Chapter 9

9-1 Eq. (9-3):

$$F = 0.707hl\tau = 0.707(5/16)(4)(20) = 17.7 \text{ kip} \quad \textit{Ans.}$$

9-2 Table 9-6: $\tau_{\text{all}} = 21.0$ kpsi

$$\begin{aligned} f &= 14.85h \text{ kip/in} \\ &= 14.85(5/16) = 4.64 \text{ kip/in} \\ F &= fl = 4.64(4) = 18.56 \text{ kip} \quad \textit{Ans.} \end{aligned}$$

9-3 Table A-20:

1018 HR: $S_{ut} = 58$ kpsi, $S_y = 32$ kpsi

1018 CR: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned} \tau_{\text{all}} &= \min(0.30S_{ut}, 0.40S_y) \\ &= \min[0.30(58), 0.40(32)] \\ &= \min(17.4, 12.8) = 12.8 \text{ kpsi} \end{aligned}$$

for both materials.

Eq. (9-3):

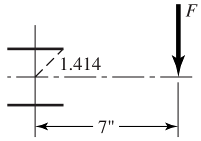
$$F = 0.707hl\tau_{\text{all}}$$

$$F = 0.707(5/16)(4)(12.8) = 11.3 \text{ kip} \quad \textit{Ans.}$$

9-4 Eq. (9-3)

$$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(32)}{(5/16)(4)(2)} = 18.1 \text{ kpsi} \quad \textit{Ans.}$$

9-5 $b = d = 2$ in



(a) *Primary shear* Table 9-1

$$\tau_y' = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.13F \text{ kpsi}$$

Secondary shear Table 9-1

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{2[(3)(2^2) + 2^2]}{6} = 5.333 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/16)(5.333) = 1.18 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.18} = 5.93F \text{ kpsi}$$

Maximum shear

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = F\sqrt{5.93^2 + (1.13 + 5.93)^2} = 9.22F \text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}}{9.22} = \frac{20}{9.22} = 2.17 \text{ kip} \quad \textit{Ans.} \qquad (1)$$

(b) For E7010 from Table 9-6, $\tau_{\text{all}} = 21$ kpsi

Table A-20:

HR 1020 Bar: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

HR 1015 Support: $S_{ut} = 50$ kpsi, $S_y = 27.5$ kpsi

Table 9-5, E7010 Electrode: $S_{ut} = 70$ kpsi, $S_y = 57$ kpsi

The support controls the design.

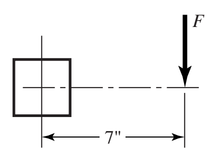
Table 9-4:

$$\tau_{\text{all}} = \min[0.30(50), 0.40(27.5)] = \min[15, 11] = 11 \text{ kpsi}$$

The allowable load from Eq. (1) is

$$F = \frac{\tau_{\text{all}}}{9.22} = \frac{11}{9.22} = 1.19 \text{ kip} \quad \textit{Ans.}$$

9-6 $b = d = 2$ in



Primary shear

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2+2)} = 0.566F$$

Secondary shear

Table 9-1:
$$J_u = \frac{(b+d)^3}{6} = \frac{(2+2)^3}{6} = 10.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/16)(10.67) = 2.36 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{(7F)(1)}{2.36} = 2.97F$$

Maximum shear

$$\tau_{\max} = \sqrt{\tau_x''^2 + (\tau_y' + \tau_y'')^2} = F\sqrt{2.97^2 + (0.556 + 2.97)^2} = 4.61F \text{ kpsi}$$

$$F = \frac{\tau_{\text{all}}}{4.61} \quad \textit{Ans.}$$

which is twice $\tau_{\max}/9.22$ of Prob. 9-5.

9-7 Weldment, subjected to alternating fatigue, has throat area of

$$A = 0.707(6)(60 + 50 + 60) = 721 \text{ mm}^2$$

Members' endurance limit: AISI 1010 steel

$$S_{ut} = 320 \text{ MPa}, \qquad S_e' = 0.5(320) = 160 \text{ MPa}$$

$$k_a = 272(320)^{-0.995} = 0.875$$

$$k_b = 1 \quad \text{(direct shear)}$$

$$k_c = 0.59 \quad \text{(shear)}$$

$$k_d = 1$$

$$k_f = \frac{1}{K_{fs}} = \frac{1}{2.7} = 0.370$$

$$S_{se} = 0.875(1)(0.59)(0.37)(160) = 30.56 \text{ MPa}$$

Electrode's endurance: 6010

$$S_{ut} = 62(6.89) = 427 \text{ MPa}$$

$$S_e' = 0.5(427) = 213.5 \text{ MPa}$$

$$k_a = 272(427)^{-0.995} = 0.657$$

$$k_b = 1 \quad \text{(direct shear)}$$

$$k_c = 0.59 \quad \text{(shear)}$$

$$k_d = 1$$

$$k_f = 1/K_{fs} = 1/2.7 = 0.370$$

$$S_{se} = 0.657(1)(0.59)(0.37)(213.5) = 30.62 \text{ MPa} \doteq 30.56$$

Thus, the members and the electrode are of equal strength. For a factor of safety of 1,

$$F_a = \tau_a A = 30.6(721)(10^{-3}) = 22.1 \text{ kN} \quad \textit{Ans.}$$

9-8 *Primary shear* $\tau' = 0$ (why?)

