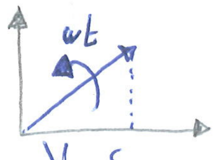
Chapter 2: Fundementals

2.1: Phases

$$v(t) = V_{max} \cdot \cos(\omega t + \delta) \xrightarrow[\text{if all signals have same frequency}]{\mathcal{P} \text{ phasor transformation}} \frac{V_{max}}{\sqrt{2}} \cdot e^{j\delta} = \frac{V_{max}}{\sqrt{2}} \angle \delta$$

$$= \frac{V_{max}}{\sqrt{2}} (\cos\delta + j\sin\delta)$$

$$v(t) = \sqrt{2} \cdot V \cdot \cos(\omega t + \delta) \xleftarrow{\mathcal{P}^{-1}}$$

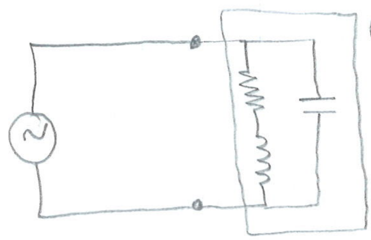


$V\cos\delta$

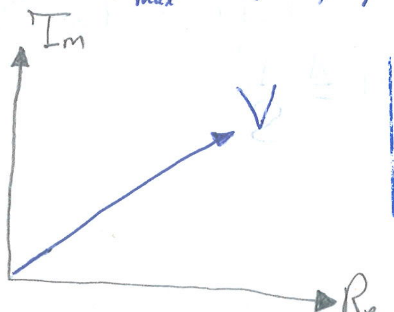
Time domain signal is obtained by rotating phasor at given regular speed ω.

2.2: Instantaneous Power in Single Phase AC circuits

General RLC Loads



* $v(t) = V_{max} \cdot \cos(\omega t + \delta)$
* $i(t) = I_{max} \cdot \cos(\omega t + \beta)$



Note: phasor domain should be in "cosine" format

I: current * capacitive leading
* inductive lagging

$$V = \frac{V_{max}}{\sqrt{2}} \qquad I = \frac{I_{max}}{\sqrt{2}}$$

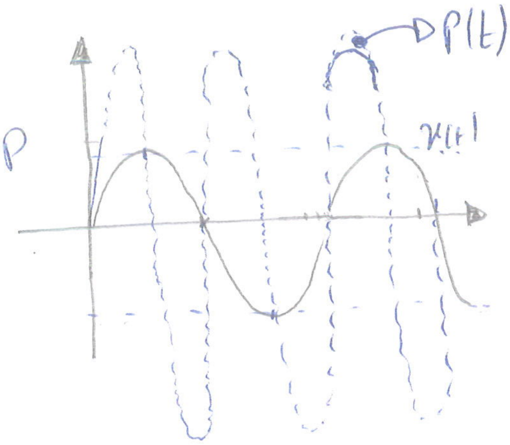
* $P(t) = v(t) \cdot i(t)$

$$= \frac{1}{2} V_{max} \cdot I_{max} \left[2 \cdot \cos(\omega t + \delta) \cdot \cos(\omega t + \beta)\right]$$

$$= \underbrace{\boxed{V \cdot I \cdot \cos(\delta - \beta)}}_{P} \left\{1 + \cos[2(\omega t + \delta)]\right\}$$

$$+ \underbrace{\boxed{V \cdot I \cdot \sin(\delta - \beta)}}_{Q} \cdot \sin[2(\omega t + \delta)]$$

$$= P\left[1 + \cos(2(\omega t + \delta))\right] + Q \cdot \sin(2(\omega t + \delta))$$



Note: Inductive Load consumes positive reactive power.

* Capacitive load consumes negative reactive power.

generate positive

2.3: Complex Power

$$\vec{V} = Ve^{j\delta}$$
$$\vec{I} = Ie^{j\beta}$$

Define Complex power S as:

$$S \triangleq V \cdot I^* = (Ve^{j\delta})(Ie^{j\beta})^*$$
$$= VIe^{j(\delta-\beta)}$$
$$= VI[\cos(\delta-\beta) + j\sin(\delta-\beta)]$$
$$= P + jQ$$

$$S = P + jQ \longrightarrow P(t) = P \cdot [1 + \cos(2(\omega t+\delta))] + Q \cdot [\sin(2(\omega t+\delta))]$$

$$V \cdot I^* = P + jQ$$

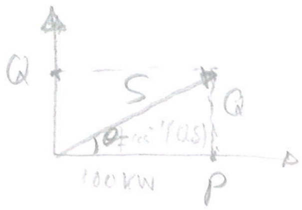
Note: power Factor $= \cos(\delta - \beta)$

