**AMERICAN UNIV. OF BEIRUT**

**QUIZ-II-MATH.201(Thomas Sec.10.8-14.5)** **Oct. 31,2013**

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NAME:. I.D..

SECTION: 27 26 (2:00F) 27 (4:00F)

| Question | 1 | 2 | 3 | 4 | 5 | 6 | | QUIZ II GRADE |
|---|---|---|---|---|---|---|---|---|
| Max. | 21 | 14 | 16 | 18 | 15 | 16 | | |
| Gr. | 19 | 8 | 13 | 18 | 15 | 14 | | ~~87~~ 89.6 |



1. ***(21 Points)*** **Answer the following three independent parts:**

a) Let $w = xe^y + y\sin z - \cos z$, with $x = r + 2t$, $y = rt^2$, and $z = t^2$.

Find $\frac{\partial w}{\partial r}$ at $(r,t) = (2,1)$.

$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$

$= (e^y)(1) + (xe^y + \sin z)(t^2) + (y\cos z + \sin z)(0)$

$= (e^2)(1) + (4e^2 + \sin(1))(1) + (2\cos(1) + \sin(1))(0)$

$= e^2 + 4e^2 + \sin 1$

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b) Let $F(x,y,z) = x^3 + y^3 + z^3 + 6xyz - 1$, and assume that the equation $F(x,y,z) = 0$, defines $z$ as a differentiable function of $x$ and $y$. Find $\frac{\partial z}{\partial x}$, at $(x,y,z) = (1,2,3)$.

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\left[\frac{3x^2 + 6yz}{0 + 0 + 3z^2 + 6xy}\right]$

$\left.\frac{\partial z}{\partial x}\right]_{(1,2,3)} = -\left[\frac{39}{39}\right] = -1$

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c) Find the slope $(\frac{dy}{dx})$ of the tangent to the graph of the cardioid:

$r = f(\theta) = 1 - \cos\theta$, at $(r,\theta) = (\frac{1}{2}, \frac{\pi}{3})$.

$\boxed{f'(\theta) = \sin\theta}$ $\quad y = r\sin\theta \quad x = r\cos\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ ; $\frac{dy}{d\theta} = \frac{dr}{d\theta}\cdot\sin\theta + r\cos\theta$

$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cdot\cos\theta - r\sin\theta$

$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$ at $(\frac{1}{2}, \frac{\pi}{3})$ $\frac{dy}{dx} = \frac{1}{\frac{-2+\sqrt{3}}{2}}$

$\frac{\sin\theta\cdot\sin\theta + (1-\cos\theta)\cos\theta}{\sin\theta\cdot\cos\theta - (1-\cos\theta)\sin\theta}$

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$9-x^2-y^2>0$

$x^2+y^2<9$

**2. (14 Points)**

*You are given the function of three variables* : $f(x,y,z)=\dfrac{5}{\sqrt{9-x^2-y^2}}$.

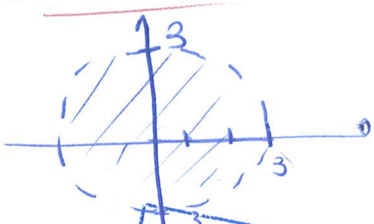
*a) Describe fully the domain of this function.*

$\sqrt{9-x^2-y^2}$ ; $9-x^2-y^2 > 0$. $-x^2-y^2 > -9$

so the domain is the ~~disk and~~ $x^2+y^2<9$ the interior of a disk of center (0,0) radius 3.

1

*b) What is the range?*



Range is [0,3].

0

*c) Does the domain have a boundary? If yes, what is it?*

Yes it has a boundary $x^2+y^2=9$ all (disk).

$1\frac{1}{2}$

*d) Is the domain open? Justify.*

~~Since at least 1 boundary point is included in the doma~~. Since no boundary point is included in the domain and all the points of the domain are interior then it is open.

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*e) Is the domain closed? Justify.*

Since not all (none of) the boundary points are included in the domain. then it is not closed.

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*f) Is the domain bounded? Justify.*

Yes since we can always have a bigger circle including the domain.

0

*g) Describe fully that level surface which contains the point P(−2,2,7).*

$\dfrac{5}{\sqrt{9-x^2-y^2}}=c$ ; $\dfrac{25}{9-x^2-y^2}=K$ where $K=c^2$

$5=c(\sqrt{9-x^2-y^2})$.