Secondary shear

Table 9-1:

$$J_u = 2\pi r^3 = 2\pi(4)^3 = 402 \text{ cm}^3$$

$$J = 0.707hJ_u = 0.707(0.5)(402) = 142 \text{ cm}^4$$

$$M = 200F \text{ N}\cdot\text{m} \quad (F \text{ in kN})$$

$$\tau'' = \frac{Mr}{2J} = \frac{(200F)(4)}{2(142)} = 2.82F \quad \text{(2 welds)}$$

$$F = \frac{\tau_{\text{all}}}{\tau''} = \frac{140}{2.82} = 49.2 \text{ kN} \quad \textit{Ans.}$$

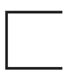
9-9

Rank

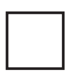
$$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833\left(\frac{a^2}{h}\right) \qquad (5)$$

$$\text{fom}' = \frac{a(3a^2 + a^2)}{6(2a)h} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right) \qquad (1)$$

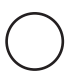
$$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083\left(\frac{a^2}{h}\right) \qquad (4)$$



$$\text{fom}' = \frac{1}{3ah}\left[\frac{8a^3 + 6a^3 + a^3}{12} - \frac{a^4}{2a + a}\right] = \frac{11}{36}\frac{a^2}{h} = 0.3056\left(\frac{a^2}{h}\right) \qquad (2)$$



$$\text{fom}' = \frac{(2a)^3}{6h}\frac{1}{4a} = \frac{8a^3}{24ah} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right) \qquad (1)$$



$$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = \frac{a^2}{4h} = 0.25\left(\frac{a^2}{h}\right) \qquad (3)$$

These rankings apply to fillet weld patterns in torsion that have a square area $a \times a$ in which to place weld metal. The object is to place as much metal as possible to the border. If your area is rectangular, your goal is the same but the rankings may change.

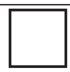
Students will be surprised that the circular weld bead does not rank first.

9-10

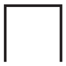
$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{a}\left(\frac{a^3}{12}\right)\left(\frac{1}{h}\right) = \frac{1}{12}\left(\frac{a^2}{h}\right) = 0.0833\left(\frac{a^2}{h}\right) \qquad (5)$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{2ah}\left(\frac{a^3}{6}\right) = 0.0833\left(\frac{a^2}{h}\right) \qquad (5)$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{2ah}\left(\frac{a^2}{2}\right) = \frac{1}{4}\left(\frac{a^2}{h}\right) = 0.25\left(\frac{a^2}{h}\right) \qquad (1)$$



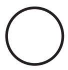
$$\text{fom}' = \frac{I_u}{lh} = \frac{1}{[2(2a)]h}\left(\frac{a^2}{6}\right)(3a+a) = \frac{1}{6}\left(\frac{a^2}{h}\right) = 0.1667\left(\frac{a^2}{h}\right) \quad 2$$



$$\bar{x} = \frac{b}{2} = \frac{a}{2}, \quad \bar{y} = \frac{d^2}{b+2d} = \frac{a^2}{3a} = \frac{a}{3}$$

$$I_u = \frac{2d^3}{3} - 2d^2\left(\frac{a}{3}\right) + (b+2d)\left(\frac{a^2}{9}\right) = \frac{2a^3}{3} - \frac{2a^3}{3} + 3a\left(\frac{a^2}{9}\right) = \frac{a^3}{3}$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{a^3/3}{3ah} = \frac{1}{9}\left(\frac{a^2}{h}\right) = 0.1111\left(\frac{a^2}{h}\right) \quad 4$$



$$I_u = \pi r^3 = \frac{\pi a^3}{8}$$

$$\text{fom}' = \frac{I_u}{lh} = \frac{\pi a^3/8}{\pi a h} = \frac{a^2}{8h} = 0.125\left(\frac{a^2}{h}\right) \quad 3$$

The CEE-section pattern was not ranked because the deflection of the beam is out-of-plane.

If you have a square area in which to place a fillet weldment pattern under bending, your objective is to place as much material as possible away from the *x*-axis. If your area is rectangular, your goal is the same, but the rankings may change.

9-11 Materials:

Attachment (1018 HR) $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36) $S_y = 36$ kpsi, S_{ut} ranges from 58 to 80 kpsi, use 58.

The member and attachment are weak compared to the E60XX electrode.

Decision Specify E6010 electrode

Controlling property: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = \min(16.6, 12.8) = 12.8$ kpsi

For a static load the parallel and transverse fillets are the same. If *n* is the number of beads,

$$\tau = \frac{F}{n(0.707)hl} = \tau_{\text{all}}$$

$$nh = \frac{F}{0.707 l \tau_{\text{all}}} = \frac{25}{0.707(3)(12.8)} = 0.921$$

Make a table.

