Chapter 7

7-1 **(a)** DE-Gerber, Eq. (7-10):

$$A = \left\{4[2.2(600)]^2 + 3[1.8(400)]^2\right\}^{1/2} = 2920 \text{ lbf} \cdot \text{in}$$

$$B = \left\{4[2.2(500)]^2 + 3[1.8(300)]^2\right\}^{1/2} = 2391 \text{ lbf} \cdot \text{in}$$

$$d = \left\{\frac{8(2)(2920)}{\pi(30\,000)}\left[1 + \left(1 + \left[\frac{2(2391)(30\,000)}{2920(100\,000)}\right]^2\right)^{1/2}\right]\right\}^{1/3}$$

$$= 1.016 \text{ in} \quad Ans.$$

(b) DE-elliptic, Eq. (7-12) can be shown to be

$$d = \left(\frac{16n}{\pi}\sqrt{\frac{A^2}{S_e^2} + \frac{B^2}{S_y^2}}\right)^{1/3}$$

$$= \left[\frac{16(2)}{\pi}\sqrt{\left(\frac{2920}{30\,000}\right)^2 + \left(\frac{2391}{80\,000}\right)^2}\right]^{1/3} = 1.012 \text{ in} \quad Ans.$$

(c) DE-Soderberg, Eq. (7-14) can be shown to be

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_y}\right)\right]^{1/3}$$

$$= \left[\frac{16(2)}{\pi}\left(\frac{2920}{30\,000} + \frac{2391}{80\,000}\right)\right]^{1/3}$$

$$= 1.090 \text{ in} \quad Ans.$$

(d) DE-Goodman: Eq. (7-8) can be shown to be

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3}$$

$$= \left[\frac{16(2)}{\pi}\left(\frac{2920}{30\,000} + \frac{2391}{100\,000}\right)\right]^{1/3} = 1.073 \text{ in} \quad Ans.$$

Criterion	d (in)	Compared to DE-Gerber	
DE-Gerber	1.016		
DE-elliptic	1.012	0.4% lower	less conservative
DE-Soderberg	1.090	7.3% higher	more conservative
DE-Goodman	1.073	5.6% higher	more conservative

7-2 This problem has to be done by successive trials, since S_e is a function of shaft size. The material is SAE 2340 for which $S_{ut} = 1226$ MPa, $S_y = 1130$ MPa, and $H_B \geq 368$.

Eq. (6-19): $k_a = 4.51(1226)^{-0.265} = 0.685$

Trial #1: Choose $d_r = 22$ mm

Eq. (6-20): $k_b = \left(\dfrac{22}{7.62}\right)^{-0.107} = 0.893$

Eq. (6-18): $S_e = 0.685(0.893)(0.5)(1226) = 375$ MPa

$$d_r = d - 2r = 0.75D - 2D/20 = 0.65D$$

$$D = \frac{d_r}{0.65} = \frac{22}{0.65} = 33.8 \text{ mm}$$

$$r = \frac{D}{20} = \frac{33.8}{20} = 1.69 \text{ mm}$$

Fig. A-15-14:

$$d = d_r + 2r = 22 + 2(1.69) = 25.4 \text{ mm}$$

$$\frac{d}{d_r} = \frac{25.4}{22} = 1.15$$

$$\frac{r}{d_r} = \frac{1.69}{22} = 0.077$$

$$K_t = 1.9$$

Fig. A-15-15: $K_{ts} = 1.5$

Fig. 6-20: $r = 1.69$ mm, $q = 0.90$

Fig. 6-21: $r = 1.69$ mm, $q_s = 0.97$

Eq. (6-32): $K_f = 1 + 0.90(1.9 - 1) = 1.81$

$$K_{fs} = 1 + 0.97(1.5 - 1) = 1.49$$

We select the DE-ASME Elliptic failure criteria.

Eq. (7-12) with d as d_r, and $M_m = T_a = 0$,

$$d_r = \left\{ \frac{16(2.5)}{\pi} \left[4\left(\frac{1.81(70)(10^3)}{375}\right)^2 + 3\left(\frac{1.49(45)(10^3)}{1130}\right)^2 \right]^{1/2} \right\}^{1/3}$$

$$= 20.6 \text{ mm}$$

Trial #2: Choose $d_r = 20.6$ mm

$$k_b = \left(\frac{20.6}{7.62}\right)^{-0.107} = 0.899$$

$$S_e = 0.685(0.899)(0.5)(1226) = 377.5 \text{ MPa}$$

$$D = \frac{d_r}{0.65} = \frac{20.6}{0.65} = 31.7 \text{ mm}$$

$$r = \frac{D}{20} = \frac{31.7}{20} = 1.59 \text{ mm}$$

Figs. A-15-14 and A-15-15:

$$d = d_r + 2r = 20.6 + 2(1.59) = 23.8 \text{ mm}$$

$$\frac{d}{d_r} = \frac{23.8}{20.6} = 1.16$$

$$\frac{r}{d_r} = \frac{1.59}{20.6} = 0.077$$

We are at the limit of readability of the figures so

$$K_t = 1.9, \quad K_{ts} = 1.5 \qquad q = 0.9, \quad q_s = 0.97$$

$$\therefore K_f = 1.81 \qquad K_{fs} = 1.49$$

Using Eq. (7-12) produces $d_r = 20.5$ mm. Further iteration produces no change.

Decisions:

$$d_r = 20.5 \text{ mm}$$

$$D = \frac{20.5}{0.65} = 31.5 \text{ mm}, \quad d = 0.75(31.5) = 23.6 \text{ mm}$$

Use $D = 32$ mm, $d = 24$ mm, $r = 1.6$ mm *Ans.*

7-3 $F \cos 20°(d/2) = T, \quad F = 2T/(d \cos 20°) = 2(3000)/(6 \cos 20°) = 1064 \text{ lbf}$

$$M_C = 1064(4) = 4257 \text{ lbf} \cdot \text{in}$$

For sharp fillet radii at the shoulders, from Table 7-1, $K_t = 2.7$, and $K_{ts} = 2.2$. Examining Figs. 6-20 and 6-21, with $S_{ut} = 80$ kpsi, conservatively estimate $q = 0.8$ and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32): $K_f = 1 + (0.8)(2.7 - 1) = 2.4$

$$K_{fs} = 1 + (0.9)(2.2 - 1) = 2.1$$

(a) Static analysis using fatigue stress concentration factors:

From Eq. (7-15) with $M = M_m$, $T = T_m$, and $M_a = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T}{\pi d^3}\right)^2\right]^{1/2}$$

Eq. (7-16): $$n = \frac{S_y}{\sigma'_{\max}} = \frac{S_y}{\left[\left(\frac{32K_f M}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T}{\pi d^3}\right)^2\right]^{1/2}}$$

Solving for d,

$$d = \left\{\frac{16n}{\pi S_y}\left[4(K_f M)^2 + 3(K_{fs}T)^2\right]^{1/2}\right\}^{1/3}$$
$$= \left\{\frac{16(2.5)}{\pi(60\,000)}\left[4(2.4)(4257)^2 + 3(2.1)(3000)^2\right]^{1/2}\right\}^{1/3}$$
$$= 1.700 \text{ in} \quad Ans.$$

