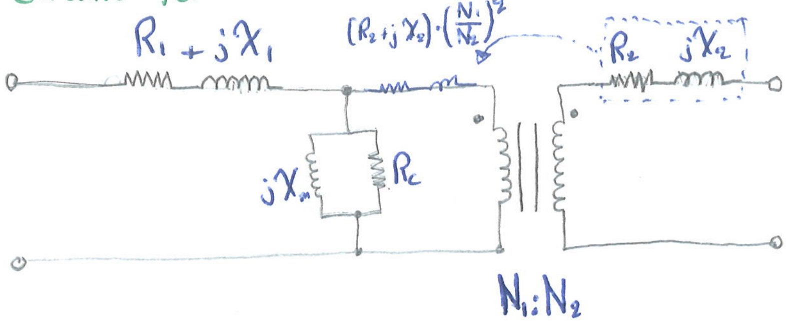
Chapter 3: Power Transformers

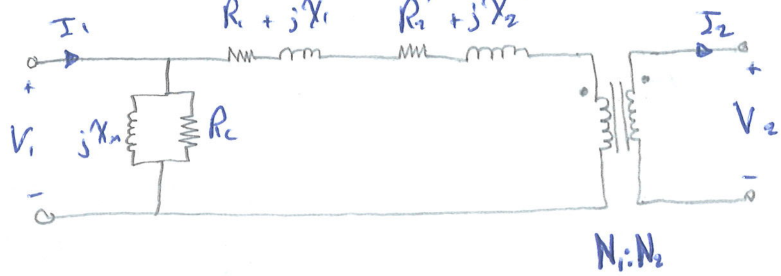
3.1: The Ideal Transformer (Self Studying)

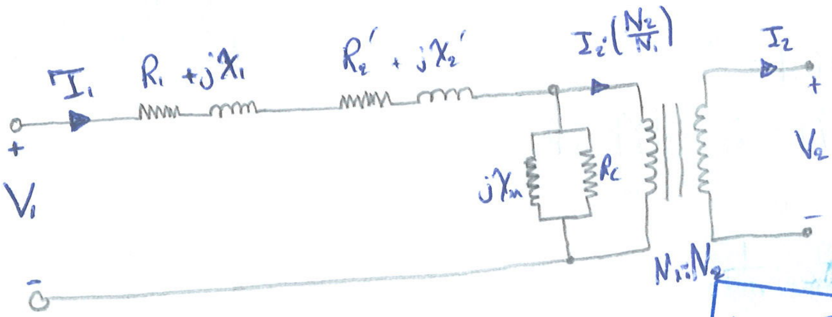
3.2: Equivalent Circuits for Practical Transformers



* High level of Approximation : Removing Excitation Branch 
  ∴ becomes only series reactance

Equivalent Circuit #1

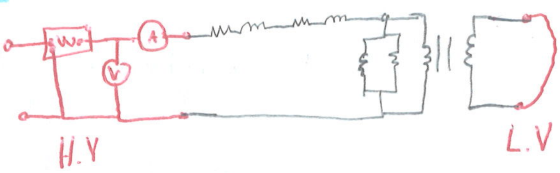




Short Circuit Test

- $V_{sc} \ll V_{rated}$
- $I_{sc} \approx I_{rated}$
- P_{sc}

get → $R_{eq} + jX_{eq} = Z_{eq}$



→ $|Z_{eq}| = \dfrac{V_{sc}}{I_{sc}}$

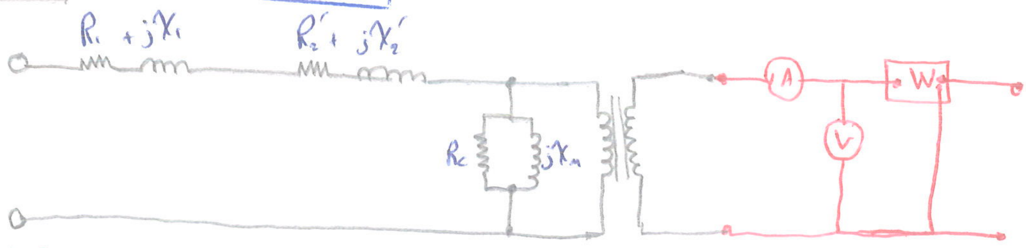
→ $\cos\phi_{sc} = \dfrac{P_{sc}}{V_{sc} \cdot I_{sc}}$; $\sin\phi_{sc}$

→ $R_{eq} + jX_{eq} = |Z_{eq}| * (\cos\phi_{sc} + j\sin\phi_{sc})$

Note: * S.C Test: S.C the L.V side & measure on H.V side

* O.C Test: O.C the H.V side & measure on L.V side

Open Circuit Test:



- $V_{oc} = V_{rated\,(L.V)}$
- $I_{oc} \ll I_{rated\,(L.V)}$
- P_{oc}

$$\left.\begin{array}{l} V_{oc},\ I_{oc},\ P_{oc} \end{array}\right\} \xrightarrow{\text{to get}} R_c + jX_m$$

$$\rightarrow Y = \frac{1}{R_c} + \frac{1}{jX_m}$$

$$\rightarrow |Y| = \frac{I_{oc}}{V_{oc}}$$

$$\rightarrow \cos\phi_{oc} = \frac{P_{oc}}{V_{oc} \cdot I_{oc}} \ ; \ \sin\phi_{oc}$$

$$\rightarrow Y = |Y_{eq}| \cdot e^{-j\phi_{oc}}$$

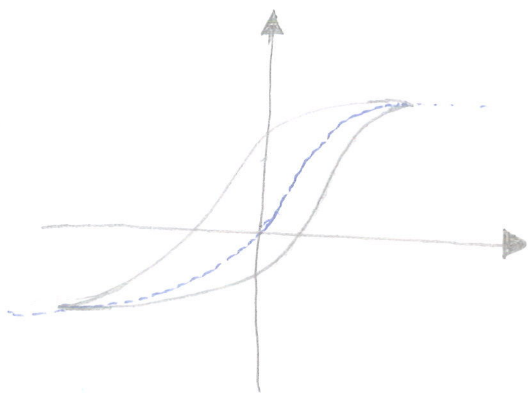
$$= \underbrace{|Y|\cos\phi_{oc}}_{G_c} - j\underbrace{|Y|\sin\phi_{o.c.}}_{B_m}$$

$$R_c = \frac{1}{G_c}$$

$$X_m = \frac{1}{B_m}$$

* Saturation:

* Saturation lead to Harmonic due to excitation current
* But its small, its neglected at calculation level but taken into consideration at design level



* Issues not represented in Equivalent Circuit
  1. Saturation
  2. Inrush Current
  3. Non sinusoidal existing current
  4. Surge phenomena

1- Saturation:

* Due to saturation trend in the BH curve of the laminations.
  This causes changes in the value of μ_r that makes $L_m (a^2 X_m)$ dependent on applied voltage.