Number of beads n	Leg size h
1	0.921
2	0.460 → 1/2"
3	0.307 → 5/16"
4	0.230 → 1/4"

Decision: Specify 1/4" leg size

Decision: Weld all-around

Weldment Specifications:

Pattern: All-around square
Electrode: E6010
Type: Two parallel fillets *Ans.*
Two transverse fillets
Length of bead: 12 in
Leg: 1/4 in

For a figure of merit of, in terms of weldbead volume, is this design optimal?

9-12 *Decision:* Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-9) and have thus reduced a synthesis problem to an analysis problem:

Table 9-1: $A = 1.414hd = 1.414(h)(3) = 4.24h \text{ in}^3$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{3000}{4.24h} = \frac{707}{h}$$

Secondary shear

Table 9-1: $$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{3[3(3^2) + 3^2]}{6} = 18 \text{ in}^3$$

$$J = 0.707(h)(18) = 12.7h \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{3000(7.5)(1.5)}{12.7h} = \frac{2657}{h} = \tau''_y$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = \frac{1}{h}\sqrt{2657^2 + (707 + 2657)^2} = \frac{4287}{h}$$

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36): $S_y = 36$ kpsi

The attachment is weaker

Decision: Use E60XX electrode

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8 \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{4287}{h} = 12\,800 \text{ psi}$$

$$h = \frac{4287}{12\,800} = 0.335 \text{ in}$$

Decision: Specify 3/8" leg size

Weldment Specifications:

Pattern: Parallel fillet welds
Electrode: E6010
Type: Fillet *Ans.*
Length of bead: 6 in
Leg size: 3/8 in

9-13 An optimal square space $(3" \times 3")$ weldment pattern is $\parallel$ or $=$ or $\square$. In Prob. 9-12, there was roundup of leg size to $3/8$ in. Consider the member material to be structural A36 steel.

Decision: Use a parallel horizontal weld bead pattern for welding optimization and convenience.

Materials:

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Member (A36): $S_y = 36$ kpsi, S_{ut} 58–80 kpsi; use 58 kpsi

From Table 9-4 AISC welding code,

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = \min(16.6, 12.8) = 12.8 \text{ kpsi}$$

Select a stronger electrode material from Table 9-3.

Decision: Specify E6010

Throat area and other properties:

$$A = 1.414hd = 1.414(h)(3) = 4.24h \text{ in}^2$$

$$\bar{x} = b/2 = 3/2 = 1.5 \text{ in}$$

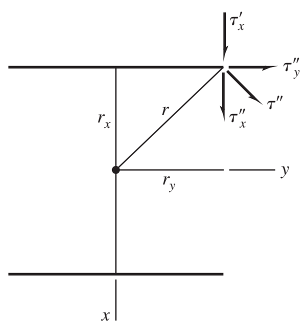
$$\bar{y} = d/2 = 3/2 = 1.5 \text{ in}$$

$$J_u = \frac{d(3b^2 + d^2)}{6} = \frac{3[3(3^2) + 3^2]}{6} = 18 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(h)(18) = 12.73h \text{ in}^4$$

Primary shear:

$$\tau'_x = \frac{V}{A} = \frac{3000}{4.24h} = \frac{707.5}{h}$$



Secondary shear:

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos 45^\circ = \frac{Mr}{J} \cos 45^\circ = \frac{Mr_x}{J}$$

$$\tau''_x = \frac{3000(6 + 1.5)(1.5)}{12.73h} = \frac{2651}{h}$$

$$\tau''_y = \tau''_x = \frac{2651}{h}$$

$$\tau_{\max} = \sqrt{(\tau_x'' + \tau_x')^2 + \tau_y''^2}$$
$$= \frac{1}{h}\sqrt{(2651 + 707.5)^2 + 2651^2}$$
$$= \frac{4279}{h} \text{ psi}$$

Relate stress and strength:

$$\tau_{\max} = \tau_{\text{all}}$$
$$\frac{4279}{h} = 12\,800$$
$$h = \frac{4279}{12\,800} = 0.334 \text{ in} \rightarrow 3/8 \text{ in}$$

Weldment Specifications:

Pattern: Horizontal parallel weld tracks
Electrode: E6010
Type of weld: Two parallel fillet welds
Length of bead: 6 in
Leg size: 3/8 in

Additional thoughts:

Since the round-up in leg size was substantial, why not investigate a backward C $\sqsupset$ weld pattern. One might then expect shorter horizontal weld beads which will have the advantage of allowing a shorter member (assuming the member has not yet been designed). This will show the inter-relationship between attachment design and supporting members.

