ELEG 071E POWER SYSTEMS FUNDAMENTALS
FINAL EXAM CORRECTION – Feb 4, 2002

PROBLEM 1

a) $r = \dfrac{0.3372}{1.605 \times 2} = 0.105\Omega / km$

$l = 2 \times 10^{-7} \ln \dfrac{D}{R_b}$

$with : D = \sqrt[3]{6 \times 6 \times 10} = 7.114m$

$R_b = \sqrt[2]{0.0229 \times 0.305 \times 0.3} = 0.0458m$

$Then : l = 2 \times 10^{-7} \ln \dfrac{7.114}{0.0458} = 1.01 \times 10^{-3} H / km$

$and : x = wl = 2\pi \times 50 \times 10.09 \times 10^{-7} \times 10^{+3} = 0.317\Omega / km$

b) $C = \dfrac{2\pi \times 8.854 \times 10^{-12}}{\ln\left(\dfrac{D}{R_{bc}}\right)} \times 10^{6} \times 10^{3} \, \mu F / km$

$with : R_{bc} = \sqrt[2]{0.009 \times 0.3} = 0.051m$

$Then : C = \dfrac{2\pi \times 8.854 \times 10^{-12}}{\ln\left(\dfrac{7.114}{0.052}\right)} \times 10^{6} \times 10^{3} = 0.0113 \mu F / km$

$and : y_c = wC = 2\pi \times 50 \times 0.0113 = 3.55 \mu S / km$

c) $I = 300 \times 0.5067 \times 2 \times 2 = 608A$

$|S| = \sqrt{3} \times 230 \times 608 \times 10^{-3} = 242MVA$

$P_{SIL} = \dfrac{V^2}{Z_C} = 177MVA$

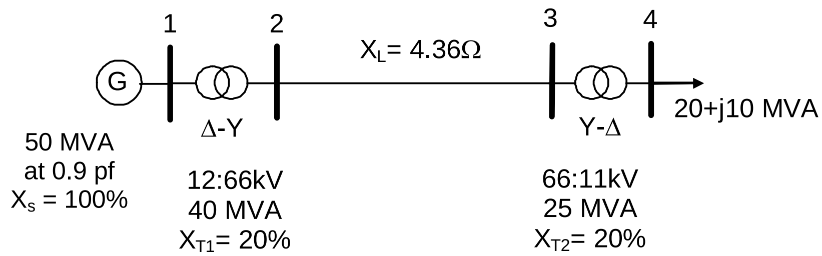
d) From the curve, for a short line of about 75 km, its power handling capability can be about $2.5 \times P_{SIL}$ which is much higher than 242 MVA

$P_{SIL} = \dfrac{V^2}{|Z_C|} = \dfrac{230^2}{299} = 177MVA$

$Then : \dfrac{P}{P_{SIL}} = \dfrac{242}{177} = 1.37$

From the line capability curve the distance is about about 200 miles.