(b) $$k_a = 2.70(80)^{-0.265} = 0.845$$

Assume $d = 2.00$ in to estimate the size factor,

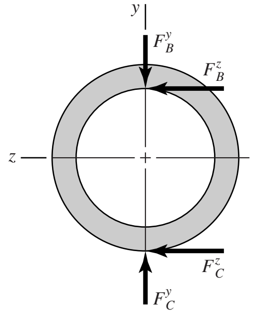
$$k_b = \left(\frac{2}{0.3}\right)^{-0.107} = 0.816$$
$$S_e = 0.845(0.816)(0.5)(80) = 27.6 \text{ kpsi}$$

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$d = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(4257)}{27\,600}\right)^2 + 3\left(\frac{2.1(3000)}{60\,000}\right)^2\right]^{1/2}\right\}^{1/3} = 2.133 \text{ in}$$

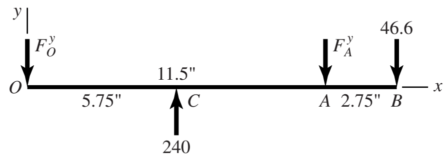
Revising k_b results in $d = 2.138$ in *Ans.*

7-4 We have a design task of identifying bending moment and torsion diagrams which are preliminary to an industrial roller shaft design.



$$F_C^y = 30(8) = 240 \text{ lbf}$$
$$F_C^z = 0.4(240) = 96 \text{ lbf}$$
$$T = F_C^z(2) = 96(2) = 192 \text{ lbf} \cdot \text{in}$$
$$F_B^z = \frac{T}{1.5} = \frac{192}{1.5} = 128 \text{ lbf}$$
$$F_B^y = F_B^z \tan 20° = 128 \tan 20° = 46.6 \text{ lbf}$$

(a) *xy-plane*



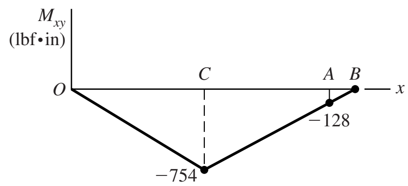
$$\sum M_O = 240(5.75) - F_A^y(11.5) - 46.6(14.25) = 0$$

$$F_A^y = \frac{240(5.75) - 46.6(14.25)}{11.5} = 62.3\,\text{lbf}$$

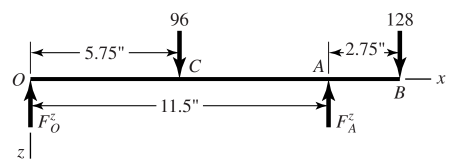
$$\sum M_A = F_O^y(11.5) - 46.6(2.75) - 240(5.75) = 0$$

$$F_O^y = \frac{240(5.75) + 46.6(2.75)}{11.5} = 131.1\,\text{lbf}$$

Bending moment diagram



xz-plane



$$\sum M_O = 0$$

$$= 96(5.75) - F_A^z(11.5) + 128(14.25)$$

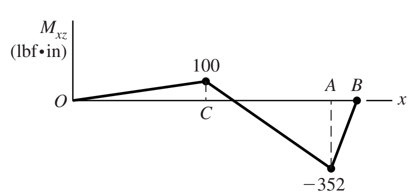
$$F_A^z = \frac{96(5.75) + 128(14.25)}{11.5} = 206.6\,\text{lbf}$$

$$\sum M_A = 0$$

$$= F_O^z(11.5) + 128(2.75) - 96(5.75)$$

$$F_O^z = \frac{96(5.75) - 128(2.75)}{11.5} = 17.4\,\text{lbf}$$

Bending moment diagram:

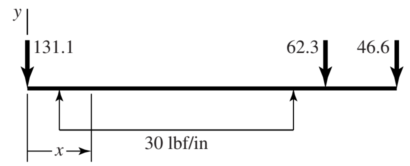


$$M_C = \sqrt{100^2 + (-754)^2} = 761\,\text{lbf}\cdot\text{in}$$

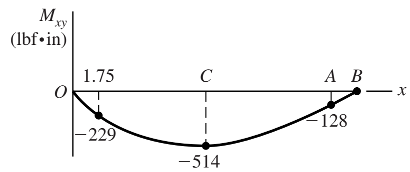
$$M_A = \sqrt{(-128)^2 + (-352)^2} = 375\,\text{lbf}\cdot\text{in}$$

This approach over-estimates the bending moment at *C*, but not at *A*.

(b) *xy-plane*



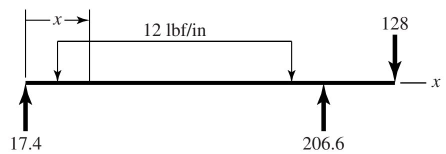
$$M_{xy} = -131.1x + 15\langle x - 1.75\rangle^2 - 15\langle x - 9.75\rangle^2 - 62.3\langle x - 11.5\rangle^1$$



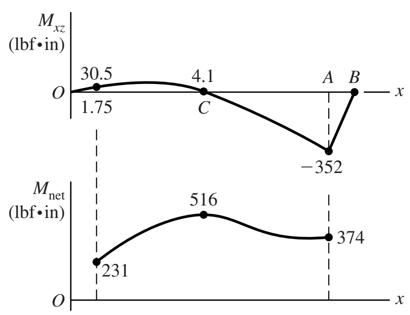
$$M_{max} \text{ occurs at } 6.12 \text{ in}$$
$$M_{max} = -516 \text{ lbf} \cdot \text{in}$$
$$M_C = 131.1(5.75) - 15(5.75 - 1.75)^2 = 514$$

Reduced from 754 lbf · in. The maximum occurs at $x = 6.12$ in rather than C, but it is close enough.

xz-plane



$$M_{xz} = 17.4x - 6\langle x - 1.75\rangle^2 + 6\langle x - 9.75\rangle^2 + 206.6\langle x - 11.5\rangle^1$$



Let $M_{net} = \sqrt{M_{xy}^2 + M_{xz}^2}$

Plot $M_{net}(x)$

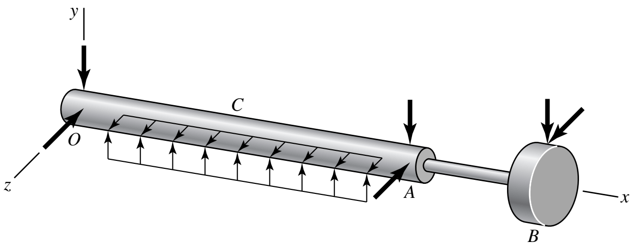
$1.75 \le x \le 11.5$ in

$M_{max} = 516$ lbf · in
at $x = 6.25$ in

Torque: In both cases the torque rises from 0 to 192 lbf · in linearly across the roller and is steady until the coupling keyway is encountered; then it falls linearly to 0 across the key. *Ans.*

7-5 This is a design problem, which can have many acceptable designs. See the solution for Problem 7-7 for an example of the design process.