2- Inrush Current:

* Consider that $B(0)$ is some finite value $(= 1.2\ Wb/m^2)$ Tesla
* Faraday's law:

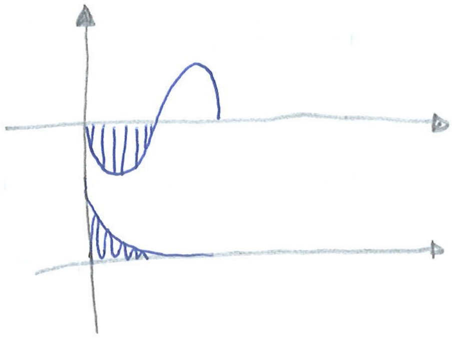
$$e = -NA\frac{dB}{dt}$$

when integrated it generates:

$$\int_0^t e\,dt = -NA\int_0^t dB$$

$$e(t) = -NA\left(B(t) - B(0)\right)$$

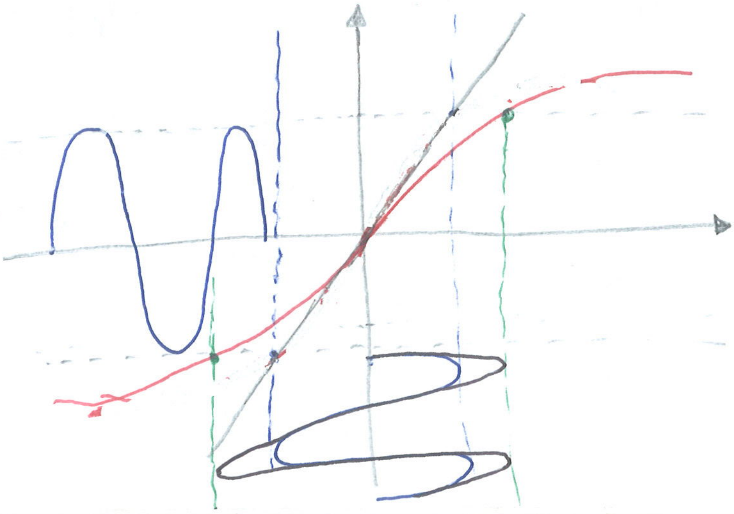
$$\rightarrow B(t) = -\frac{1}{NA}\int_0^t e(t) + B(0)$$



* This causes $B(t)$ to be larger than usual, resulting to a very high current flowing - distortion

3- Non sinusoidal exciting current

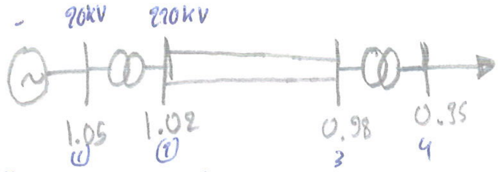
* Non-sinusoidal exciting current contains fundamental and harmonics. The dominant harmonic is the 3rd, with about 40%. This is neglecting in steady state analysis, because the excitation current is a 5% of the rated current.



3.3: The Per Unit System

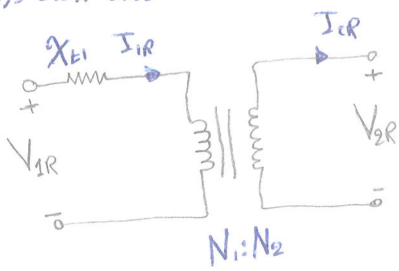
Advantages:

* By proper selection of base quantities, the transformer equivalent circuit is simplified (the ideal transformer dissappears)



Actual voltage (1) = 90K * 1.05
Actual voltage (2) = 220K * 1.02
Actual voltage (3) = 220K * 0.98

$$S_R = V_{1R} \cdot I_{1R} = V_{2R} \cdot I_{2R}$$



$V_{1R} = 11\,KV$

$V_{2R} = 220\,KV$

$S_R = 10\,MVA \Rightarrow I_{1R} = \dfrac{10\,000\,\cancel{K}}{11\,K} = 909.1\,A = 0.909\,KA$

$I_{2R} = \dfrac{10\,000\,K}{220\,K} = 45.5\,A$

1) Select a scale for power: S_B (arbitrary)
→ usually S_R

2) Select a base for the voltage on one side: V_{B1}
$V_{B1} = V_{1R}$ (usually selected at rated voltage but it is in general arbitrary)

3) Voltage base on side 2 is selected according to transformer turns rating: $V_{B2} = V_{B1} \cdot \dfrac{N_2}{N_1}$ (no choice)
→ in this case ($V_{B2} = V_{2R}$)

Per unit Calculations:

* $E_1(kV) = 10.5\,kV$
↳ $E_1(pu) = \dfrac{10.5K}{11K} = \underline{0.95\,pu}$

* $E_2(kV) = 10.5 \times \dfrac{220}{11}$
↳ $E_2(pu) = \left(10.5 \times \dfrac{\cancel{220}}{\cancel{11}}\right) \cdot \dfrac{1}{11 \cdot \frac{\cancel{220}}{\cancel{11}}} = \dfrac{10.5}{11} = \underline{0.95\,pu}$

$E_1(pu) = E_2(pu)$

$E_2(pu) = \dfrac{E_2(kV)}{V_{B2}}$

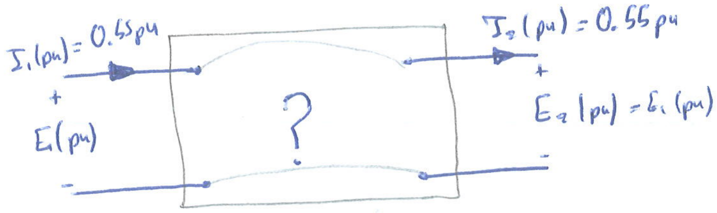
$= \left(E_1(kV) \cdot \dfrac{\cancel{N_2}}{\cancel{N_1}}\right) \cdot \dfrac{1}{V_{B1} \cdot \left(\frac{\cancel{N_2}}{\cancel{N_1}}\right)}$

