q_c = 0$$

$$q = CV \Rightarrow \quad C_a = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad F/m$$

* Eample of Transmission Line Design:



- Load $\quad P + jQ = 100 + j50$
  - 100 MV @ 0.9 lag.
- voltage: 220 KV (2 bundles)
- Conductor temperature allowed $\leq 75°C$
- Transmission Distance: 120 km
- Frequency: 50 Hz
- Percent Losses $\leq 2\%$
- Determine resistance of line from loss considerations

$$3I^2R < 0.02 * 100$$

where $$I = \frac{P}{\sqrt{3} * V * \cos\phi} = \frac{100M}{\sqrt{3} * 220k * 0.9} = 0.292 \, kA$$

so, $$R_{75} \leq \frac{2M}{0.292k^2 * 3} = 7.82 \, \Omega/\text{phase}$$

$$R_b \leq 7.82 * 2 = 15.64 \, \Omega/\text{bundle} \quad \text{(parallel resistance)}$$

$$R_{b50} = R_{b75} * \left(\frac{50 + 228.1}{75 + 228.1}\right) = 14.35 \, \Omega$$

$$r_{b_{50}} = 14.35 \times \frac{\overbrace{1.609}^{\text{mile}}}{190\text{km}} = 0.192\ \Omega/\text{mile}$$

Table $\rightarrow$ Eagle has $r_b(50) = 0.1859\ \Omega/\text{mile}$
or (Dove)

$$R_{e_{75}} = 0.1859 \times \frac{190\text{km}}{\underbrace{1.609}_{\text{mile}}} \times \frac{1}{\underset{\text{bundle}}{(2)}} \times \underbrace{1.09}_{50°C \to 75°C} = 7.62\ \Omega/\text{phase}$$

N.B.: $r' = re^{-1/4}$ not for stranded conductors, only for cylindrical conductors.

* $X = \omega L \cdot l = x l$

$$L = \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{D_{eq}}{D_{SL}}\right)$$

2×10^{-7}

$$D_{eq} = \sqrt[3]{10 \times 10 \times 20} = 12.6\text{ m}$$

$$D_{SL} = \sqrt[2]{gmr \cdot d} = 0.064\text{ m}$$

d: 0.4

gmr: from table: $0.0398\text{ ft} \times 0.315\ \frac{\text{m}}{\text{ft}}$

$$L = 2 \times 10^{-7} \cdot \ln\left(\frac{12.6}{0.064}\right) = 1.06 \times 10^{-6}\text{ H/m} = 1.06 \times 10^{-3}\text{ H/km}$$

$$X = x \cdot l = \underbrace{2\pi \times 50}_{f} \times 1.06 \times 10^{-3}\ \frac{\text{H}}{\text{Km}} \times 190\text{ Km} = 40\ \Omega$$

* $Y = \omega C l = c l$

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)} = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln\left(\frac{12.6}{0.07}\right)} = 1.07 \times 10^{-11}\text{ F/m} = 1.07 \times 10^{-8}\text{ F/Km}$$

$\hookrightarrow D_{eq}$ = same = 12.6m

$$\hookrightarrow D_{sc} = \sqrt[2]{r \cdot d} = \sqrt{\frac{0.953}{2}\text{inch} \times 0.0254\ \frac{\text{m}}{\text{inch}} \times 0.4} = 0.07\text{m}$$

r: from table: $\frac{\text{Outside diam.}}{2}$

$$y = \omega C = 314.16 \times 1.07 \times 10^{-8}\text{ F/Km} = 3.36 \times 10^{-6}\text{ S/Km}$$

$$Y = 3.36 \times 10^{-6}\ \frac{\text{S}}{\text{Km}} \times 120\text{ Km} = 403\ \mu\text{S}$$

* Find P.U values: $V_{base} = 220\,kV$

$S_{base} = 100\,MVA$

so $Z_{base} = \frac{220^2}{100} = 484\,\Omega$

$R(p.u) = \frac{7.62}{484} = 0.015$ p.u

$X(p.u) = \frac{40}{484} = 0.0826$ p.u.

$Y(p.u.) = 403 \times 10^{-6} \times 484 = 0.195$ p.u.

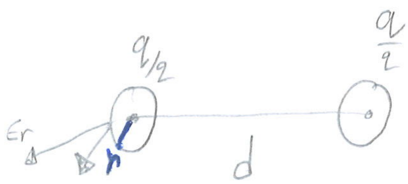
4.12: Electric Field Strength at Surfaces and Ground Levels

- 1 conductor per phase:

$$E_r = \frac{q}{2\pi \varepsilon r}$$

- 2 conductors per phase:

$$E_{r_{max}} = \frac{q}{2\pi\varepsilon}\left(\frac{1}{2r} + \frac{1}{2(r+d)}\right)$$



$< E_{r_{max}}$ of a conductor

- 4 conductors perphase (4 bundles)

$$E_{r_{max}} = \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{r} + \frac{1}{4dr^2} + \frac{r_2}{2d}\right)$$