**Fundamentals of Power Systems Analysis (EECE 471)**
**FORMULAE**

- Chapter 2: **Basic Principles**

$$Z_Y = \frac{Z_\Delta}{3}$$

- Chapter 3: **Transformers and the Per-Unit System**

$$V_{a'n'} = K \quad V_{an} \qquad \text{and} \qquad I_{a'n'} = \frac{1}{K^*} \quad I_{an}$$

$$\Delta - Y : K = \frac{\sqrt{3}}{a} e^{j\frac{\pi}{6}} \qquad Y - \Delta : K = \frac{1}{a\sqrt{3}} \quad e^{-j\frac{\pi}{6}} \qquad a = \frac{N_1}{N_2}$$

$$Z_B = \frac{{V_B^{ll}}^2}{S_B^{3\Phi}} = \frac{{V_B}^2}{S_B} \qquad Z_{actual} = Z_{pu} Z_B \qquad Z_{pu}^n = \frac{Z_{actual}}{Z_B^n} = Z_{pu}^o \frac{Z_B^o}{Z_B^n}$$

- Chapter 4: **Transmission-Line Parameters**

$$\rho_2 = \rho_1 \left( \frac{T_2 + T}{T_1 + T} \right)$$

$$l = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_{SL}} \quad H/m \qquad \varepsilon = 8.854 \times 10^{-12} \quad F/m$$

$$D_{eq} = \sqrt[3]{D_{ab} D_{ac} D_{bc}} \qquad D_{SL} = \sqrt[4]{r' d_{12} d_{13} d_{14}}$$

$$c = \frac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_{SC}}} \qquad D_{SC} = \sqrt[4]{r \quad d_{12} d_{13} d_{14}}$$

$$E_{r\max} = \frac{q}{2\pi\varepsilon r} \left( \frac{1}{2} + \frac{r}{2d} \right) \qquad \text{2-bundles}$$

$$E_{r\max} = \frac{q}{2\pi\varepsilon r} \left( \frac{1}{4} + \frac{r}{d\sqrt{2}} + \frac{r\sqrt{2}}{d} \right) \qquad \text{4-bundles}$$

$$E = \left( \frac{q}{2\pi\varepsilon} \right) \frac{2\cos\theta}{\sqrt{y^2 + x^2}}$$

- Ch. 5: **Transmission Line Modeling**
  - $z = r + j\omega l \quad \Omega / m \qquad y = j\omega c \quad S/m$
  - $\gamma = \sqrt{yz} \qquad Z_c = \sqrt{\frac{z}{y}}$

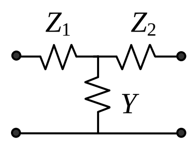
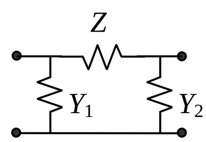
- $V_1 = V_2 \quad \cosh \gamma l + Z_c I_2 \quad \sinh \gamma l = AV_2 + BI_2$

  $I_1 = I_2 \quad \cosh \gamma l + \dfrac{V_2}{Z_c} \quad \sinh \gamma l = CV_2 + DI_2$

- $A = D = 1 + \dfrac{ZY}{2} \qquad B = Z \qquad C = Y\left(1 + \dfrac{ZY}{4}\right)$

- $\dfrac{Y'}{2} = \dfrac{Y}{2}\dfrac{\tanh(\gamma l/2)}{(\gamma l/2)} \qquad Z' = Z\dfrac{\sinh(\gamma l)}{(\gamma l)}$

- $T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ and $T^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$

| | |
|---|---|
|   | $\begin{bmatrix} (1+YZ_1) & (Z_1 + Z_2 + YZ_1Z_2) \\ Y & (1+YZ_2) \end{bmatrix}$ |
|   | $\begin{bmatrix} (1+Y_2Z) & Z \\ (Y_1 + Y_2 + Y_1Y_2Z) & (1+Y_1Z) \end{bmatrix}$ |

- Complex Power Flow at the sending end on Medium Line:

  $$S_{12} = \frac{Y^*}{2} \quad |V_1|^2 + \frac{|V_1|^2}{Z^*} - \frac{|V_1||V_2|}{Z^*} e^{j\theta_{12}}$$

  For $S_{21}$ exchange indices 1 and 2 in above equation. For a long line use *Y'* and *Z'*.

- Power Flow on a short loss-less line:

  $$P_{12} = -P_{21} = \frac{|V_1||V_2|}{X}\sin\theta_{12} \qquad Z = z \times l = R + jX$$

  $$Q_{12} = \frac{|V_1|^2}{X} - \frac{|V_1||V_2|}{X}\cos\theta_{12} \qquad Y = y \times l$$

  $$Q_{21} = \frac{|V_2|^2}{X} - \frac{|V_1||V_2|}{X}\cos\theta_{12}$$ 

  *l*: length of line

- Power transmission capability:

  $$P_{12} = V_{1pu}V_{2pu}\frac{|V_R|^2}{Z_c}\frac{\sin\theta_{12}}{\sin\beta l} = V_{1pu}V_{2pu}P_{SIL}\frac{\sin\theta_{12}}{\sin\beta l}$$