7-6 If students have access to finite element or beam analysis software, have them model the shaft to check deflections. If not, solve a simpler version of shaft. The 1" diameter sections will not affect the results much, so model the 1" diameter as 1.25". Also, ignore the step in *AB*.



From Prob. 18-10, integrate M_{xy} and M_{xz}

xy plane, with $dy/dx = y'$

$$EIy' = -\frac{131.1}{2}(x^2) + 5\langle x - 1.75\rangle^3 - 5\langle x - 9.75\rangle^3 - \frac{62.3}{2}\langle x - 11.5\rangle^2 + C_1 \qquad (1)$$

$$EIy = -\frac{131.1}{6}(x^3) + \frac{5}{4}\langle x - 1.75\rangle^4 - \frac{5}{4}\langle x - 9.75\rangle^4 - \frac{62.3}{6}\langle x - 11.5\rangle^3 + C_1 x + C_2$$

$$y = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0$$

$$y = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_1 = 1908.4 \text{ lbf} \cdot \text{in}^3$$

From (1)

$$x = 0: \quad EIy' = 1908.4$$

$$x = 11.5: \quad EIy' = -2153.1$$

xz plane (treating $z \uparrow +$)

$$EIz' = \frac{17.4}{2}(x^2) - 2\langle x - 1.75\rangle^3 + 2\langle x - 9.75\rangle^3 + \frac{206.6}{2}\langle x - 11.5\rangle^2 + C_3 \qquad (2)$$

$$EIz = \frac{17.4}{6}(x^3) - \frac{1}{2}\langle x - 1.75\rangle^4 + \frac{1}{2}\langle x - 9.75\rangle^4 + \frac{206.6}{6}\langle x - 11.5\rangle^3 + C_3 x + C_4$$

$$z = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_4 = 0$$

$$z = 0 \text{ at } x = 11.5 \quad \Rightarrow \quad C_3 = 8.975 \text{ lbf} \cdot \text{in}^3$$

From (2)

$$x = 0: \quad EIz' = 8.975$$

$$x = 11.5: \quad EIz' = -683.5$$

At *O*: $EI\theta = \sqrt{1908.4^2 + 8.975^2} = 1908.4 \text{ lbf} \cdot \text{in}^3$

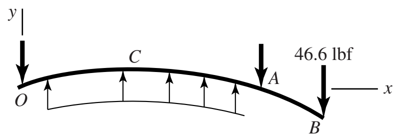
A: $EI\theta = \sqrt{(-2153.1)^2 + (-683.5)^2} = 2259 \text{ lbf} \cdot \text{in}^3$ (dictates size)

$$\theta = \frac{2259}{30(10^6)(\pi/64)(1.25^4)} = 0.000\,628 \text{ rad}$$

$$n = \frac{0.001}{0.000\,628} = 1.59$$

At gear mesh, B

xy plane



With $I = I_1$ in section *OCA*,

$$y'_A = -2153.1/EI_1$$

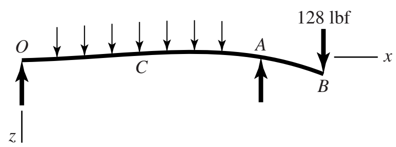
Since $y'_{B/A}$ is a cantilever, from Table A-9-1, with $I = I_2$ in section *AB*

$$y'_{B/A} = \frac{Fx(x-2l)}{2EI_2} = \frac{46.6}{2EI_2}(2.75)[2.75 - 2(2.75)] = -176.2/EI_2$$

$$\therefore y'_B = y'_A + y'_{B/A} = -\frac{2153.1}{30(10^6)(\pi/64)(1.25^4)} - \frac{176.2}{30(10^6)(\pi/64)(0.875^4)}$$

$$= -0.000\,803 \text{ rad} \quad \text{(magnitude greater than 0.0005 rad)}$$

xz plane



$$z'_A = -\frac{683.5}{EI_1}, \quad z'_{B/A} = -\frac{128(2.75^2)}{2EI_2} = -\frac{484}{EI_2}$$

$$z'_B = -\frac{683.5}{30(10^6)(\pi/64)(1.25^4)} - \frac{484}{30(10^6)(\pi/64)(0.875^4)} = -0.000\,751 \text{ rad}$$

$$\theta_B = \sqrt{(-0.000\,803)^2 + (0.000\,751)^2} = 0.001\,10 \text{ rad}$$

Crowned teeth must be used.

Finite element results:	Error in simplified model
$\theta_O = 5.47(10^{-4})$ rad	3.0%
$\theta_A = 7.09(10^{-4})$ rad	11.4%
$\theta_B = 1.10(10^{-3})$ rad	0.0%

The simplified model yielded reasonable results.

Strength $\quad S_{ut} = 72$ kpsi, $\quad S_y = 39.5$ kpsi

At the shoulder at A, $x = 10.75$ in. From Prob. 7-4,

$$M_{xy} = -209.3 \text{ lbf} \cdot \text{in}, \quad M_{xz} = -293.0 \text{ lbf} \cdot \text{in}, \quad T = 192 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{(-209.3)^2 + (-293)^2} = 360.0 \text{ lbf} \cdot \text{in}$$

$$S'_e = 0.5(72) = 36 \text{ kpsi}$$

$$k_a = 2.70(72)^{-0.265} = 0.869$$

$$k_b = \left(\frac{1}{0.3}\right)^{-0.107} = 0.879$$

$$k_c = k_d = k_e = k_f = 1$$

$$S_e = 0.869(0.879)(36) = 27.5 \text{ kpsi}$$

From Fig. A-15-8 with $D/d = 1.25$ and $r/d = 0.03$, $K_{ts} = 1.8$.

From Fig. A-15-9 with $D/d = 1.25$ and $r/d = 0.03$, $K_t = 2.3$

From Fig. 6-20 with $r = 0.03$ in, $q = 0.65$.

From Fig. 6-21 with $r = 0.03$ in, $q_s = 0.83$

Eq. (6-31):

$$K_f = 1 + 0.65(2.3 - 1) = 1.85$$

$$K_{fs} = 1 + 0.83(1.8 - 1) = 1.66$$

Using DE-elliptic, Eq. (7-11) with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi(1^3)}\left\{4\left[\frac{1.85(360)}{27\,500}\right]^2 + 3\left[\frac{1.66(192)}{39\,500}\right]^2\right\}^{1/2}$$

$$n = 3.89$$

Perform a similar analysis at the profile keyway under the gear.

The main problem with the design is the undersized shaft overhang with excessive slope at the gear. The use of crowned-teeth in the gears will eliminate this problem.

7-7 **(a)** One possible shaft layout is shown. Both bearings and the gear will be located against shoulders. The gear and the motor will transmit the torque through keys. The bearings can be lightly pressed onto the shaft. The left bearing will locate the shaft in the housing, while the right bearing will float in the housing.