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$25=K(9-x^2-y^2)$ ; $\dfrac{25}{K}-9=-x^2-y^2$.

so the level curve are circles of center (0,0) and radius less than 3. $\boxed{-\dfrac{25}{K}+9=x^2+y^2}$

3. (**16 Points**)

*You are given a function $f(x,y)$ and the point $P_0(3,2)$.*

*The directional derivative of $f$ at $P_0$, in the direction of $P_1(6,-2)$ is 0.4.*

*The directional derivative of $f$ at $P_0$, in the direction of $P_2(7,-1)$ is $-0.2$.*

*Find the gradient of $f$ at $P_0$.*

$D_u f|_{(6,-2)} = 0.4 = \vec{\nabla f}|_{(6,-2)} \cdot (\vec{P_1P_0})$.

$\boxed{\vec{P_1P_0} = -3\vec{i} + 4\vec{j}}$

let $\vec{\nabla f} = A\vec{i} + B\vec{j}$.

so $\boxed{0.4 = -3A + 4B}$

so $\cancel{\vec{\nabla f} = \frac{4}{135}\vec{i} - \frac{7}{9}\vec{j}}$

make them unit!!

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$\cancel{\frac{0.4 + 4B}{3} = A}$.

(continued on the back Page)

$\cdot\ P_2P_0 = -4\vec{i} + 3\vec{j}$.

so $-0.2 = -4A + 3B$; $\cancel{-0.2 = -4\left(\frac{0.4}{3} + \frac{4B}{3}\right) + 3B}$

$\cancel{-0.2 = -\frac{8}{15} - \frac{16B}{3} + 3B;\ -\frac{7B}{3} = \frac{1}{3}}$ $\cancel{B = -\frac{1}{9}}$ $\cancel{A = -\frac{4}{135}}$

4. (**18 Points**)

*Evaluate the following, if they exist:*

a) $\lim\limits_{(x,y)\to(0,0)} \tan^{-1}\left(\frac{\sin(xy)}{xy}\right)$ (using Taylor Series).

$\sin(xy) = \sum \frac{(-1)^n (xy)^{2n+1}}{(2n+1)!} = xy - \frac{(xy)^3}{3!} + \frac{(xy)^5}{5!} \ldots$

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so $\lim\limits_{(x,y)\to(0,0)} \frac{\sin xy}{xy} = \frac{xy - \frac{(xy)^3}{3!}}{xy} = 1$ then $\tan(1) = \frac{\pi}{4}$

so $\lim\limits_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} = \frac{\pi}{4}$.

b) $\lim\limits_{(x,y)\to(0,0)} \left[\frac{x^2y^2}{(x^2+y^2)^{\frac{3}{2}}}\right]$

$\frac{x^2y^2}{(x^2+y^2)\sqrt{x^2+y^2}} = \frac{r^4\cos^2\theta \cdot \sin^2\theta}{(r^2)^{3/2}} = \frac{r^4\cos^2\theta \cdot \sin^2\theta}{r^3} = r\cos^2\theta\sin^2\theta$

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but $\cos\theta$ and $\sin\theta$ are limited $-1 < \sin\theta \cdot \cos\theta < 1$ then $\lim\limits_{r\to 0} \frac{x^2y^2}{(x^2+y^2)^{3/2}} = 0$

$= 0$

c) $\lim\limits_{(x,y)\to(0,0)} \left(\frac{x^3y}{x^6+y^2}\right)$

~~let $y = mx$~~ (On the back of the page).

$\cancel{\lim\limits_{(x,y)\to(0,0)} \frac{x^3 m x}{x^4(1+x^2)}}$ ~~(the 2 path test fails).~~ $\cancel{\text{since } x^3 - \sqrt{x}}$

$\cancel{\text{but } \frac{x^3y}{x^6+y^2} < 1}$

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$\cancel{\text{then } \frac{x^3y}{x^6+y^2} < y \text{ but } \lim\limits_{(x,y)\to(0,0)} y = 0 \text{ so}}$

c) ~~by sandwich theorem~~

~~$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2} = 0$~~

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$$ let = 0

along $y = mx^3$

$$= \lim_{(x,y)\to(0,0)} \frac{x^3 \cdot mx^3}{x^6 + m^2x^6} \quad \lim \frac{m\cancel{x^6}}{\cancel{x^6}(1+m^2)}$$

$$= \lim \frac{m}{(1+m^2)}$$

since the limit depends on m so we will get many values for the limit, thus there is No limit.