9-14 *Materials:*

Member (A36): $S_y = 36$ kpsi, $S_{ut} = 58$ to 80 kpsi; use $S_{ut} = 58$ kpsi

Attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

$$\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8 \text{ kpsi}$$

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi,
$\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = 20$ kpsi

Decision: Since A36 and 1018 HR are weld metals to an unknown extent, use
$\tau_{\text{all}} = 12.8$ kpsi

Decision: Use the most efficient weld pattern–square, weld-all-around. Choose 6" × 6" size.

Attachment length:

$$l_1 = 6 + a = 6 + 6.25 = 12.25 \text{ in}$$

Throat area and other properties:

$$A = 1.414h(b + d) = 1.414(h)(6 + 6) = 17.0h$$
$$\bar{x} = \frac{b}{2} = \frac{6}{2} = 3 \text{ in}, \quad \bar{y} = \frac{d}{2} = \frac{6}{2} = 3 \text{ in}$$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{F}{A} = \frac{20\,000}{17h} = \frac{1176}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b+d)^3}{6} = \frac{(6+6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{20\,000(6.25+3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau''_y + \tau'_y)^2} = \frac{1}{h}\sqrt{2726^2 + (2726+1176)^2} = \frac{4760}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{\max} = \tau_{\text{all}}$$

$$\frac{4760}{h} = 12\,800$$

$$h = \frac{4760}{12\,800} = 0.372 \text{ in}$$

Decision:
Specify 3/8 in leg size

Specifications:
Pattern: All-around square weld bead track
Electrode: E6010
Type of weld: Fillet
Weld bead length: 24 in
Leg size: 3/8 in
Attachment length: 12.25 in

9-15 This is a good analysis task to test the students' understanding

(1) Solicit information related to a priori decisions.

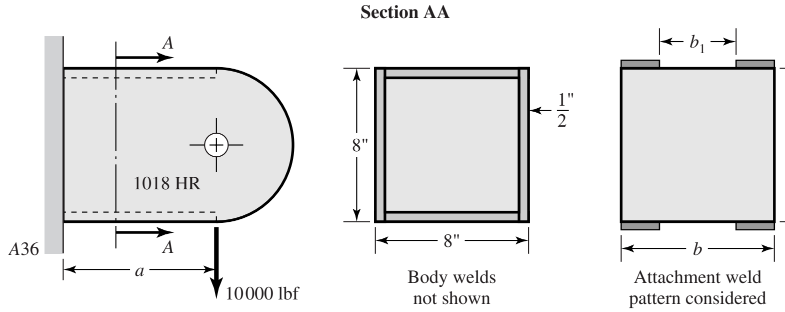
(2) Solicit design variables b and d.

(3) Find h and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.

(4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-16 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-3 can be added or subtracted to obtain the properties of a comtemplated weld pattern. The instructor can control the level of complication. I have left the presentation of the drawing to you. Here is one possibility. Study the problem's opportunities, then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1. From Table 9-3, category 3:

$$A = 1.414h(b - b_1)$$

$$\bar{x} = b/2, \quad \bar{y} = d/2$$

$$I_u = \frac{bd^2}{2} - \frac{b_1 d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{\text{all}} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8 - 2) = 8.48h \text{ in}^2$$

$$I_u = (8 - 2)(8^2/2) = 192 \text{ in}^3$$

$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$

$$\tau'' = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h} \text{ psi}$$

$$\tau_{\max} = \frac{1}{h}\sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800$$

from which $h = 0.248$ in. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5$$

$$A = 8.48(0.248) = 2.10 \text{ in}^2$$

$$I = 135.7(0.248) = 33.65 \text{ in}^4$$

$$\text{vol} = \frac{h^2}{2}l = \frac{0.248^2}{2}12 = 0.369\,\text{in}^3$$

$$\frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 = \text{eff}$$

$$\tau' = \frac{1179}{0.248} = 4754\,\text{psi}$$

$$\tau'' = \frac{2948}{0.248} = 11\,887\,\text{psi}$$

$$\tau_{\text{max}} = \frac{4127}{0.248} \doteq 12\,800\,\text{psi}$$

Now consider the case of uninterrupted welds,

$$b_1 = 0$$

$$A = 1.414(h)(8-0) = 11.31h$$

$$I_u = (8-0)(8^2/2) = 256\,\text{in}^3$$

$$I = 0.707(256)h = 181h\,\text{in}^4$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$

$$\tau'' = \frac{10\,000(10)(8/2)}{181h} = \frac{2210}{h}$$

$$\tau_{\text{max}} = \frac{1}{h}\sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$

$$h = \frac{\tau_{\text{max}}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186\,\text{in}$$

Do not round off h.

$$A = 11.31(0.186) = 2.10\,\text{in}^2$$

$$I = 181(0.186) = 33.67$$

$$\tau' = \frac{884}{0.186} = 4753\,\text{psi}, \quad \text{vol} = \frac{0.186^2}{2}16 = 0.277\,\text{in}^3$$

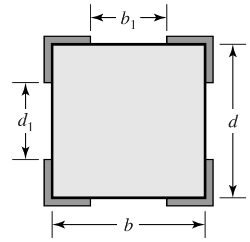
$$\tau'' = \frac{2210}{0.186} = 11\,882\,\text{psi}$$

$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0$$

$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7$$

Conclusions: To meet allowable stress limitations, I and A do not change, nor do τ and σ. To meet the shortened bead length, h is increased proportionately. However, volume of bead laid down increases as h^2. The uninterrupted bead is superior. In this example, we did not round h and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a followup task analyzing an alternative weld pattern.