(b) From summing moments around the shaft axis, the tangential transmitted load through the gear will be

$$W_t = T/(d/2) = 2500/(4/2) = 1250 \text{ lbf}$$

The radial component of gear force is related by the pressure angle.

$$W_r = W_t \tan\phi = 1250 \tan 20^\circ = 455 \text{ lbf}$$

$$W = [W_r^2 + W_t^2]^{1/2} = (455^2 + 1250^2)^{1/2} = 1330 \text{ lbf}$$

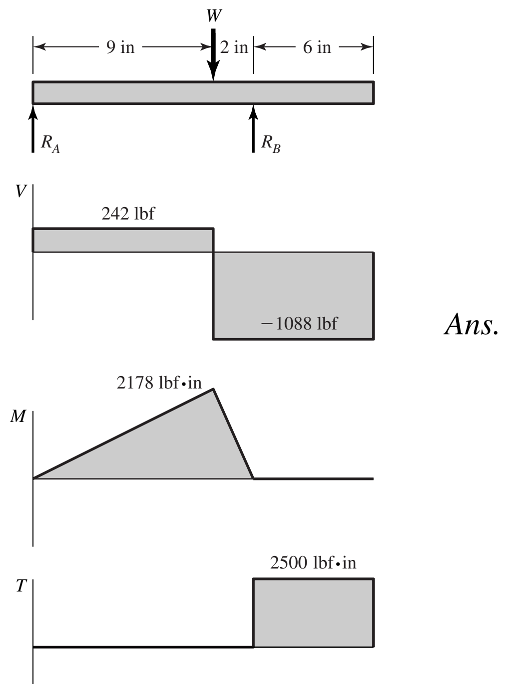
Reactions R_A and R_B, and the load W are all in the same plane. From force and moment balance,

$$R_A = 1330(2/11) = 242 \text{ lbf}$$

$$R_B = 1330(9/11) = 1088 \text{ lbf}$$

$$M_{\max} = R_A(9) = (242)(9) = 2178 \text{ lbf} \cdot \text{in}$$

Shear force, bending moment, and torque diagrams can now be obtained.



(c) Potential critical locations occur at each stress concentration (shoulders and keyways). To be thorough, the stress at each potentially critical location should be evaluated. For now, we will choose the most likely critical location, by observation of the loading situation, to be in the keyway for the gear. At this point there is a large stress concentration, a large bending moment, and the torque is present. The other locations either have small bending moments, or no torque. The stress concentration for the keyway is highest at the ends. For simplicity, and to be conservative, we will use the maximum bending moment, even though it will have dropped off a little at the end of the keyway.

(d) At the gear keyway, approximately 9 in from the left end of the shaft, the bending is completely reversed and the torque is steady.

$$M_a = 2178 \text{ lbf} \cdot \text{in} \qquad T_m = 2500 \text{ lbf} \cdot \text{in} \qquad M_m = T_a = 0$$

From Table 7-1, estimate stress concentrations for the end-milled keyseat to be $K_t = 2.2$ and $K_{ts} = 3.0$. For the relatively low strength steel specified (AISI 1020 CD), estimate notch sensitivities of $q = 0.75$ and $q_s = 0.9$, obtained by observation of Figs. 6-20 and 6-21. Assuming a typical radius at the bottom of the keyseat of $r/d = 0.02$ (p. 361), these estimates for notch sensitivity are good for up to about 3 in shaft diameter.

Eq. (6-32): $K_f = 1 + 0.75(2.2 - 1) = 1.9$

$K_{fs} = 1 + 0.9(3.0 - 1) = 2.8$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

For estimating k_b, guess $d = 2$ in.

$k_b = (2/0.3)^{-0.107} = 0.816$

$S_e = (0.883)(0.816)(0.5)(68) = 24.5$ kpsi

Selecting the DE-Goodman criteria for a conservative first design,

Eq. (7-8):
$$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4(K_f M_a)^2\right]^{1/2}}{S_e} + \frac{\left[3(K_{fs} T_m)^2\right]^{1/2}}{S_{ut}} \right\} \right]^{1/3}$$

$$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4(1.9 \cdot 2178)^2\right]^{1/2}}{24\,500} + \frac{\left[3(2.8 \cdot 2500)^2\right]^{1/2}}{68\,000} \right\} \right]^{1/3}$$

$$d = 1.58 \text{ in} \quad Ans.$$

With this diameter, the estimates for notch sensitivity and size factor were conservative, but close enough for a first iteration until deflections are checked.

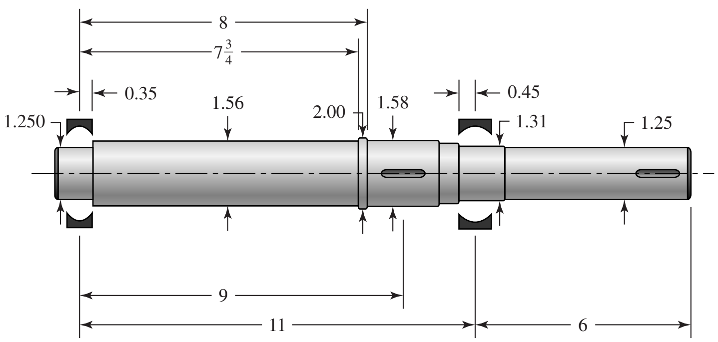
Check for static failure.

Eq. (7-15):
$$\sigma'_{\max} = \left[\left(\frac{32 K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16 K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma'_{\max} = \left[\left(\frac{32(1.9)(2178)}{\pi (1.58)^3} \right)^2 + 3 \left(\frac{16(2.8)(2500)}{\pi (1.58)^3} \right)^2 \right]^{1/2} = 19.0 \text{ kpsi}$$

$$n_y = S_y / \sigma'_{\max} = 57/19.0 = 3.0 \quad Ans.$$

(e) Now estimate other diameters to provide typical shoulder supports for the gear and bearings (p. 360). Also, estimate the gear and bearing widths.



(f) Entering this shaft geometry into beam analysis software (or Finite Element software), the following deflections are determined:

Left bearing slope:	0.000532 rad
Right bearing slope:	−0.000850 rad
Gear slope:	−0.000545 rad
Right end of shaft slope:	−0.000850 rad
Gear deflection:	−0.00145 in
Right end of shaft deflection:	0.00510 in

Comparing these deflections to the recommendations in Table 7-2, everything is within typical range except the gear slope is a little high for an uncrowned gear.

(g) To use a non-crowned gear, the gear slope is recommended to be less than 0.0005 rad. Since all other deflections are acceptable, we will target an increase in diameter only for the long section between the left bearing and the gear. Increasing this diameter from the proposed 1.56 in to 1.75 in, produces a gear slope of –0.000401 rad. All other deflections are improved as well.

7-8 **(a)** Use the distortion-energy elliptic failure locus. The torque and moment loadings on the shaft are shown in the solution to Prob. 7-7.