$= \dfrac{E_1(kV)}{V_{B1}} = E_1(pu)$

* $I_1 = 500\,A$
↳ $I_1(pu) = \dfrac{I_1}{I_{B1}}$; $I_{B1} = \dfrac{S_B}{V_{B1}} = \dfrac{10\,000\,K}{11\,K} = 909\,A$

$= \dfrac{500}{909} = 0.55\,pu$

$I_2(pu) = \dfrac{I_2(A)}{I_{B2}} = \left(I_1(A) \cdot \dfrac{\cancel{N_1}}{\cancel{N_2}}\right) \cdot \dfrac{1}{\left(I_{B1} \cdot \frac{\cancel{N_1}}{\cancel{N_2}}\right)} = \dfrac{I_1(A)}{I_{B1}} = I_1(pu)$



- $X_{B1} = X_1 \cdot \frac{V_{B1}}{I_{B1}}$

  ↳ $\boxed{Z_B = \frac{V_{Bn}^2}{S_B}}$

if $S'_B = 2S_B$

$E_1(pu)$ same

$E_2(pu)$ same

$I'_{B1} = \frac{S'_B}{V_B}$; ×2

↳ $I_1(pu) = \frac{I_1(A)}{I'_{B1}}$; ÷2

↳ $I_2(pu)$ ÷2

$X'_{B1} = \frac{V_{B1}^2}{S'_B}$; ÷2

↳ $X_{L1}(pu) = \frac{X_{L\Omega}}{X'_{B1}}$; ×2

Per Unit System for 1-Φ circuits:

- S_B selected
- V_{B1} selected
  - ↳ $V_{B2} = V_{B1}\left(\frac{N_2}{N_1}\right) = V_{B1}\left(\frac{V_{R2}}{V_{R1}}\right)$
  - ↳ $I_{B1} = \frac{S_B}{V_{B1}}$
  - ↳ $I_{B2} = \frac{S_B}{V_{B2}} = I_{B1}\left(\frac{V_{B1}}{V_{B2}}\right) = I_{B1}\cdot\left(\frac{V_{R1}}{V_{R2}}\right)$
  - ↳ $Z_{B1} = \frac{V_{B1}}{I_{B1}} = \frac{V_{B1}^2}{S_B}$
  - ↳ $Z_{B2} = \frac{V_{B2}^2}{S_B}$

- $\frac{Z_{B2}}{Z_{B1}} = \frac{V_{B2}^2}{V_{B1}^2}$
- $Z_{p.u} = \frac{Z_{\Omega 1}}{Z_{B1}} = \frac{Z_{\Omega 2}}{Z_{B2}}$

N.B: 3 Φ voltage quoted is the (line-to-line) voltage

Z_{pu} to V_{B1} ; S_{B1}

$Z_{p.u}$ to V'_{B1} ; S'_{B1}

$Z'_{p.u} = Z_{pu} \cdot \frac{V_{B1}^2}{S_{B1}} \cdot \frac{S'_{B1}}{V'^2_{B1}}$

Per Unit System for 3-Φ circuits

- $S_{B(3\Phi)}$; selected arbitrary
- $V_{B1} = V_{BLL}$; selected arbitrary
  - ↳ $V_{B2} = V_{B1}\left(\frac{V_{R2}}{V_{R1}}\right)$
  - ↳ $I_{B1} = \frac{S_{B(3\Phi)}}{3 \cdot \frac{V_{B1}}{\sqrt{3}}} = \frac{S_{B(3\Phi)}}{\sqrt{3}\,V_{B1}}$; base value of line current $I_{p.u.}^{(\Delta)} = I_{pu}^{(Y)}$
  - ↳ $Z_{B1} = \frac{\frac{V_{B1}}{\cancel{\sqrt{3}}}}{\frac{S_{B(3\Phi)}}{\cancel{\sqrt{3}}\,V_{B1}}} = \frac{V_{B1}^2}{S_{B(3\Phi)}}$; Z_B in the Y

$\hookrightarrow Z_{B1}(\Delta) = \dfrac{V_{B1}}{I_{B1}(\Delta)}$

$I_{B1} = \sqrt{3}\, I_{B1}(\Delta)$

- $I_{B1}(\Delta) = \dfrac{S_B(3\phi)}{3 V_{B1}}$

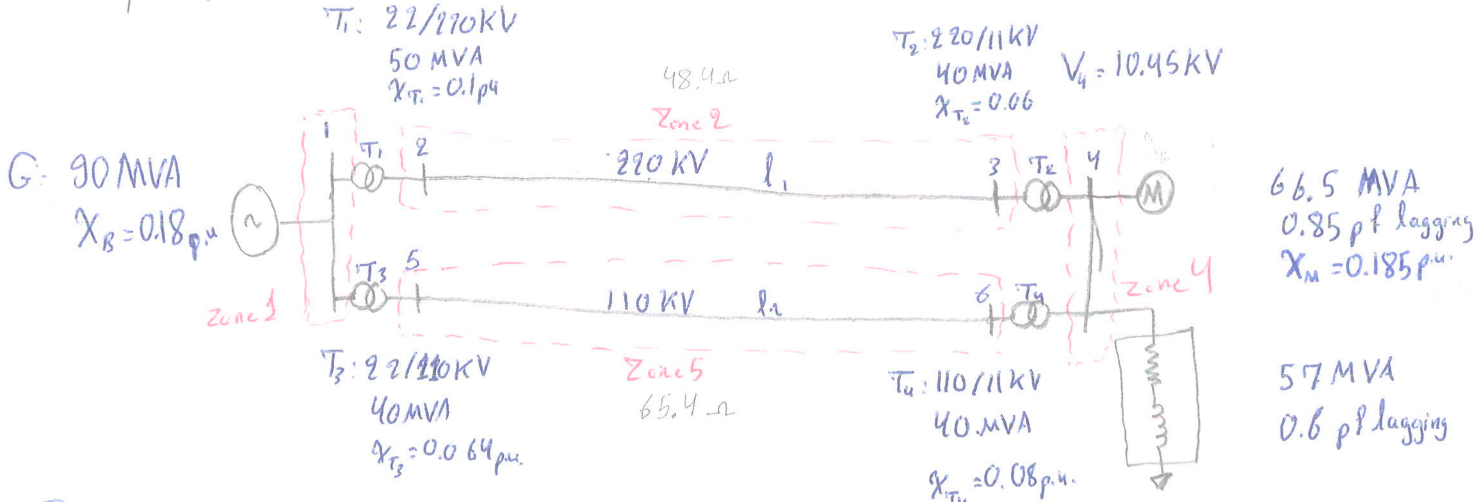
- $Z_{B1}(\Delta) = \dfrac{V_{B1}}{I_{B1}(\Delta)} = \dfrac{V_{B1}}{I_{B1}(\Delta)} = \dfrac{\sqrt{3}\, V_{B1(l-n)}}{\frac{I_{B1}}{\sqrt{3}}} = 3 Z_{B1}$