9-17 From Table 9-2

For the box $A = 1.414h(b + d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414\,h(b - b_1 + d - d_1)$$

$$I_u = \frac{d^2}{6}(3b + d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2}$$

$$= \frac{1}{2}(b - b_1)d^2 + \frac{1}{6}\left(d^3 - d_1^3\right)$$

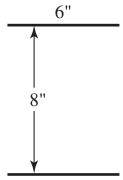
length of bead $l = 2(b - b_1 + d - d_1)$

$$\text{fom} = I_u/hl$$

9-18 Computer programs will vary.

9-19 $\tau_{\text{all}} = 12\,800$ psi. Use Fig. 9-17(a) for general geometry, but employ $\overline{\underline{\;\;}}$ beads and then $||$ beads.

Horizontal parallel weld bead pattern



$b = 6$ in
$d = 8$ in

From Table 9-2, category 3

$$A = 1.414\,hb = 1.414(h)(6) = 8.48\,h\text{ in}^2$$

$$\bar{x} = b/2 = 6/2 = 3\text{ in}, \qquad \bar{y} = d/2 = 8/2 = 4\text{ in}$$

$$I_u = \frac{bd^2}{2} = \frac{6(8)^2}{2} = 192\text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(192) = 135.7h\text{ in}^4$$

$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h}\text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h}\ \text{psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}(1179^2 + 2948^2)^{1/2} = \frac{3175}{h}\ \text{psi}$$

Equate the maximum and allowable shear stresses.

$$\tau_{\max} = \tau_{\text{all}} = \frac{3175}{h} = 12\,800$$

from which $h = 0.248$ in. It follows that

$$I = 135.7(0.248) = 33.65\ \text{in}^4$$

The volume of the weld metal is

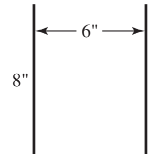
$$\text{vol} = \frac{h^2 l}{2} = \frac{0.248^2(6+6)}{2} = 0.369\ \text{in}^3$$

The effectiveness, $(\text{eff})_{\text{H}}$, is

$$(\text{eff})_{\text{H}} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2\ \text{in}$$

$$(\text{fom}')_{\text{H}} = \frac{I_u}{hl} = \frac{192}{0.248(6+6)} = 64.5\ \text{in}$$

Vertical parallel weld beads



$$b = 6\ \text{in}$$
$$d = 8\ \text{in}$$

From Table 9-2, category 2

$$A = 1.414hd = 1.414(h)(8) = 11.31h\ \text{in}^2$$

$$\bar{x} = b/2 = 6/2 = 3\ \text{in}, \qquad \bar{y} = d/2 = 8/2 = 4\ \text{in}$$

$$I_u = \frac{d^3}{6} = \frac{8^3}{6} = 85.33\ \text{in}^3$$

$$I = 0.707hI_u = 0.707(h)(85.33) = 60.3h$$

$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}\ \text{psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10\,000(10)(8/2)}{60.3\,h} = \frac{6633}{h}\ \text{psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}(884^2 + 6633^2)^{1/2}$$

$$= \frac{6692}{h}\ \text{psi}$$

Equating $\tau_{\max}$ to τ_{all} gives $h = 0.523$ in. It follows that

$$I = 60.3(0.523) = 31.5\text{ in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.523^2}{2}(8+8) = 2.19\text{ in}^3$$

$$(\text{eff})_V = \frac{I}{\text{vol}} = \frac{31.6}{2.19} = 14.4\text{ in}$$

$$(\text{fom}')_V = \frac{I_u}{hl} = \frac{85.33}{0.523(8+8)} = 10.2\text{ in}$$

The ratio of $(\text{eff})_V/(\text{eff})_H$ is $14.4/91.2 = 0.158$. The ratio $(\text{fom}')_V/(\text{fom}')_H$ is $10.2/64.5 = 0.158$. This is not surprising since

$$\text{eff} = \frac{I}{\text{vol}} = \frac{I}{(h^2/2)l} = \frac{0.707\,hI_u}{(h^2/2)l} = 1.414\frac{I_u}{hl} = 1.414\text{ fom}'$$

The ratios $(\text{eff})_V/(\text{eff})_H$ and $(\text{fom}')_V/(\text{fom}')_H$ give the same information.