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3. $0.4 = -3A + 4B$ (continued).

$-0.2 = -4A + 3B$

so $\frac{-0.2 + 4A}{3} = B$

$B = -\frac{1}{15} + \frac{4A}{3}$

$0.4 = -3A + \frac{4}{15} + \frac{16A}{3}$ so $\nabla f = \frac{2}{7}\vec{i} + \frac{11}{35}\vec{j}$

$\frac{2}{3} = \frac{7A}{3}$; $\boxed{A = \frac{2}{7}}$

$\boxed{B = \frac{11}{35}}$

5. (*15 Points*)

*a) Use series to find the value(s) of k for which* $L = \lim_{x \to 0}\left(\frac{\cos(kx) + \frac{3x^2}{2} - \cos x}{x^4}\right)$ *exists.*

$\cos(Kx) = \sum \frac{(-1)^n (Kx)^{2n}}{(2n)!}$

$= 1 - \frac{K^2x^2}{2!} + \frac{K^4x^4}{4!} - \frac{K^6x^6}{6!} \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots$

$\lim_{x\to 0}\left(\frac{\cancel{1} - \frac{K^2x^2}{2!} + \frac{K^4x^4}{4!} \dots + \frac{3x^2}{2} - \cancel{1} + \frac{x^2}{2!} - \frac{x^4}{4!} \dots}{x^4}\right)$

$= \lim \left[\frac{\frac{x^2}{2!}\left[-K^2 + \frac{3}{2}\cdot + 1\right) + \frac{x^4}{4!}\left[K^4 - 1\right] + \frac{x^6}{6!}\left[-K^6 + 1\right]}{x^4}\right]$

$= \left[\frac{(-K^2+1)}{2!\,x^2} + \frac{1}{4!}\left[K^4 - 1\right] \dots\right]$

so that the limit exist $K^4 - 1 = 0$. ~~$K^4 = 1$ then $K = 1$~~

$K^4 - 1 \neq 0$ so $K^4 \neq 1$ so $K \neq 1$.

an

*b) Now find L.*

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~~so~~ ~~lim so $L = \frac{K^4 - 1}{4!}$ when $K^4 \neq 1$.~~

a) So; $K^4 \neq 1$ $K \neq$ sind $K^4 - 1 \neq 0$.

but also $-K^2 + 3 + 1 = 0$. so that we won't have an $x$ in the demoniator

so $-K^2 = -4$; $\boxed{K = 2}$ or $\boxed{K = -2}$

b) $L = \left[0 + \frac{1}{4!}\left[(2)^4 - 1\right]\right] = \frac{5}{8}$

6. (*16 Points*)

*Apply the binomial series to find the value of* $J = \int_0^1 \sqrt{1+x^4}\,dx$

*with an error of magnitude less than* 0.04. *(Use the smallest number of terms).*

$$J = (1+x^4)^{1/2} = 1 + \sum \binom{1/2}{n} (x^4)^n$$

$$= 1 + \frac{(1/2)\,x^4}{1} + \frac{(1/2)(-1/2)(x^8)}{2!} + \frac{3}{8}\frac{x^{12}}{3!} \cdots$$

$$\int \sqrt{1+x^4} = x + \sum \binom{1/2}{n} \frac{x^{4n+1}}{4n+1}$$

$$\int \sqrt{1+x^4} = x + (1/2)\frac{x^5}{5} - \frac{x^9}{4 \times 2! \times 9} + \frac{3x^{13}}{8 \times 3! \, 13} \cdots$$

$$\int_0^1 \sqrt{1+x^4} = 1 + \frac{1}{10} - \frac{1}{72} + \frac{3}{8 \times 3! \times 13} \cdots$$

cut it here

since $\frac{3x^{13}}{8 \times 3! \times 13} < 0.04$.

So the Binomial series is.

$$J = \int_0^1 \sqrt{1+x^4} = 1 + \frac{1}{10} - \frac{1}{72}$$

cut since $\frac{1}{72} < 0.04$

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Ans 1.1