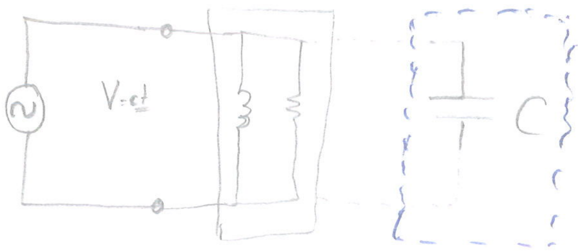
$$pF \triangleq \frac{P}{S}$$

Example 2.3

* 100 KW at 0.8 pf lagging
* $Q_c = ??$ to make pf = 0.95 lagging

Before Adding Capacitor





$$\theta_L = \delta - \beta_L = \cos^{-1}(0.8) = 36.9°$$
$$Q_L = P_L \cdot \tan(\theta_L) = 100 \cdot \tan(36.9) = 75 \text{ KVAr}$$
$$S_L = \frac{P}{\cos(\theta_L)} = 125 \text{ KVA}$$

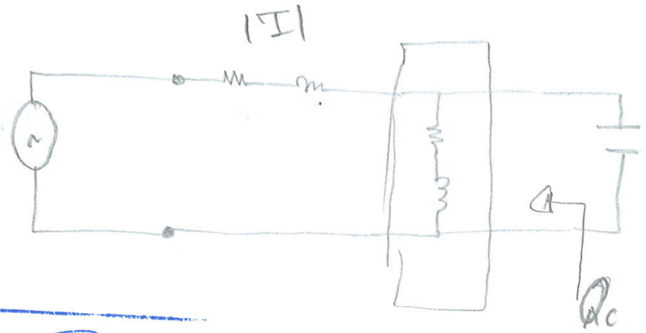
After Adding Capacitor

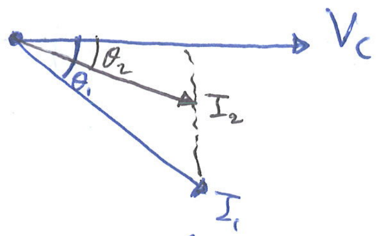
$$\theta_{system} = \delta - \beta_s = \cos^{-1}(0.95) = 18.2^\circ$$

$$Q_s = \overset{\text{same}}{P_L} \cdot \tan(\theta_s) = 32.89\,\text{KVAr}$$

$$|S_s| = \frac{P_L}{\cos\theta_s} = 105.3\,\text{KVA}$$

$$Q_c = Q_L - Q_s = 75 - 32.89 = 42.1\,\text{KVAr}$$



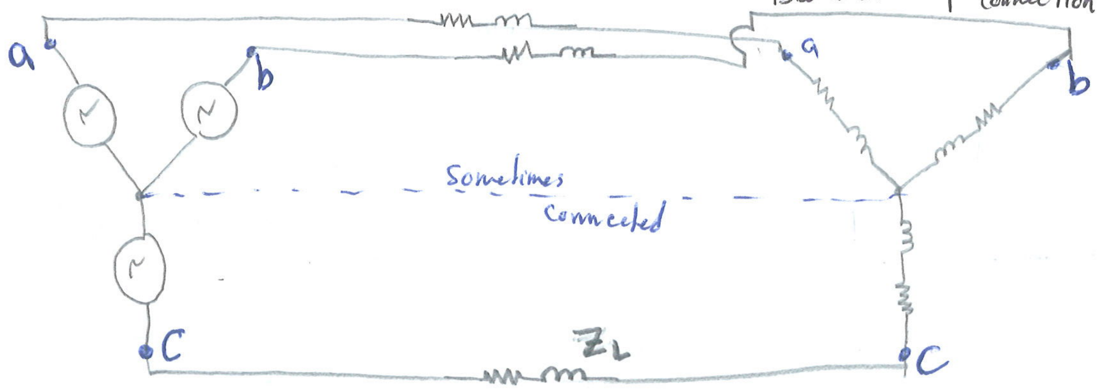


- $|I_2|^2 R < |I_1|^2 R$
- $P_1 = V_L (I_{L_1} \cos\theta_1)$

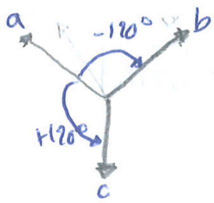
Vecl $\quad P_2 = V_L \cdot I_{L_2} \cos\theta_2$