9-20 Because the loading is pure torsion, there is no primary shear. From Table 9-1, category 6:

$$J_u = 2\pi r^3 = 2\pi(1)^3 = 6.28\text{ in}^3$$

$$J = 0.707\,hJ_u = 0.707(0.25)(6.28) = 1.11\text{ in}^4$$

$$\tau = \frac{Tr}{J} = \frac{20(1)}{1.11} = 18.0\text{ kpsi} \quad \textit{Ans.}$$

9-21 $h = 0.375$ in, $d = 8$ in, $b = 1$ in

From Table 9-2, category 2:

$$A = 1.414(0.375)(8) = 4.24\text{ in}^2$$

$$I_u = \frac{d^3}{6} = \frac{8^3}{6} = 85.3\text{ in}^3$$

$$I = 0.707hI_u = 0.707(0.375)(85.3) = 22.6\text{ in}^4$$

$$\tau' = \frac{F}{A} = \frac{5}{4.24} = 1.18\text{ kpsi}$$

$$M = 5(6) = 30\text{ kip}\cdot\text{in}$$

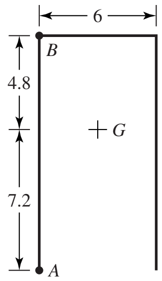
$$c = (1+8+1-2)/2 = 4\text{ in}$$

$$\tau'' = \frac{Mc}{I} = \frac{30(4)}{22.6} = 5.31\text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1.18^2 + 5.31^2} = 5.44\text{ kpsi} \quad \textit{Ans.}$$

9-22 $h = 0.6\,\text{cm}, \quad b = 6\,\text{cm}, \quad d = 12\,\text{cm}.$

Table 9-3, category 5:



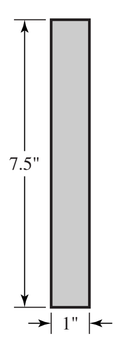
$$A = 0.707h(b + 2d)$$
$$= 0.707(0.6)[6 + 2(12)] = 12.7\,\text{cm}^2$$
$$\bar{y} = \frac{d^2}{b + 2d} = \frac{12^2}{6 + 2(12)} = 4.8\,\text{cm}$$
$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2$$
$$= \frac{2(12)^3}{3} - 2(12^2)(4.8) + [6 + 2(12)]4.8^2$$
$$= 461\,\text{cm}^3$$
$$I = 0.707hI_u = 0.707(0.6)(461) = 196\,\text{cm}^4$$
$$\tau' = \frac{F}{A} = \frac{7.5(10^3)}{12.7(10^2)} = 5.91\text{ MPa}$$
$$M = 7.5(120) = 900\,\text{N}\cdot\text{m}$$
$$c_A = 7.2\,\text{cm}, \qquad c_B = 4.8\,\text{cm}$$

The critical location is at A.

$$\tau''_A = \frac{Mc_A}{I} = \frac{900(7.2)}{196} = 33.1\,\text{MPa}$$
$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = (5.91^2 + 33.1^2)^{1/2} = 33.6\,\text{MPa}$$
$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{120}{33.6} = 3.57 \quad \textit{Ans.}$$

9-23 The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment.

Use a rectangular, weld-all-around pattern – Table 9-2, category 6:



$$A = 1.414\,h(b + d)$$
$$= 1.414(1/16)(1 + 7.5)$$
$$= 0.751\,\text{in}^2$$
$$\bar{x} = b/2 = 0.5\,\text{in}$$
$$\bar{y} = \frac{d}{2} = \frac{7.5}{2} = 3.75\,\text{in}$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{7.5^2}{6}[3(1) + 7.5] = 98.4\,\text{in}^3$$

$$I = 0.707hI_u = 0.707(1/16)(98.4) = 4.35\,\text{in}^4$$

$$M = (3.75 + 0.5)W = 4.25W$$

$$\tau' = \frac{V}{A} = \frac{W}{0.751} = 1.332W$$

$$\tau'' = \frac{Mc}{I} = \frac{4.25W(7.5/2)}{4.35} = 3.664W$$

$$\tau_{\text{max}} = \sqrt{\tau'^2 + \tau''^2} = W\sqrt{1.332^2 + 3.664^2} = 3.90W$$

Material properties: The allowable stress given is low. Let's demonstrate that.

For the A36 structural steel member, $S_y = 36$ kpsi and $S_{ut} = 58$ kpsi. For the 1020 CD attachment, use HR properties of $S_y = 30$ kpsi and $S_{ut} = 55$. The E6010 electrode has strengths of $S_y = 50$ and $S_{ut} = 62$ kpsi.

Allowable stresses:

A36:
$$\tau_{\text{all}} = \min[0.3(58), 0.4(36)] = \min(17.4, 14.4) = 14.4\,\text{kpsi}$$

1020:
$$\tau_{\text{all}} = \min[0.3(55), 0.4(30)]$$
$$\tau_{\text{all}} = \min(16.5, 12) = 12\,\text{kpsi}$$

E6010:
$$\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = \min(18.6, 20) = 18.6\,\text{kpsi}$$

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value.