PROBLEM 2



a) $S_B 10MVA$

$V_{B1} = 12KV$ line to line *(l-l)*

$V_{B2} = V_{B3} = 66KV(l-l)$

$V_{B4} = 11KV(l-l)$

$$Z_{B1} = \frac{12^2}{10} = 14.4\Omega$$

$$Z_{B2} = \frac{66^2}{10} = 435.6\Omega = Z_{B3}$$

$$Z_{B4} = \frac{11^2}{10} = 12.1\Omega$$

b) $P_D = 2 + j1 p.u.$

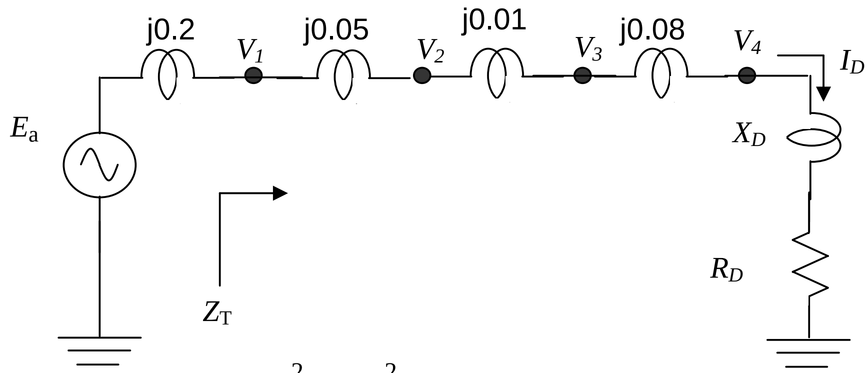
$$X_{t2} = 0.2 \times \frac{11^2}{25} \times \frac{10}{11^2} = 0.08 p.u.$$

$$X_{t1} = 0.2 \times \frac{12^2}{40} \times \frac{10}{12^2} = 0.05 p.u.$$

$$X_l = 4.36 \times \frac{10}{66^2} = 0.01 p.u.$$

$$X_S = 1.0 \times \frac{12^2}{50} \times \frac{10}{12^2} = 0.2 p.u.$$

c)



$$Z_D = R_D + jX_D = \frac{V_4}{I_D} = \frac{V_4^2}{V_4^* I_D} = \frac{V_4^2}{S_D^*} = \frac{1}{2 - j1} = 0.4 + j0.2\,p.u.$$

Total impedance seen by the generator is:

$$Z_T = 0.4 + j(0.05 + 0.01 + 0.08 + 0.2) = 0.4 + j0.34$$

Generator current in p.u.:

$$I_G = \frac{V_1}{Z_T} = \frac{1}{0.4 + j0.34} = 1.45 - j1.23\,p.u.$$

Total active and reactive power supplied by generator:

$$P_G + jQ_G = V_1 I_G^* = (1 + j0)(1.45 + j1.23)\,p.u.$$

In real quantities:

$$P_G + jQ_G = 14.5 + j12.3\,MVA$$

d) $V_4 = I_G \times Z_D = (1.45 - j1.23)(0.4 + j0.2) = 0.826 - j0.202 = 0.8503\angle -13.74^\circ\ p.u.$

$$Z_D = 0.5\,p.u.$$

$$Z_T = 0.5 + j(0.05 + 0.01 + 0.08) = 0.5 + j0.14$$

$$I_G = \frac{V_1}{Z_T} = \frac{1}{0.5 + j0.14} = 1.85 - j0.52$$

$$P_G + jQ_G = 18.5 + j5.2\,MVA$$

$$V_4 = (1.85 - j0.52)(0.5) = 0.927 - j0.26 = 0.963\angle -15.6^\circ\ p.u.$$

e) $P_{rated} = 50 \times 0.9 = 45\,MW = 4.5\,p.u.$

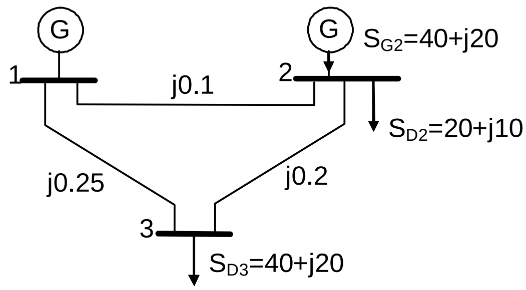
$$Q_G^{\max} = 50 \times \sin\left(\cos^{-1}(0.9)\right) = 50 \times 0.436 = 21.8\,MVAr$$

$$Q_G^{\min} = \frac{P_G}{\tan\delta_{\max}} - \frac{V_a^2}{X_S} = \frac{4.5}{1} - \frac{1}{0.2} = -0.5$$

$$\tan(45^\circ) = 1$$

$$Q_G^{\min} = -5\,MVAr$$

PROBLEM 3



a)

$$Y_{BUS} = j\begin{bmatrix} -14 & +10 & +4 \\ +10 & -15 & +5 \\ +4 & +5 & -9 \end{bmatrix}$$