2.5 Balanced 3-Φ circuits

Balanced Y connection



- $E_a = E\angle 0$
- $E_b = E\angle -120^\circ$
- $E_c = E\angle +120^\circ$



$$I_a = \frac{E_a}{Z_L + Z_Y} \qquad I_a = I\angle -\theta_2$$

$$I_b = \frac{E_b}{Z_L + Z_Y} \qquad I_b = I\angle -120^\circ - \theta_2$$

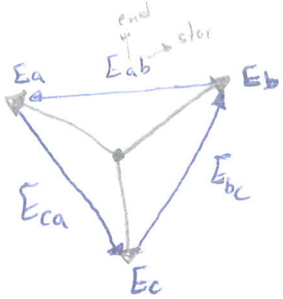
$$I_c = \frac{E_c}{Z_L + Z_Y} \qquad I_c = I\angle -240^\circ - \theta_2$$

example

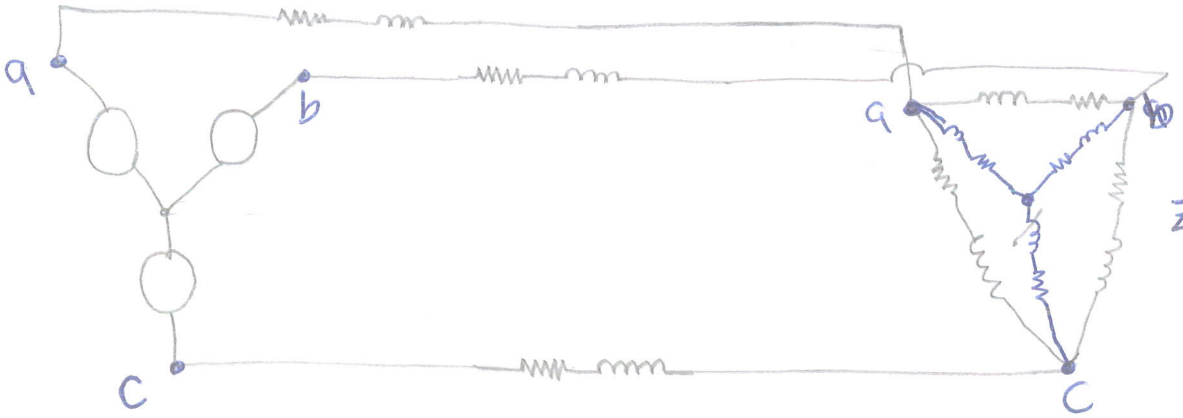
$$E_{ab} = \sqrt{3}\cdot E_a e^{j30} = \sqrt{3}\, E \angle 30^\circ = E_{an} - E_{bn}$$

$$E_{bc} = \sqrt{3}\cdot E\, e^{j(30-120)} = \sqrt{3}\, E \angle 30-120$$

$$E_{ca} = \sqrt{3}\, E \angle 30-240$$



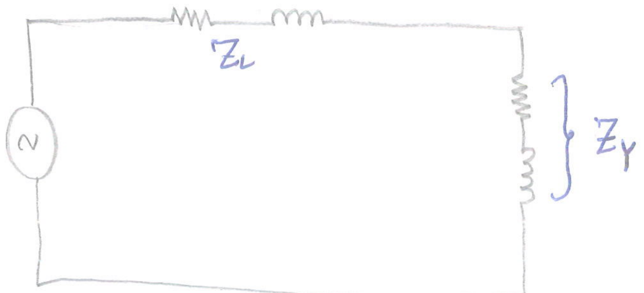
Balanced Δ Connection



$$Z_\Delta = 3 Z_Y$$

Verify that $I_a + I_b + I_c = 0$ for a Δ-connected Load by considering KCL at Nodes A, B, C at load side

$$I_a = I_{AB} - I_{CA}$$



or equivalent Z_Y of Δ-load

2.6: Power in Balanced 3-Φ Circuit

$$v_{an}(t) = \sqrt{2}\, V_{ln} \cos(\omega t + \delta)$$
b, c: -120, -240

$$i_a(t) = \sqrt{2}\, I_l \cos(\omega t + \beta)$$
b, c: -120, -240

$$P(t) = v_{an} i_a + v_{bn} i_b + v_{cn} i_c$$
$$= 3 V_{ln} I_l \cos(\delta - \beta) + V_{ln} I_l \left[\cos(2\omega t + \delta + \beta) + \cos(2\omega t + \delta + \beta - 120^\circ) + \cos(2\omega t + \delta + \beta - 240^\circ)\right]$$

0 zero $\forall t$

Complex Power

$$V_{an} = V_{ln} \angle \delta \qquad I_a = I_l \angle \beta$$

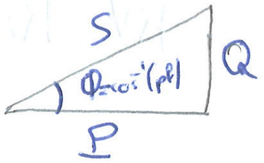
$$* \; S_a = V_{an} I_a^* = V_{ln} I_l \angle \delta - \beta$$
$$* \; S_b = V_{bn} I_b^* = V_{ln} I_l \angle \delta - \beta$$
$$* \; S_c = V_{cn} I_c^* = V_{ln} I_l \angle \delta - \beta$$