$Z_{B1}(\Delta) = 3 Z_{B1}$

- $Z_{p.u}(Y) = \dfrac{Z_Y}{Z_{B1}} = \dfrac{Z_\Delta}{3} \cdot \dfrac{3}{Z_{B1}(\Delta)} = Z_{p.u}(\Delta)$

$Z_{p.u}(\Delta) = Z_{p.u}(Y)$

Example 3.23



- Base Quantities

$S_B = 100$ MVA
$V_{B1} = 22$ KV } free choices

$V_{B2} = 220$ KV
$V_{B4} = 11$ kV
$V_{B5} = 110$ kV

Z_{Base} for each zone

$Z_{B1} = \dfrac{22^2}{100} = 4.84\ \Omega$

$Z_{B2} = \dfrac{220^2}{100} = 484\ \Omega$

$Z_{B4} = \dfrac{11^2}{100} = 1.21\ \Omega$

$Z_{B5} = \dfrac{110^2}{100} = 121\ \Omega$

- Impedance in Per Unit

$\underset{S=100}{X_B(p.u)} = \underset{p.u.\ S=90}{0.18} * \underbrace{\dfrac{1}{\frac{90}{22^2}}}_{X_B \text{ in } (\Omega)} * \dfrac{1}{\frac{22^2}{100}} = 0.2$ p.u.

- $X_{T_1}(p.u.) = \underbrace{0.1 * \dfrac{220^2}{50} \cdot \dfrac{1}{\frac{220^2}{100}}}_{\text{reffered to H.V side}} = 0.2$ p.u $= \underbrace{0.1 * \dfrac{22^2}{50} * \dfrac{1}{\frac{22^2}{100}}}_{\text{reffered to L.V. side}}$

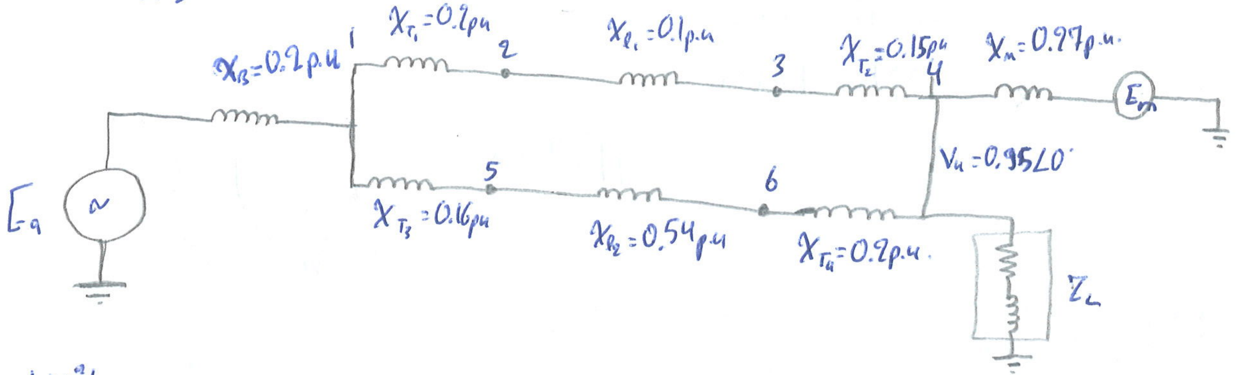
- $X_{T_2}(p.u) = 0.06 * \dfrac{220^2}{40} * \dfrac{1}{\frac{220^2}{100}} = 0.15$ p.u.

- $X_{T_3}(p.u.) = 0.064 * \dfrac{110^2}{40} \cdot \dfrac{1}{\frac{110^2}{100}} = 0.16$ pu

- $x_{T_4}(p.u) = 0.08 \times \frac{110^2}{40} \cdot \frac{1}{\frac{110^2}{40}} = 0.2\ p.u.$
- $x_{\ell_1}(p.u.) = \frac{48.4}{484} = 0.1\,p.u$
- $x_{\ell_2}(p.u) = \frac{65.4}{121} = 0.54\,p.u$
- $x_u(p.u) = 0.185 \times \frac{100}{66.5} = 0.27\,p.u.$

Voltage in p.u. on motor & load (node 4)

$$V_4 = \frac{10.45}{11} = 0.95\ p.u.$$



$$Z_L = \frac{|V^2|}{S^*}\ (\Omega)$$

$$S_L = 57 \times [0.6 + j\sin[\cos^{-1}(0.6)]]$$

$$Z_L = \frac{0.95^2}{0.57(0.6 - j0.8)}$$

? lagging? No → $P + jQ = V \cdot I^*$

$P - jQ = \frac{V^2}{Z_L}$

$$Z_M = \frac{|V_m|^2}{S_m^*} = \frac{0.95^2}{\frac{66.5}{100}(0.85 - j0.53)}$$

× Calculate Active & Reactive Current delivered by the Generator

- $I_T = I_M + I_L = I_{\ell_1} + I_{\ell_2} = I_G$

$$= \frac{0.95 \times \frac{66.5}{100}(0.85 - j0.53)}{0.95^2} + \frac{0.57(0.6 - j0.8)}{0.95}$$

$$= 0.955 - j0.851$$

- $S_G = V_1 \cdot I_G^*$

$V_1 = V_4 + \Delta V_B = 0.9 + 0.253 + j0.287 = 1.203 + j0.287 = 1.24\angle 14^\circ$