b) $\Delta P_2 = V_1V_2 10\sin(\theta_2-\theta_1) + V_2V_3 5\sin(\theta_2-\theta_3) - 0.2$

$$\Delta P_3 = V_3V_1 4\sin(\theta_3-\theta_1) + V_3V_2 5\sin(\theta_3-\theta_2) + 0.4$$

$$\Delta Q_2 = -V_2V_1 10\cos(\theta_2-\theta_1) + V_2^2 15 - V_2V_3 5\cos(\theta_2-\theta_3) - 0.1$$

$$\Delta Q_2 = -V_3V_1 10\cos(\theta_3-\theta_1) - V_2V_3 5\cos(\theta_3-\theta_2) + V_3^2 9 + 0.2$$

c) $\underline{\theta} = \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}$ and $\underline{V} = \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$

$$\begin{bmatrix} \theta_2^{r+1} \\ \theta_3^{r+1} \end{bmatrix} = \begin{bmatrix} \theta_2^{r} \\ \theta_3^{r} \end{bmatrix} - \begin{bmatrix} 15 & -5 \\ -5 & 9 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2^r \\ \Delta P_3^r \end{bmatrix}$$

$$\begin{bmatrix} V_2^{r+1} \\ V_3^{r+1} \end{bmatrix} = \begin{bmatrix} V_2^{r} \\ V_3^{r} \end{bmatrix} - \begin{bmatrix} 15 & -5 \\ -5 & 9 \end{bmatrix}^{-1} \begin{bmatrix} \Delta Q_2^r \\ \Delta Q_3^r \end{bmatrix}$$

$$J^{-1} = \frac{1}{10}\begin{bmatrix} 9 & 5 \\ 5 & 15 \end{bmatrix} = \begin{bmatrix} 0.082 & 0.045 \\ 0.045 & 0.136 \end{bmatrix}$$

$\Delta P_2^\circ = -0.2$ $\quad$ $\Delta Q_2^\circ = -0.1$

$\Delta P_3^\circ = -0.4$ $\quad$ $\Delta Q_3^\circ = 0.2$

$$\begin{bmatrix} \Delta\theta_2^1 \\ \Delta\theta_3^1 \end{bmatrix} = J^{-1}\begin{bmatrix} -0.2 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.0016 \\ 0.045 \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_2^1 \\ \Delta V_3^1 \end{bmatrix} = J^{-1}\begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix} = \begin{bmatrix} 0.0008 \\ 0.0227 \end{bmatrix}$$

$$V_2 = 0.9992\angle -0.0016$$

$$V_3 = 0.9773\angle -0.045$$

PROBLEM 4

a)

$$7+\left(2\times 0.012P_{G1}\right)-\lambda=0 \qquad (1)$$
$$8+\left(2\times 0.018P_{G2}\right)-\lambda=0 \qquad (2)$$
$$P_{G1}+P_{G2}=700$$
$$P_{G1}=\frac{\lambda-8}{2\times 0.012} \qquad P_{G2}=\frac{\lambda-7}{2\times 0.018}$$
$$\Rightarrow 700=\lambda\left(\frac{1}{2\times 0.012}+\frac{1}{2\times 0.018}\right)-\left(\frac{7}{2\times 0.012}+\frac{8}{2\times 0.018}\right)=\lambda(69.44)-513.9$$
$$\lambda=\frac{700+513.9}{69.44}=17.48\$/MWh$$
$$(1)\Rightarrow P_{G1}=436.7MW$$
$$(2)\Rightarrow P_{G2}=263.3MW$$

b)

$$\lambda=\frac{1000+513.9}{69.44}=21.8\$/MWh$$
$$P_{G1}=616.7MW$$
$$P_{G2}=383.3MW$$

P_{G1} is above limit, so set it to limit.

$$P_{G1}=600MW$$
$$P_{G2}=400MW$$
$$\lambda=8+\left(2\times 0.018\times 400\right)=22.4\$/MWh$$