  $$\beta = \mathrm{Im}(\gamma) \cong \frac{2\pi}{\lambda}$$

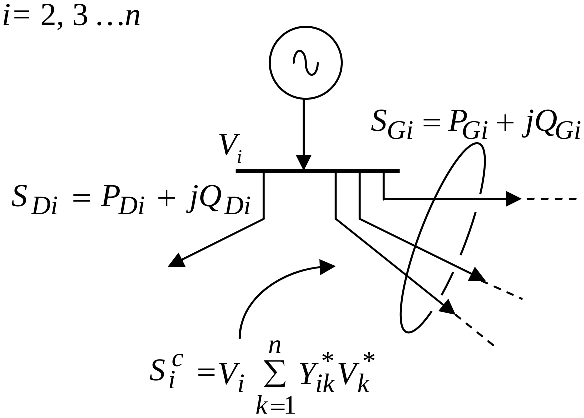
- Ch. 6: **Power Flow Analysis**

$$S_i = V_i I_i^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^* \qquad i = 2, 3 \ldots n$$

$$S_i \stackrel{\Delta}{=} S_{Gi} - S_{Di}$$

$$Y_{ik} = G_{ik} + jB_{ik}$$

$$\theta_{ik} = \theta_i - \theta_k$$




$$P_i^c = \sum_{k=1}^{n} V_i V_k \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right)$$

$$Q_i^c = \sum_{k=1}^{n} V_i V_k \left(G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}\right)$$

- Gauss-Seidel Load Flow at iteration $r$:

$$V_i^r = \frac{1}{Y_{ii}} \left[ \frac{S_i^*}{\left[V_i^{r-1}\right]^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^r - \sum_{k=i+1}^{n} Y_{ik} V_k^{r-1} \right] \qquad i = 2, 3 \ldots n$$

- Newton Power Flow:

$$\begin{bmatrix} \theta^{k+1} \\ V^{k+1} \end{bmatrix} = \begin{bmatrix} \theta^{k} \\ V^{k} \end{bmatrix} - \left[ \begin{array}{c|c} J_1 & J_2 \\ J_3 & J_4 \end{array} \right]^{-1} \begin{bmatrix} \Delta P\left(\theta^k, V^k\right) \\ \Delta Q\left(\theta^k, V^k\right) \end{bmatrix}$$

$$\Delta P = P_c - \left(P_G - P_D\right) \quad \text{and} \quad \Delta Q = Q_c - \left(Q_G - Q_D\right)$$

- $J_{ik}^1 = \dfrac{\partial \Delta P_i}{\partial \theta_k} = V_i V_k \left(G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}\right)$
- $J_{ii}^1 = \dfrac{\partial \Delta P_i}{\partial \theta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} V_i V_k \left(-G_{ik} \sin\theta_{ik} + B_{ik} \cos\theta_{ik}\right)$
- $J_{ik}^2 = \dfrac{\partial \Delta P_i}{\partial V_k} = V_i \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right)$
- $J_{ii}^2 = \dfrac{\partial \Delta P_i}{\partial V_i} = 2 V_i G_{ii} + \sum_{k \neq i} V_k \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right)$
- $J_{ik}^3 = \dfrac{\partial \Delta Q_i}{\partial \theta_k} = -V_i V_k \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right)$
- $J_{ii}^3 = \dfrac{\partial \Delta Q_i}{\partial \theta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} V_i V_k \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right)$

- $J_{ik}^{4} = \dfrac{\partial \Delta Q_i}{\partial V_k} = V_i\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right)$
- $J_{ii}^{4} = \dfrac{\partial \Delta Q_i}{\partial V_i} = -2V_i B_{ii} + \sum_{k\neq i} V_k\left(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}\right)$

- **Operating Limit on Synchronous Generators**
  - $|S_G| \le 1$
  - $P_G + j\left(Q_G + \dfrac{|V_a|^2}{X_s}\right) = \dfrac{|V_a|E_{\max}}{X_s}(\sin\delta_m + j\cos\delta_m)$
  - $Q_G = \dfrac{1}{\tan\delta_{\max}}P_G - \dfrac{|V_a|^2}{X_S}$

- Ch. 7: **Three-Phase Short Circuit**

  For a fault at node *k*:

  - $$\begin{bmatrix} V_{F1} \\ V_{F2} \\ \vdots \\ V_{Fk} \\ \vdots \\ V_{Fn} \end{bmatrix} + \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & & Z_{2k} & & Z_{2n} \\ \vdots & & \ddots & & & \vdots \\ Z_{k1} & Z_{k2} & & Z_{kk} & & Z_{kn} \\ \vdots & & & & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I_{Fk} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_k \\ \vdots \\ E_n \end{bmatrix}$$

- Ch.11: **Economic Operation of Power Systems**
  - Lines Losses and Generators' Limits:

    Let $C_i(P_{Gi}) = \alpha + \beta P_{Gi} + \gamma P_{Gi}^{\;2}$

    Minimize: $C_T = \sum_{i=1}^{m} C_i(P_{Gi})$

    Subject to: $\sum_{i=1}^{m} P_{Gi} - P_D - P_L(P_{G1},...,P_{Gm}) = 0$

    And $P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}$ *i*= 1…*m*

    $\lambda = \dfrac{dC_i(P_{Gi})}{dP_{Gi}}(L_i)$ $L_i = \dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_{Gi}}}$ *i*= 1…*m*

  - Power loss in MW on a line of impedance $R + jX$ in per unit:

    $P_{LOSS} = \dfrac{X^2 P^2 R}{S_B(R^2 + X^2)}$