$\Delta V_B = j\,x_{\ell(eq)} \times I_T = 0.253 + j\,0.287$

$x_{\ell(eq)} = (x_{T_1} + x_{\ell_1} + x_{T_2}) // (x_{T_3} + x_{\ell_2} + x_{T_4})$

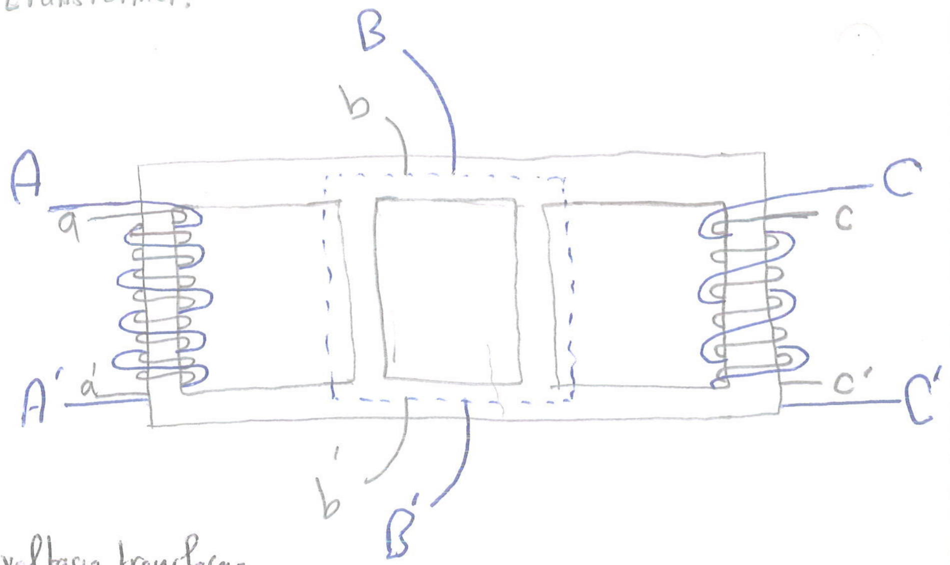
$= 0.3\,p.u$

- $S_G(MVA) = S_G \times S_B$

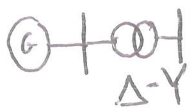
3.4: Three Phase Transformers Connection and Phase Shift

* Four Ways to Connect transformer:

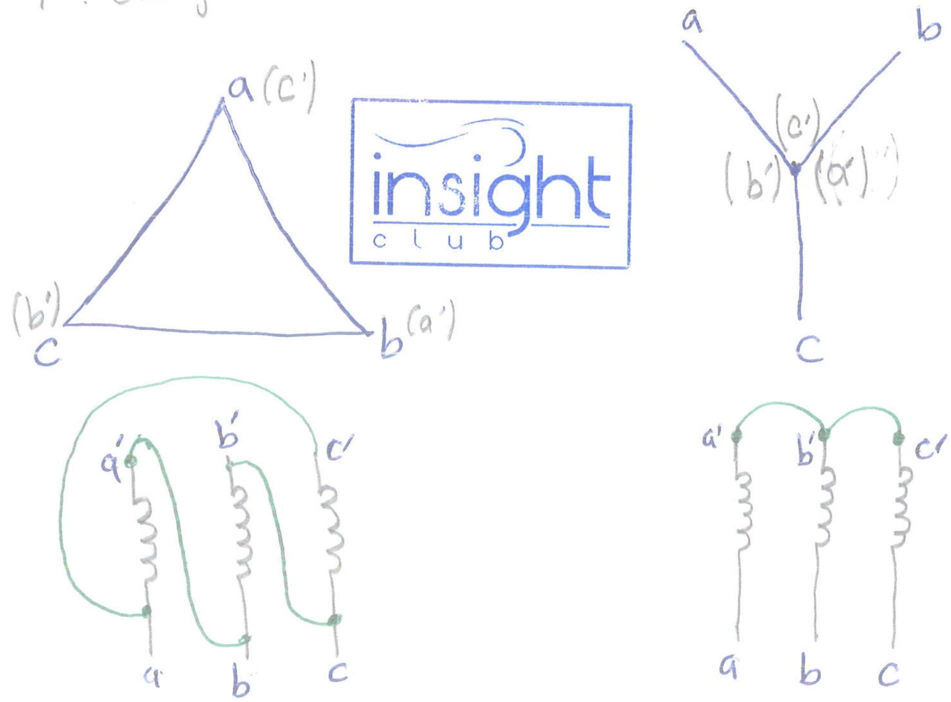
- Wye - Wye
- Delta - Delta
- Delta - Wye
- Wye - Delta

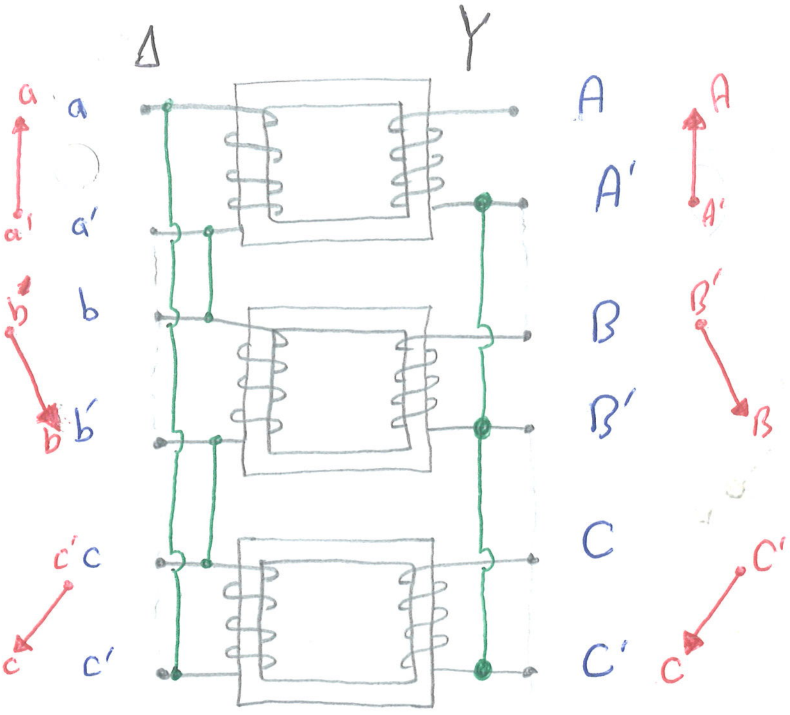


N.B: * Y-Y → Too extra High voltage transforme
→ $a = \frac{N_1}{N_2} = \left(\frac{t}{n}\right)$
→ no phase shift