Candidate critical locations for strength:

- Pinion seat keyway
- Right bearing shoulder
- Coupling keyway

Table A-20 for 1030 HR: $S_{ut} = 68$ kpsi, $S_y = 37.5$ kpsi, $H_B = 137$

Eq. (6-8): $S'_e = 0.5(68) = 34.0$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

$k_c = k_d = k_e = 1$

Pinion seat keyway

See Table 7-1 for keyway stress concentration factors

$$\left.\begin{aligned} K_t &= 2.2 \\ K_{ts} &= 3.0 \end{aligned}\right\} \text{Profile keyway}$$

For an end-mill profile keyway cutter of 0.010 in radius,

From Fig. 6-20: $q = 0.50$

From Fig. 6-21: $q_s = 0.65$

Eq. (6-32):

$$\begin{aligned} K_{fs} &= 1 + q_s(K_{ts} - 1) \\ &= 1 + 0.65(3.0 - 1) = 2.3 \\ K_f &= 1 + 0.50(2.2 - 1) = 1.6 \end{aligned}$$

Eq. (6-20): $k_b = \left(\dfrac{1.875}{0.30}\right)^{-0.107} = 0.822$

Eq. (6-18): $S_e = 0.883(0.822)(34.0) = 24.7$ kpsi

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi(1.875^3)}\left\{4\left[\frac{1.6(2178)}{24\,700}\right]^2 + 3\left[\frac{2.3(2500)}{37\,500}\right]^2\right\}^{1/2}$$

$$= 0.353, \quad \text{from which } n = 2.83$$

Right-hand bearing shoulder

The text does not give minimum and maximum shoulder diameters for 03-series bearings (roller). Use $D = 1.75$ in.

$$\frac{r}{d} = \frac{0.030}{1.574} = 0.019, \quad \frac{D}{d} = \frac{1.75}{1.574} = 1.11$$

From Fig. A-15-9,

$$K_t = 2.4$$

From Fig. A-15-8,

$$K_{ts} = 1.6$$

From Fig. 6-20, $q = 0.65$

From Fig. 6-21, $q_s = 0.83$

$$K_f = 1 + 0.65(2.4 - 1) = 1.91$$

$$K_{fs} = 1 + 0.83(1.6 - 1) = 1.50$$

$$M = 2178\left(\frac{0.453}{2}\right) = 493 \text{ lbf} \cdot \text{in}$$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi(1.574^3)}\left[4\left(\frac{1.91(493)}{24\,700}\right)^2 + 3\left(\frac{1.50(2500)}{37\,500}\right)^2\right]^{1/2}$$

$$= 0.247, \quad \text{from which } n = 4.05$$

Overhanging coupling keyway

There is no bending moment, thus Eq. (7-11) reduces to:

$$\frac{1}{n} = \frac{16\sqrt{3}K_{fs}T_m}{\pi d^3 S_y} = \frac{16\sqrt{3}(1.50)(2500)}{\pi(1.5^3)(37\,500)}$$

$$= 0.261 \quad \text{from which } n = 3.83$$

(b) One could take pains to model this shaft exactly, using say finite element software. However, for the bearings and the gear, the shaft is basically of uniform diameter, 1.875 in. The reductions in diameter at the bearings will change the results insignificantly. Use $E = 30(10^6)$ psi.

To the left of the load:

$$\theta_{AB} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$

$$= \frac{1449(2)(3x^2 + 2^2 - 11^2)}{6(30)(10^6)(\pi/64)(1.825^4)(11)}$$

$$= 2.4124(10^{-6})(3x^2 - 117)$$

At $x = 0$: $\theta = -2.823(10^{-4})$ rad

At $x = 9$ in: $\theta = 3.040(10^{-4})$ rad

At $x = 11$ in:

$$\theta = \frac{1449(9)(11^2 - 9^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)} = 4.342(10^{-4}) \text{ rad}$$

Obtain allowable slopes from Table 7-2.

Left bearing:

$$n_{fs} = \frac{\text{Allowable slope}}{\text{Actual slope}} = \frac{0.001}{0.000\,282\,3} = 3.54$$

Right bearing:

$$n_{fs} = \frac{0.0008}{0.000\,434\,2} = 1.84$$

Gear mesh slope:

Table 7-2 recommends a minimum relative slope of 0.0005 rad. While we don't know the slope on the next shaft, we know that it will need to have a larger diameter and be stiffer. At the moment we can say

$$n_{fs} < \frac{0.0005}{0.000\,304} = 1.64$$

7-9 The solution to Problem 7-8 may be used as an example of the analysis process for a similar situation.

7-10 If you have a finite element program available, it is highly recommended. Beam deflection programs can be implemented but this is time consuming and the programs have narrow applications. Here we will demonstrate how the problem can be simplified and solved using singularity functions.

Deflection: First we will ignore the steps near the bearings where the bending moments are low. Thus let the 30 mm dia. be 35 mm. Secondly, the 55 mm dia. is very thin, 10 mm. The full bending stresses will not develop at the outer fibers so full stiffness will not develop either. Thus, ignore this step and let the diameter be 45 mm.

Statics: Left support: $R_1 = 7(315 - 140)/315 = 3.889$ kN

Right support: $R_2 = 7(140)/315 = 3.111$ kN

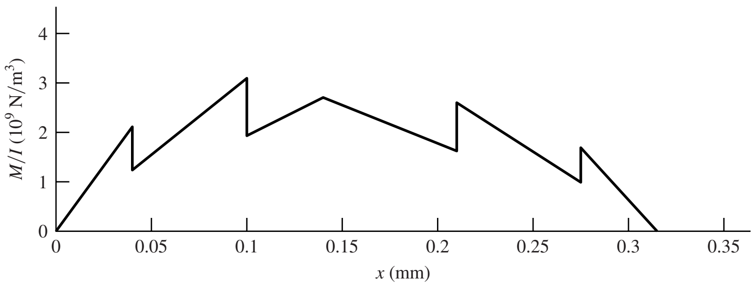
Determine the bending moment at each step.

x(mm)	0	40	100	140	210	275	315
M(N · m)	0	155.56	388.89	544.44	326.67	124.44	0

$I_{35} = (\pi/64)(0.035^4) = 7.366(10^{-8})\text{ m}^4,\ I_{40} = 1.257(10^{-7})\text{ m}^4,\ I_{45} = 2.013(10^{-7})\text{ m}^4$

Plot M/I as a function of x.

x (m)	$M/I\,(10^9\ \text{N/m}^3)$	Step	Slope	Δ Slope
0	0		52.8	
0.04	2.112			
0.04	1.2375	−0.8745	30.942	−21.86
0.1	3.094			
0.1	1.932	−1.162	19.325	−11.617
0.14	2.705			
0.14	2.705	0	−15.457	−34.78
0.21	1.623			
0.21	2.6	0.977	−24.769	−9.312
0.275	0.99			
0.275	1.6894	0.6994	−42.235	−17.47
0.315	0			



The steps and the change of slopes are evaluated in the table. From these, the function M/I can be generated:

$$\begin{aligned} M/I = \big[&52.8x - 0.8745\langle x-0.04\rangle^0 - 21.86\langle x-0.04\rangle^1 - 1.162\langle x-0.1\rangle^0 \\ &- 11.617\langle x-0.1\rangle^1 - 34.78\langle x-0.14\rangle^1 + 0.977\langle x-0.21\rangle^0 \\ &- 9.312\langle x-0.21\rangle^1 + 0.6994\langle x-0.275\rangle^0 - 17.47\langle x-0.275\rangle^1\big]\,10^9 \end{aligned}$$