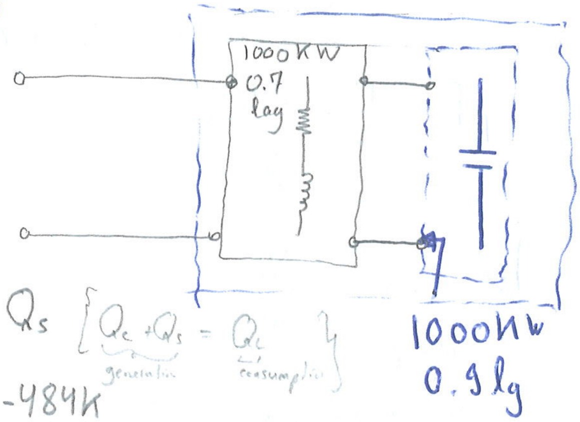
Same on all phases.

Problems

Problem 2.21

a) $Q_c = ??!$





Load

$\cdot \; \dfrac{Q_L}{P} = \tan\phi = \tan[\cos^{-1}(0.7)]$

$\hookrightarrow Q_L = 1000K * \tan[\cos^{-1}(0.7)]$
$= 1020\,KVAr$

$\cdot \; Q_s = 1000K * \tan[\cos^{-1}(0.9)]$
$= 484\,KVAr$

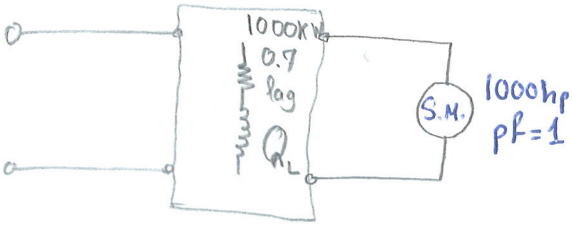
$Q_c = Q_L - Q_s \quad \{\underbrace{Q_c + Q_s}_{generation} = \underbrace{Q_L}_{consumption}\}$
$= 1020K - 484K$
$= 536\,KVAr$ generated by capacitor

or

$Q_c = Q_s - Q_L \quad \{\underbrace{Q_c + Q_L}_{consumed} = \underbrace{Q_s}_{generated}\}$
$= 484K - 1020K$
$= -536\,KVAr$ consumed by capacitor

b) $pf' = ?!$

New System (with S.M.)

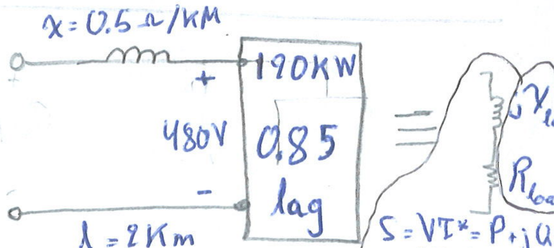


$S_s = S_L + S_{S.M.}$
$= (1000KW + j1020KVAr) + (746K + j0)$
$= (1746K + j1020K)$
$\hookrightarrow pf' = \cos\left[\tan^{-1}\left(\frac{1020}{1746}\right)\right]$
$= 0.864$

Note: $1000hp = 746KW$

Problem 2.26:

a) Use Complex Power



$P + jQ = VI^*$

$\hookrightarrow I = \dfrac{V^*_{Load}}{P_{Load} - jQ_{Load}} = \dfrac{480}{120K[1 - j0.6]}$ $\triangleright \tan[\cos^{-1}(0.85)]$

$\boxed{|I| = 0.294KA}$

$S = VI^* = P + jQ$

$\dfrac{V \cdot V^*}{Z^*} = \dfrac{|V|^2}{Z^*}$

$\boxed{Z = \dfrac{|V|^2}{P - jQ}}$

OR

$$VI^* = S \longrightarrow |I| = \frac{|S|}{V} = \frac{120}{\underbrace{\cos\varphi}_{0.85}} \cdot \frac{1}{480} = 0.294\,KA$$

$$Q_{Line} = (294)^2 \cdot 1 = 86436$$
$$= 86.436\ KVAr$$

$$Q_{system} = Q_{Load} + Q_{Line} = 120 \cdot 0.62 + 86.436$$
$$= 160.8\ KVAr$$

$$pf = \frac{P}{S} = \frac{120K}{\sqrt{(120K)^2 + (160.8K)^2}} = 0.6$$

Note: Consistant set of Unit

V	A	VA	W
KV	KA	MVA	MW

b) i) $Z = \frac{|V|^2}{P - jQ}$

ii) Calculate I

ii) Calculate $Q_{Load} = I^2 X_{Load}$

$Q_{Line} = I^2 \cdot X_{Line}$

iv) $Q_{Total} = Q_{sys} = Q_{Load} + Q_{Line}$

v) $pf = \cos\left[\tan^{-1}\left(\frac{Q_s}{P_{Load}}\right)\right]$