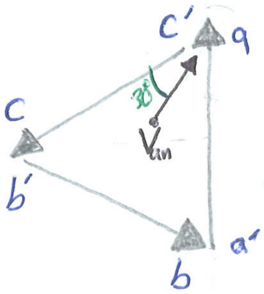
* 

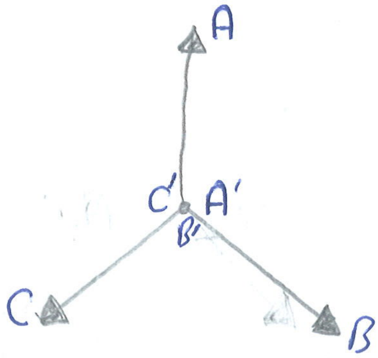
* Δ - Y : Configuration





N.B: $c'c$ inphase $C'C$
$b'b$ inphase $B'B$

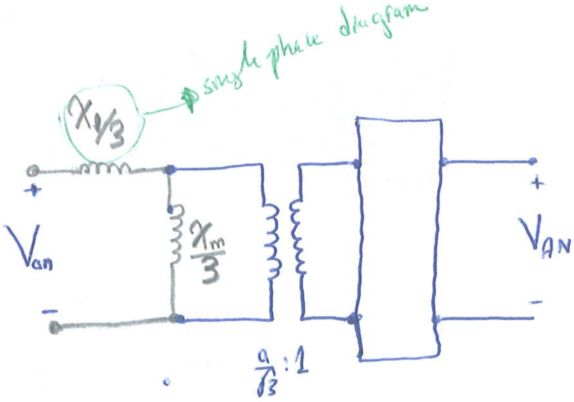


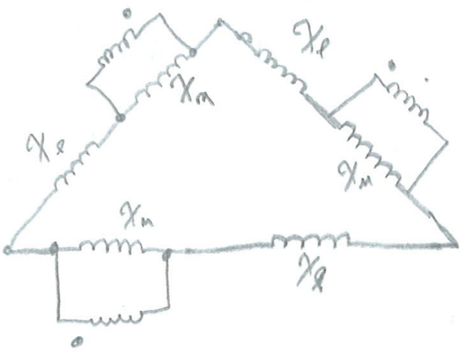


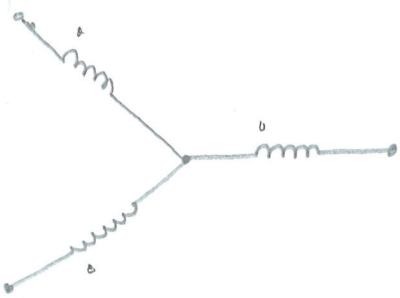
N.B.:

$$V_{ab} = a V_{AN}$$

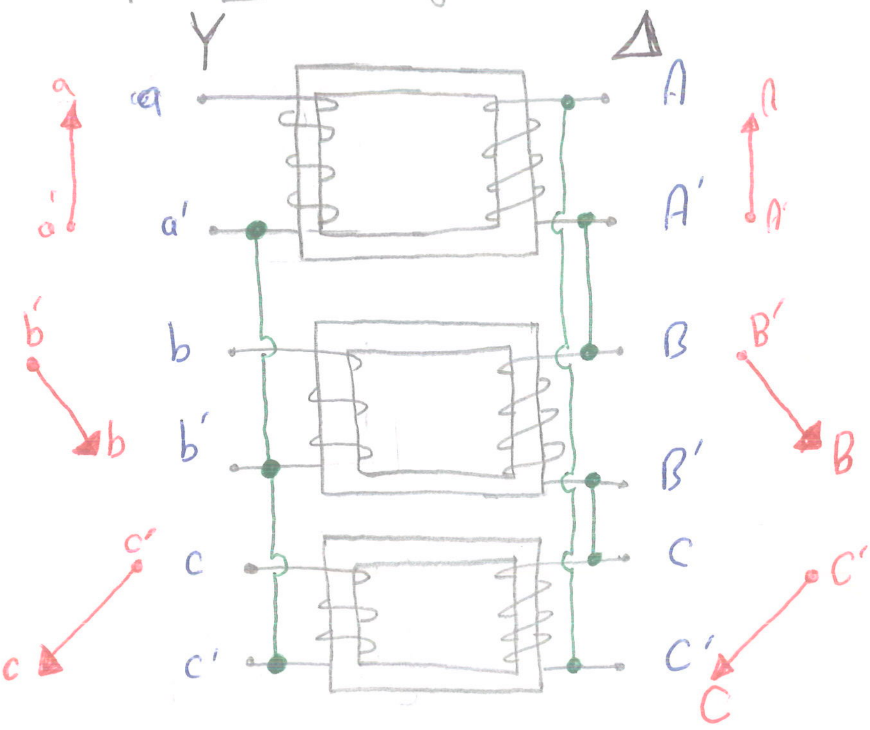
$$V_{an}\sqrt{3}e^{j30^\circ} = a V_{AN}$$

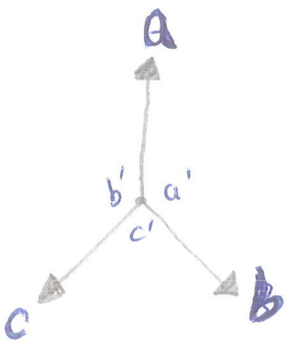


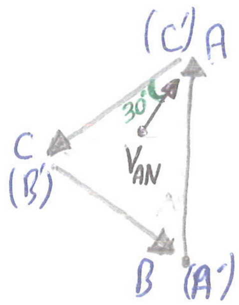




* Y – Δ : Configuration

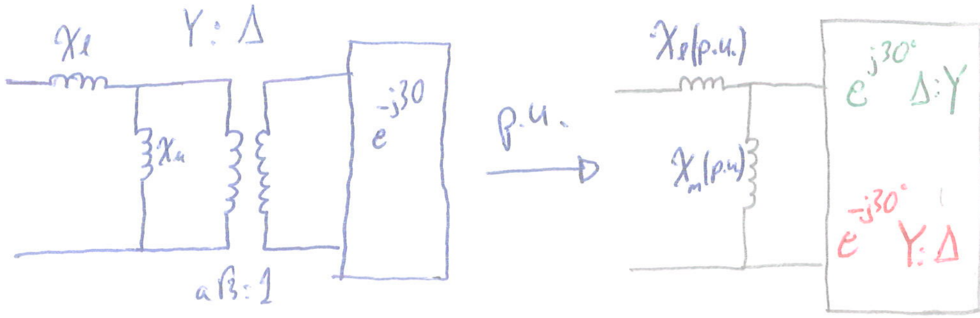






N.B.:

- $V_{aa'} = a V_{AA'}$
- $V_{an} = a V_{AN} \cdot \sqrt{3}\, e^{j30}$
- If H.V. side leads L.V side by 30°, then its American Standard.



* Three Winding Transformers

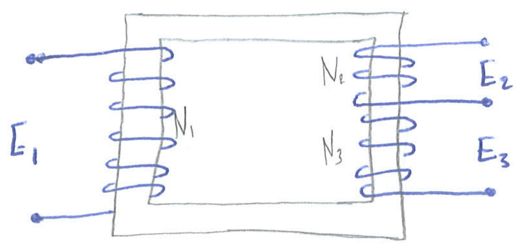
used for connection of shunt capacitors and inductors

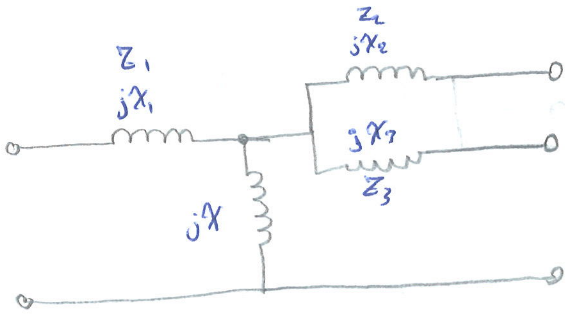
* $Q = \frac{V^2_{(kV)}}{X_{(\Omega)}} \, (MVAr)$

$= V^2 \cdot \underset{\omega C}{Y}$; when $V \nearrow$; $Y \searrow$ then $C \searrow$

* $N_1 I_1 = N_2 I_2 = N_3 I_3$

$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3}$





* Short Circuit Test $\longrightarrow$ leakage impedance

$Z_{12} = Z_1 + Z_2$ p.u. leakage impedance
measured from coil w1 with w2 shorted with w3 opened

$Z_{13} = Z_1 + Z_3$ p.u leakage impedance
measured from coil w_1 with w3 shorted with w2 opened

$Z_{23} = Z_2 + Z_3$ p.u. leakage impedance
measured from coil w2 with w_3 shorted with w_1 opened

* $Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23})$
* $Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13})$
* $Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12})$

Example 3.9 : Single Phase 3winding Transformer

w1: 300 MVA ; 13.8 KV

w2 : 300 MVA ; 199.2KV

w3 : 50 MVA ; 19.92 KV

- $X_{12} = 0.1$ p.u. on 300 MVA ; 13.8KV base
- $X_{13} = 0.16$ p.u. on 50 MVA ; 13.8KV base
- $X_{23} = 0.14$ p.u. on 50 MVA ; 199.2 KV base

- Find X_1 ; X_2 ; X_3 refered to $\boxed{S_B = 300\,MVA}$

- $Z_{B(1)} = \frac{13.8^2}{300} = 0.634\,\Omega$
- $Z_{B(2)} = \frac{13.8^2}{50} = 3.8\,\Omega$
- $Z_{B(3)} = \frac{199.2^2}{50} = 793.6\,\Omega$

$S_B = 300$ MVA

→ $X_{13} = 0.16 * \frac{300}{50} = 0.96$ $Z'_{B1} = \frac{13.8^2}{300} = 0.634$

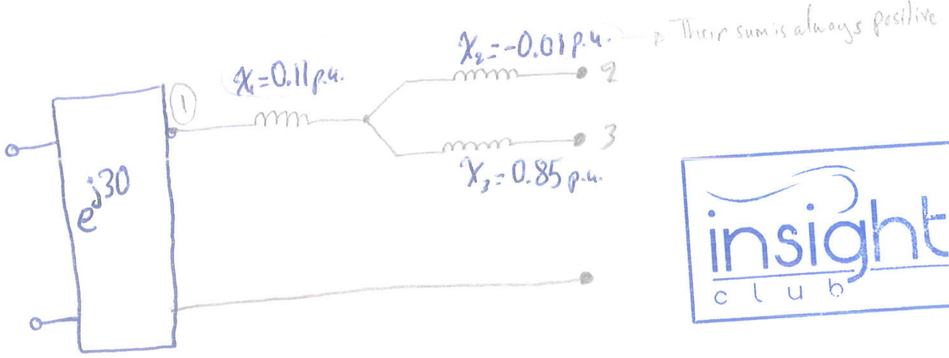
→ $X_{23} = 0.14 * \frac{300}{50} = 0.84$

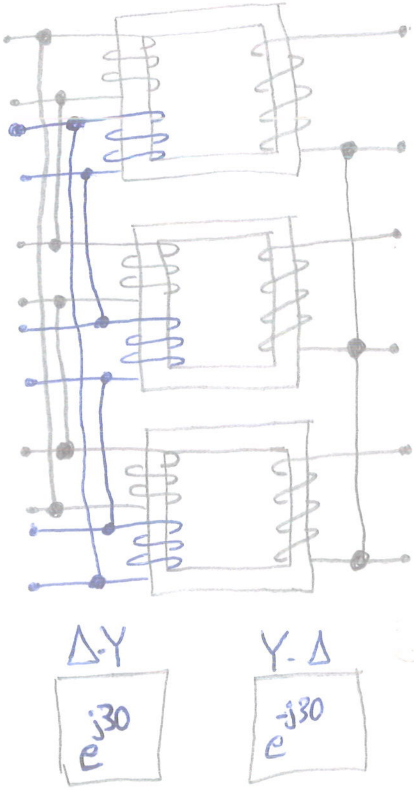
Example 3.10

13.8KV winding ($\overset{1}{X}$) : Δ to generator

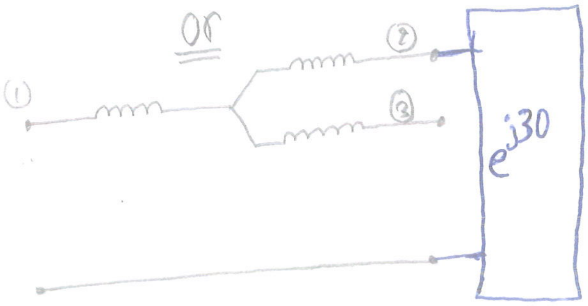
199.2KV winding ($\overset{2}{H}$) : Y - grounded connected to 345 KV line $(199.2\sqrt{3} = 345)$