Integrate twice:

$$\begin{aligned} E\frac{dy}{dx} = \big[&26.4x^2 - 0.8745\langle x-0.04\rangle^1 - 10.93\langle x-0.04\rangle^2 - 1.162\langle x-0.1\rangle^1 \\ &- 5.81\langle x-0.1\rangle^2 - 17.39\langle x-0.14\rangle^2 + 0.977\langle x-0.21\rangle^1 \\ &- 4.655\langle x-0.21\rangle^2 + 0.6994\langle x-0.275\rangle^1 - 8.735\langle x-0.275\rangle^2 + C_1\big]\,10^9 \end{aligned} \qquad (1)$$

$$\begin{aligned} Ey = \big[&8.8x^3 - 0.4373\langle x-0.04\rangle^2 - 3.643\langle x-0.04\rangle^3 - 0.581\langle x-0.1\rangle^2 \\ &- 1.937\langle x-0.1\rangle^3 - 5.797\langle x-0.14\rangle^3 + 0.4885\langle x-0.21\rangle^2 \\ &- 1.552\langle x-0.21\rangle^3 + 0.3497\langle x-0.275\rangle^2 - 2.912\langle x-0.275\rangle^3 + C_1x + C_2\big]\,10^9 \end{aligned}$$

Boundary conditions: $y = 0$ at $x = 0$ yields $C_2 = 0$;

$y = 0$ at $x = 0.315$ m yields $C_1 = -0.295\,25\ \text{N/m}^2$.

Equation (1) with $C_1 = -0.295\,25$ provides the slopes at the bearings and gear. The following table gives the results in the second column. The third column gives the results from a similar finite element model. The fourth column gives the result of a full model which models the 35 and 55 mm diameter steps.

x (mm)	θ (rad)	F.E. Model	Full F.E. Model
0	−0.001 4260	−0.001 4270	−0.001 4160
140	−0.000 1466	−0.000 1467	−0.000 1646
315	0.001 3120	0.001 3280	0.001 3150

The main discrepancy between the results is at the gear location ($x = 140$ mm). The larger value in the full model is caused by the stiffer 55 mm diameter step. As was stated earlier, this step is not as stiff as modeling implicates, so the exact answer is somewhere between the full model and the simplified model which in any event is a small value. As expected, modeling the 30 mm dia. as 35 mm does not affect the results much.

It can be seen that the allowable slopes at the bearings are exceeded. Thus, either the load has to be reduced or the shaft "beefed" up. If the allowable slope is 0.001 rad, then the maximum load should be $F_{\max} = (0.001/0.001\,46)7 = 4.79$ kN. With a design factor this would be reduced further.

To increase the stiffness of the shaft, increase the diameters by $(0.001\,46/0.001)^{1/4} = 1.097$, from Eq. (7-18). Form a table:

Old d, mm	20.00	30.00	35.00	40.00	45.00	55.00
New ideal d, mm	21.95	32.92	38.41	43.89	49.38	60.35
Rounded up d, mm	22.00	34.00	40.00	44.00	50.00	62.00

Repeating the full finite element model results in

$$x = 0: \quad \theta = -9.30 \times 10^{-4} \text{ rad}$$

$$x = 140 \text{ mm}: \quad \theta = -1.09 \times 10^{-4} \text{ rad}$$

$$x = 315 \text{ mm}: \quad \theta = 8.65 \times 10^{-4} \text{ rad}$$

Well within our goal. Have the students try a goal of 0.0005 rad at the bearings.

Strength: Due to stress concentrations and reduced shaft diameters, there are a number of locations to look at. A table of nominal stresses is given below. Note that torsion is only to the right of the 7 kN load. Using $\sigma = 32M/(\pi d^3)$ and $\tau = 16T/(\pi d^3)$,

x (mm)	0	15	40	100	110	140	210	275	300	330
σ (MPa)	0	22.0	37.0	61.9	47.8	60.9	52.0	39.6	17.6	0
τ (MPa)	0	0	0	0	0	6	8.5	12.7	20.2	68.1
σ' (MPa)	0	22.0	37.0	61.9	47.8	61.8	53.1	45.3	39.2	118.0

Table A-20 for AISI 1020 CD steel: $S_{ut} = 470$ MPa, $S_y = 390$ MPa

At $x = 210$ mm:

$$k_a = 4.51(470)^{-0.265} = 0.883, \quad k_b = (40/7.62)^{-0.107} = 0.837$$

$$S_e = 0.883(0.837)(0.5)(470) = 174 \text{ MPa}$$

$$D/d = 45/40 = 1.125, \ r/d = 2/40 = 0.05.$$

From Figs. A-15-8 and A-15-9, $K_t = 1.9$ and $K_{ts} = 1.32$.

From Figs. 6-20 and 6-21, $q = 0.75$ and $q_s = 0.92$,

$$K_f = 1 + 0.75(1.9 - 1) = 1.68, \text{ and } K_{fs} = 1 + 0.92(1.32 - 1) = 1.29.$$

From Eq. (7-11), with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi(0.04)^3}\left\{4\left[\frac{1.68(326.67)}{174(10^6)}\right]^2 + 3\left[\frac{1.29(107)}{390(10^6)}\right]^2\right\}^{1/2}$$

$$n = 1.98$$

At $x = 330$ mm: The von Mises stress is the highest but it comes from the steady torque only.

$$D/d = 30/20 = 1.5, \quad r/d = 2/20 = 0.1 \quad \Rightarrow \quad K_{ts} = 1.42,$$

$$q_s = 0.92 \quad \Rightarrow \quad K_{fs} = 1.39$$

$$\frac{1}{n} = \frac{16}{\pi(0.02)^3}\left(\sqrt{3}\right)\left[\frac{1.39(107)}{390(10^6)}\right]$$

$$n = 2.38$$

Check the other locations.

If worse-case is at $x = 210$ mm, the changes discussed for the slope criterion will improve the strength issue.

7-11 and 7-12 With these design tasks each student will travel different paths and almost all details will differ. The important points are

- The student gets a blank piece of paper, a statement of function, and some constraints–explicit and implied. At this point in the course, this is a good experience.
- It is a good preparation for the capstone design course.
- The adequacy of their design must be demonstrated and possibly include a designer's notebook.
- Many of the fundaments of the course, based on this text and this course, are useful. The student will find them useful and notice that he/she is doing it.
- Don't let the students create a time sink for themselves. Tell them how far you want them to go.

7-13 I used this task as a final exam when all of the students in the course had consistent test scores going into the final examination; it was my expectation that they would not change things much by taking the examination.

This problem is a learning experience. Following the task statement, the following guidance was added.

- Take the first half hour, resisting the temptation of putting pencil to paper, and decide what the problem really is.
- Take another twenty minutes to list several possible remedies.
- Pick one, and show your instructor how you would implement it.

The students' initial reaction is that he/she does not know much from the problem statement. Then, slowly the realization sets in that they do know some important things that the designer did not. They knew how it failed, where it failed, and that the design wasn't good enough; it was close, though.