Therefore, the allowable shear stress is

$$\tau_{\text{all}} = \min(14.4, 12, 18.0) = 12\,\text{kpsi}$$

However, the allowable stress in the problem statement is 0.9 kpsi which is low from the weldment perspective. The load associated with this strength is

$$\tau_{\text{max}} = \tau_{\text{all}} = 3.90W = 900$$

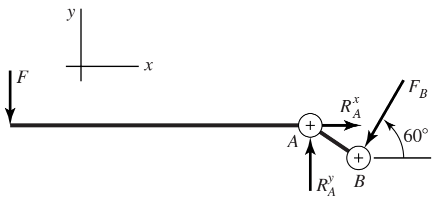
$$W = \frac{900}{3.90} = 231\,\text{lbf}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load $W = 3047$ lbf. Can the bracket carry such a load?

There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

9-24



$$F = 100\,\text{lbf}, \qquad \tau_{\text{all}} = 3\,\text{kpsi}$$

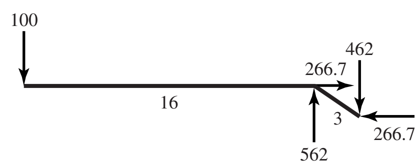
$$F_B = 100(16/3) = 533.3\,\text{lbf}$$

$$F_B^x = -533.3\cos 60^\circ = -266.7\,\text{lbf}$$

$$F_B^y = -533.3\cos 30^\circ = -462\,\text{lbf}$$

It follows that $R_A^y = 562\,\text{lbf}$ and $R_A^x = 266.7\,\text{lbf}$, $R_A = 622\,\text{lbf}$

$$M = 100(16) = 1600\,\text{lbf}\cdot\text{in}$$



The OD of the tubes is 1 in. From Table 9-1, category 6:

$$A = 1.414(\pi h r)(2)$$

$$= 2(1.414)(\pi h)(1/2) = 4.44h\,\text{in}^2$$

$$J_u = 2\pi r^3 = 2\pi(1/2)^3 = 0.785\,\text{in}^3$$

$$J = 2(0.707)hJ_u = 1.414(0.785)h = 1.11h\,\text{in}^4$$

$$\tau' = \frac{V}{A} = \frac{622}{4.44h} = \frac{140}{h}$$

$$\tau'' = \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.11h} = \frac{720.7}{h}$$

The shear stresses, τ' and τ'', are additive algebraically

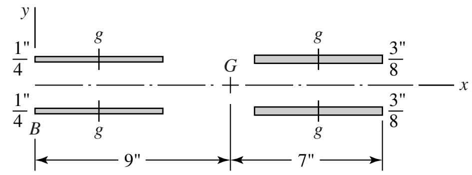
$$\tau_{\max} = \frac{1}{h}(140 + 720.7) = \frac{861}{h}\ \text{psi}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{861}{h} = 3000$$

$$h = \frac{861}{3000} = 0.287 \rightarrow 5/16"$$

Decision: Use $5/16$ in fillet welds *Ans.*

9-25



For the pattern in bending shown, find the centroid G of the weld group.

$$\bar{x} = \frac{6(0.707)(1/4)(3) + 6(0.707)(3/8)(13)}{6(0.707)(1/4) + 6(0.707)(3/8)}$$
$$= 9 \text{ in}$$
$$I_{1/4} = 2\left(I_G + A_{\bar{x}}^2\right)$$
$$= 2\left[\frac{0.707(1/4)(6^3)}{12} + 0.707(1/4)(6)(6^2)\right]$$
$$= 82.7 \text{ in}^4$$
$$I_{3/8} = 2\left[\frac{0.707(3/8)(6^3)}{12} + 0.707(3/8)(6)(4^2)\right]$$
$$= 60.4 \text{ in}^4$$
$$I = I_{1/4} + I_{3/8} = 82.7 + 60.4 = 143.1 \text{ in}^4$$

The critical location is at B. From Eq. (9-3),

$$\tau' = \frac{F}{2[6(0.707)(3/8 + 1/4)]} = 0.189F$$
$$\tau'' = \frac{Mc}{I} = \frac{(8F)(9)}{143.1} = 0.503F$$
$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.189^2 + 0.503^2} = 0.537F$$

Materials:

A36 Member: $S_y = 36$ kpsi

1015 HR Attachment: $S_y = 27.5$ kpsi

E6010 Electrode: $S_y = 50$ kpsi

$$\tau_{\text{all}} = 0.577 \min(36, 27.5, 50) = 15.9 \text{ kpsi}$$
$$F = \frac{\tau_{\text{all}}/n}{0.537} = \frac{15.9/2}{0.537} = 14.8 \text{ kip} \quad \textit{Ans.}$$

9-26 Figure P9-26*b* is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439 \text{ lbf} \cdot \text{in}$ *Ans.*

(b) $F_y = 1200 \sin 30^\circ = 600 \text{ lbf}$ *Ans.*

(c) $F_x = 1200 \cos 30^\circ = 1039 \text{ lbf}$ *Ans.*

(d) From Table 9-2, category 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through G and parallel to z is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \quad Ans.$$