19.92KV winding ($\overset{3}{M}$) : Y - grounded to 34.5 KV



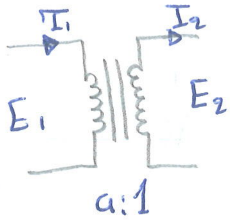


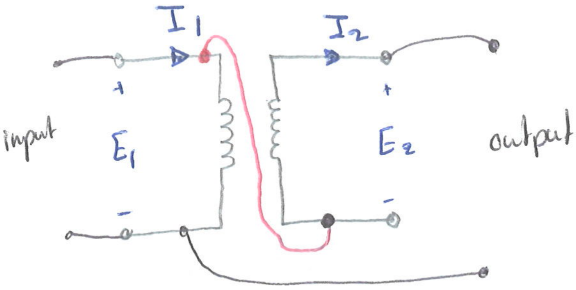
or

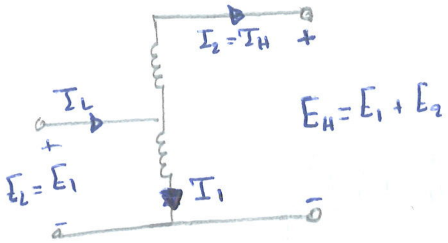


3.7: Auto transformers

$S_t = E_1 I_1 = E_2 I_2$ (rated VA of transformer)

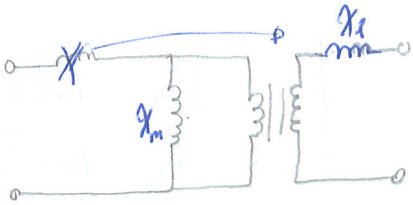


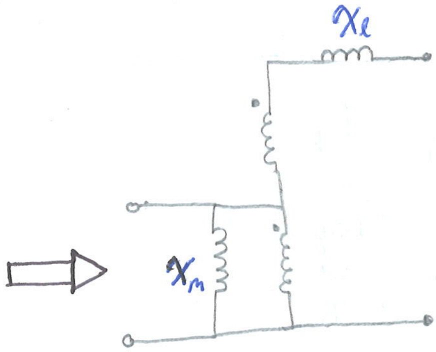




- $S_a = E_1 (\overbrace{I_1 + I_2}^{I_L}) = E_1 (I_1 + a I_1) = E_1 I_1 (1 + a)$

$$\boxed{S_a = S_t (1+a)}$$





$X_l(p.u) = X_l \cdot \frac{I_2}{E_2}$

$X_l(p.u) = X_l \cdot \frac{I_2}{E_1 + E_2} = \frac{I_2}{E_2(1+a)}$

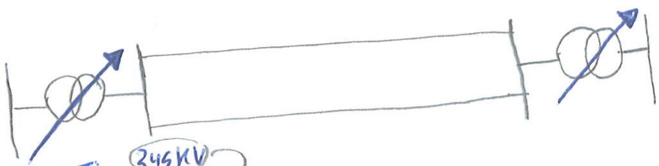
$\frac{E_1}{E_2} = a$

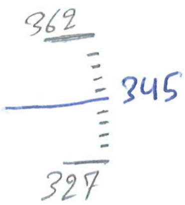
$X_m(p.u) = X_m \cdot \frac{I_1}{E_1}$

$X_m(p.u.) = X_m \cdot \frac{I_1}{E_1}(1+a)$

3.8: Transformers with off-Nominal turns Ratio.

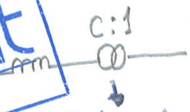
13.8/345 KV (American Standards)





345KV

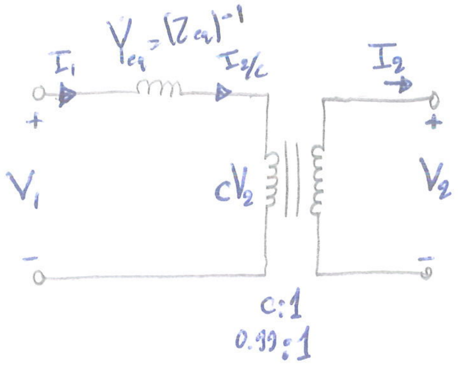
348KV

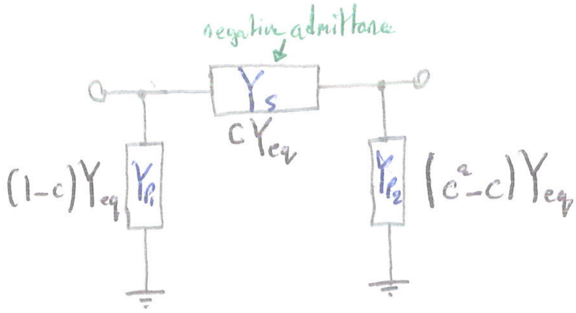


fictitious transformer to raise voltage

$\therefore c = \frac{345}{348} = 0.99$

on-nominal tap ratio





$$\frac{I_2}{c} = (V_1 - cV_2)\,Y_{eq}$$

$$I_2 = (cV_1 - c^2 V_2)\,Y_{eq}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{eq} & -cY_{eq} \\ cY_{eq} & -c^2 Y_{eq} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- $Y_{11} = Y_s + Y_{P_1} = Y_{eq}$
- $Y_{12} = Y_{21} = -cY_{eq} = -Y_s$
- $Y_{22} = Y_s + Y_{P_2} = c^2 Y_{eq}$

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ -Y_{21} & -Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$