Also, a fix at the bearing seat lead-in could transfer the problem to the shoulder fillet, and the problem may not be solved.

To many students' credit, they chose to keep the shaft geometry, and selected a new material to realize about twice the Brinell hardness.

7-14 In Eq. (7-24) set

$$I = \frac{\pi d^4}{64}, \quad A = \frac{\pi d^2}{4}$$

to obtain

$$\omega = \left(\frac{\pi}{l}\right)^2 \left(\frac{d}{4}\right) \sqrt{\frac{gE}{\gamma}} \tag{1}$$

or

$$d = \frac{4l^2\omega}{\pi^2} \sqrt{\frac{\gamma}{gE}} \tag{2}$$

(a) From Eq. (1) and Table A-5,

$$\omega = \left(\frac{\pi}{24}\right)^2 \left(\frac{1}{4}\right) \sqrt{\frac{386(30)(10^6)}{0.282}} = 868 \text{ rad/s} \quad \textit{Ans.}$$

(b) From Eq. (2),

$$d = \frac{4(24)^2(2)(868)}{\pi^2} \sqrt{\frac{0.282}{386(30)(10^6)}} = 2 \text{ in} \quad \textit{Ans.}$$

(c) From Eq. (2),

$$l\omega = \frac{\pi^2}{4} \frac{d}{l} \sqrt{\frac{gE}{\gamma}}$$

Since d/l is the same regardless of the scale.

$$l\omega = \text{constant} = 24(868) = 20\,832$$

$$\omega = \frac{20\,832}{12} = 1736 \text{ rad/s} \quad \textit{Ans.}$$

Thus the first critical speed doubles.

7-15 From Prob. 7-14, $\omega = 868$ rad/s

$$A = 0.7854 \text{ in}^2, \quad I = 0.04909 \text{ in}^4, \quad \gamma = 0.282 \text{ lbf/in}^3,$$

$$E = 30(10^6) \text{ psi}, \quad w = A\gamma l = 0.7854(0.282)(24) = 5.316 \text{ lbf}$$

One element:

Eq. (7-24) $$\delta_{11} = \frac{12(12)(24^2 - 12^2 - 12^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.956(10^{-4}) \text{ in/lbf}$$

$$y_1 = w_1\delta_{11} = 5.316(1.956)(10^{-4}) = 1.0398(10^{-3}) \text{ in}$$

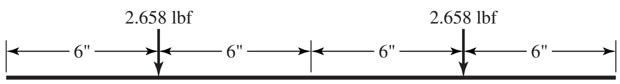
$$y_1^2 = 1.0812(10^{-6})$$

$$\sum wy = 5.316(1.0398)(10^{-3}) = 5.528(10^{-3})$$

$$\sum wy^2 = 5.316(1.0812)(10^{-6}) = 5.748(10^{-6})$$

$$\omega_1 = \sqrt{g\frac{\sum wy}{\sum wy^2}} = \sqrt{386\left[\frac{5.528(10^{-3})}{5.748(10^{-6})}\right]} = 609 \text{ rad/s} \quad (30\% \text{ low})$$

Two elements:



$$\delta_{11} = \delta_{22} = \frac{18(6)(24^2 - 18^2 - 6^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.100(10^{-4}) \text{ in/lbf}$$

$$\delta_{12} = \delta_{21} = \frac{6(6)(24^2 - 6^2 - 6^2)}{6(30)(10^6)(0.049\,09)(24)} = 8.556(10^{-5}) \text{ in/lbf}$$

$$y_1 = w_1\delta_{11} + w_2\delta_{12} = 2.658(1.100)(10^{-4}) + 2.658(8.556)(10^{-5})$$
$$= 5.198(10^{-4}) \text{ in} = y_2,$$

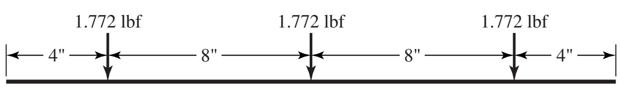
$$y_1^2 = y_2^2 = 2.702(10^{-7}) \text{ in}^2$$

$$\sum wy = 2(2.658)(5.198)(10^{-4}) = 2.763(10^{-3})$$

$$\sum wy^2 = 2(2.658)(2.702)(10^{-7}) = 1.436(10^{-6})$$

$$\omega_1 = \sqrt{386\left[\frac{2.763(10^{-3})}{1.436(10^{-6})}\right]} = 862 \text{ rad/s} \quad (0.7\% \text{ low})$$

Three elements:



$$\delta_{11} = \delta_{33} = \frac{20(4)(24^2 - 20^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 6.036(10^{-5}) \text{ in/lbf}$$

$$\delta_{22} = \frac{12(12)(24^2 - 12^2 - 12^2)}{6(30)(10^6)(0.049\,09)(24)} = 1.956(10^{-4}) \text{ in/lbf}$$

$$\delta_{12} = \delta_{32} = \frac{12(4)(24^2 - 12^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 9.416(10^{-5}) \text{ in/lbf}$$

$$\delta_{13} = \frac{4(4)(24^2 - 4^2 - 4^2)}{6(30)(10^6)(0.049\,09)(24)} = 4.104(10^{-5}) \text{ in/lbf}$$

$$y_1 = 1.772[6.036(10^{-5}) + 9.416(10^{-5}) + 4.104(10^{-5})] = 3.465(10^{-4}) \text{ in}$$
$$y_2 = 1.772[9.416(10^{-5}) + 1.956(10^{-4}) + 9.416(10^{-5})] = 6.803(10^{-4}) \text{ in}$$
$$y_3 = 1.772[4.104(10^{-5}) + 9.416(10^{-5}) + 6.036(10^{-5})] = 3.465(10^{-4}) \text{ in}$$
$$\sum wy = 2.433(10^{-3}), \quad \sum wy^2 = 1.246(10^{-6})$$
$$\omega_1 = \sqrt{386\left[\frac{2.433(10^{-3})}{1.246(10^{-6})}\right]} = 868 \text{ rad/s} \qquad \text{(same as in Prob. 7-14)}$$

The point was to show that convergence is rapid using a static deflection beam equation. The method works because:

- If a deflection curve is chosen which meets the boundary conditions of moment-free and deflection-free ends, and in this problem, of symmetry, the strain energy is not very sensitive to the equation used.
- Since the static bending equation is available, and meets the moment-free and deflection-free ends, it works.