(e) Refer to Fig. P.9-26*b*. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \left(\tau_1^2 + \tau_2^2\right)^{1/2} = (617^2 + 1069^2)^{1/2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{max} = (\tau'^2 + \tau''^2)^{1/2} = (1234^2 + 916^2)^{1/2} = 1537 \text{ psi} \quad Ans.$$

(f) *Materials:*

1018 HR Member: $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi (Table A-20)

E6010 Electrode: $S_y = 50$ kpsi (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{max}} = \frac{0.577S_y}{\tau_{max}} = \frac{0.577(32)}{1.537} = 12.0 \quad Ans.$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to *bh*.

$$A_1 \doteq bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location A

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \left(\sigma_y^2 + 3\tau_{xy}^2\right)^{1/2} = [2648^2 + 3(1662)^2]^{1/2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad Ans.$$

The clip on the mooring line bears against the side of the 1/2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad Ans.$$

Further investigation of this situation requires more detail than is included in the task statement.

(h) In shear fatigue, the weakest constituent of the weld melt is 1018 with $S_{ut} = 58$ kpsi

$$S'_e = 0.5S_{ut} = 0.5(58) = 29 \text{ kpsi}$$

Table 7-4:

$$k_a = 14.4(58)^{-0.718} = 0.780$$

For the size factor estimate, we first employ Eq. (7-24) for the equivalent diameter.

$$d_e = 0.808\sqrt{0.707hb} = 0.808\sqrt{0.707(2.5)(0.25)} = 0.537 \text{ in}$$

Eq. (7-19) is used next to find k_b

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.537}{0.30}\right)^{-0.107} = 0.940$$

The load factor for shear k_c, is

$$k_c = 0.59$$

The endurance strength in shear is

$$S_{se} = 0.780(0.940)(0.59)(29) = 12.5 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied.

$$\tau_a = \tau_m = K_{fs}\frac{\tau_{\max}}{2} = 2.7\frac{1.537}{2} = 2.07 \text{ kpsi}$$

Table 7-10: Gerber factor of safety n_f, adjusted for shear, with $S_{su} = 0.67S_{ut}$

$$n_f = \frac{1}{2}\left[\frac{0.67(58)}{2.07}\right]^2\left(\frac{2.07}{12.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(2.07)(12.5)}{0.67(58)(2.07)}\right]^2}\right\} = 5.52 \quad Ans.$$

Attachment metal should be checked for bending fatigue.

9-27 Use $b = d = 4$ in. Since $h = 5/8$ in, the primary shear is

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.283F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$

$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.9 \text{ in}^4$$

$$\tau_x'' = \tau_y'' = \frac{Mr_y}{J} = \frac{14F(2)}{18.9} = 1.48F$$

Thus, the maximum shear and allowable load are:

$$\tau_{\max} = F\sqrt{1.48^2 + (0.283 + 1.48)^2} = 2.30F$$

$$F = \frac{\tau_{\text{all}}}{2.30} = \frac{20}{2.30} = 8.70 \text{ kip} \quad \textit{Ans.}$$

From Prob. 9-5*b*, $\tau_{\text{all}} = 11$ kpsi

$$F_{\text{all}} = \frac{\tau_{\text{all}}}{2.30} = \frac{11}{2.30} = 4.78 \text{ kip}$$

The allowable load has thus increased by a factor of 1.8 *Ans.*

9-28 Purchase the hook having the design shown in Fig. P9-28*b*. Referring to text Fig. 9-32*a*, this design reduces peel stresses.

9-29 **(a)**

$$\bar{\tau} = \frac{1}{l}\int_{-l/2}^{l/2} \frac{P\omega\cosh(\omega x)}{4b\sinh(\omega l/2)}\,dx$$

$$= A_1\int_{-l/2}^{l/2} \cosh(\omega x)\,dx$$

$$= \frac{A_1}{\omega}\sinh(\omega x)\Big|_{-l/2}^{l/2}$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - \sinh(-\omega l/2)]$$

$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - (-\sinh(\omega l/2))]$$

$$= \frac{2A_1\sinh(\omega l/2)}{\omega}$$

$$= \frac{P\omega}{4bl\sinh(\omega l/2)}[2\sinh(\omega l/2)]$$

$$\bar{\tau} = \frac{P}{2bl} \quad \textit{Ans.}$$

(b)

$$\tau(l/2) = \frac{P\omega \cosh(\omega l/2)}{4b \sinh(\omega l/2)} = \frac{P\omega}{4b \tanh(\omega l/2)} \quad Ans.$$

(c)

$$K = \frac{\tau(l/2)}{\bar{\tau}} = \frac{P\omega}{4b \sinh(\omega l/2)} \left(\frac{2bl}{P}\right)$$

$$K = \frac{\omega l/2}{\tanh(\omega l/2)} \quad Ans.$$

For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

$$K = \frac{\omega l}{2} \frac{\exp(\omega l/2) - \exp(-\omega l/2)}{\exp(\omega l/2) + \exp(-\omega l/2)} \quad Ans.$$

9-30 This is a computer programming exercise. All programs will vary.