7-16 **(a)** For two bodies, Eq. (7-26) is

$$\begin{vmatrix} (m_1\delta_{11} - 1/\omega^2) & m_2\delta_{12} \\ m_1\delta_{21} & (m_2\delta_{22} - 1/\omega^2) \end{vmatrix} = 0$$

Expanding the determinant yields,

$$\left(\frac{1}{\omega^2}\right)^2 - (m_1\delta_{11} + m_2\delta_{22})\left(\frac{1}{\omega_1^2}\right) + m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) = 0 \qquad (1)$$

Eq. (1) has two roots $1/\omega_1^2$ and $1/\omega_2^2$. Thus

$$\left(\frac{1}{\omega^2} - \frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega^2} - \frac{1}{\omega_2^2}\right) = 0$$

or,

$$\left(\frac{1}{\omega^2}\right)^2 + \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\left(\frac{1}{\omega}\right)^2 + \left(\frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega_2^2}\right) = 0 \qquad (2)$$

Equate the third terms of Eqs. (1) and (2), which must be identical.

$$\frac{1}{\omega_1^2}\frac{1}{\omega_2^2} = m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) \quad \Rightarrow \quad \frac{1}{\omega_2^2} = \omega_1^2 m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})$$

and it follows that

$$\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{g^2}{w_1w_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})}} \quad Ans.$$

(b) In Ex. 7-5, Part (b) the first critical speed of the two-disk shaft ($w_1 = 35$ lbf, $w_2 = 55$ lbf) is $\omega_1 = 124.7$ rad/s. From part (a), using influence coefficients

$$\omega_2 = \frac{1}{124.7}\sqrt{\frac{386^2}{35(55)[2.061(3.534) - 2.234^2](10^{-8})}} = 466 \text{ rad/s} \quad Ans.$$

7-17 In Eq. (7-22) the term $\sqrt{I/A}$ appears. For a hollow unform diameter shaft,

$$\sqrt{\frac{I}{A}} = \sqrt{\frac{\pi\left(d_o^4 - d_i^4\right)/64}{\pi\left(d_o^2 - d_i^2\right)/4}} = \sqrt{\frac{1}{16}\frac{\left(d_o^2 + d_i^2\right)\left(d_o^2 - d_i^2\right)}{d_o^2 - d_i^2}} = \frac{1}{4}\sqrt{d_o^2 + d_i^2}$$

This means that when a solid shaft is hollowed out, the critical speed increases beyond that of the solid shaft. By how much?

$$\frac{\frac{1}{4}\sqrt{d_o^2 + d_i^2}}{\frac{1}{4}\sqrt{d_o^2}} = \sqrt{1 + \left(\frac{d_i}{d_o}\right)^2}$$

The possible values of d_i are $0 \le d_i \le d_o$, so the range of critical speeds is

$$\omega_s\sqrt{1+0} \text{ to about } \omega_s\sqrt{1+1}$$

or from ω_s to $\sqrt{2}\omega_s$. *Ans.*

7-18 All steps will be modeled using singularity functions with a spreadsheet. Programming both loads will enable the user to first set the left load to 1, the right load to 0 and calculate δ_{11} and δ_{21}. Then setting left load to 0 and the right to 1 to get δ_{12} and δ_{22}. The spreadsheet shown on the next page shows the δ_{11} and δ_{21} calculation. Table for M/I vs x is easy to make. The equation for M/I is:

$$\begin{aligned} M/I = D13x &+ C15\langle x-1\rangle^0 + E15\langle x-1\rangle^1 + E17\langle x-2\rangle^1 \\ &+ C19\langle x-9\rangle^0 + E19\langle x-9\rangle^1 + E21\langle x-14\rangle^1 \\ &+ C23\langle x-15\rangle^0 + E23\langle x-15\rangle^1 \end{aligned}$$

Integrating twice gives the equation for Ey. Boundary conditions $y = 0$ at $x = 0$ and at $x = 16$ inches provide integration constants ($C_2 = 0$). Substitution back into the deflection equation at $x = 2, 14$ inches provides the δ's. The results are: $\delta_{11} - 2.917(10^{-7})$, $\delta_{12} = \delta_{21} = 1.627(10^{-7})$, $\delta_{22} = 2.231(10^{-7})$. This can be verified by finite element analysis.

$$y_1 = 20(2.917)(10^{-7}) + 35(1.627)(10^{-7}) = 1.153(10^{-5})$$

$$y_2 = 20(1.627)(10^{-7}) + 35(2.231)(10^{-7}) = 1.106(10^{-5})$$

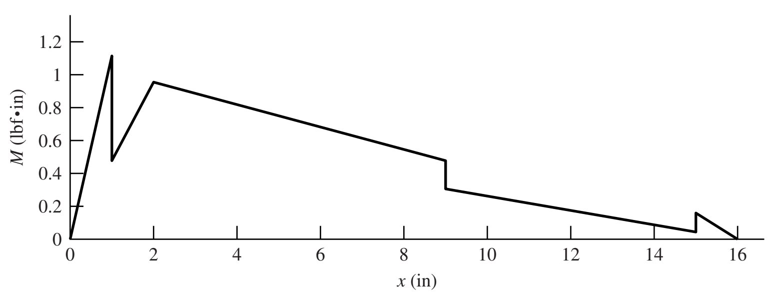
$$y_1^2 = 1.329(10^{-10}), \quad y_2^2 = 1.224(10^{-10})$$

$$\sum wy = 6.177(10^{-4}), \quad \sum wy^2 = 6.942(10^{-9})$$

Neglecting the shaft, Eq. (7-23) gives

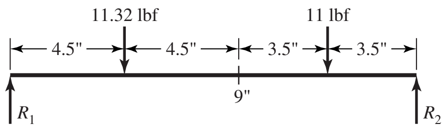
$$\omega_1 = \sqrt{386\left[\frac{6.177(10^{-4})}{6.942(10^{-9})}\right]} = 5860 \text{ rad/s} \quad \text{or} \quad 55\,970 \text{ rev/min} \quad \textit{Ans.}$$

	A	B	C	D	E	F	G	H	I
1	$F_1 = 1$		$F_2 = 0$		$R_1 = 0.875$ (left reaction)				
2									
3	x	M		$I_1 = I_4 = 0.7854$					
4	0	0		$I_2 = 1.833$					
5	1	0.875		$I_3 = 2.861$					
6	2	1.75							
7	9	0.875							
8	14	0.25							
9	15	0.125							
10	16	0							
11									
12	x	M/I	step	slope	Δ slope				
13	0	0		1.114 082					
14	1	1.114 082							
15	1	0.477 36	−0.636 722 477	0.477 36	−0.636 72				
16	2	0.954 719							
17	2	0.954 719	0	−0.068 19	−0.545 55				
18	9	0.477 36							
19	9	0.305 837	−0.171 522 4	−0.043 69	0.024 503				
20	14	0.087 382							
21	14	0.087 382	0	−0.043 69	0				
22	15	0.043 691							
23	15	0.159 155	0.115 463 554	−0.159 15	−0.115 46				
24	16	0							
25									
26		$C_1 = -4.906\,001\,093$							
27									
28									
29		$\delta_{11} = 2.91701\text{E-}07$							
30		$\delta_{21} = 1.6266\text{E-}07$							



Repeat for $F_1 = 0$ and $F_2 = 1$.

Modeling the shaft separately using 2 elements gives approximately



The spreadsheet can be easily modified to give

$$\delta_{11} = 9.605(10^{-7}), \quad \delta_{12} = \delta_{21} = 5.718(10^{-7}), \quad \delta_{22} = 5.472(10^{-7})$$
$$y_1 = 1.716(10^{-5}), \quad y_2 = 1.249(10^{-5}), \quad y_1^2 = 2.946(10^{-10}),$$
$$y_2^2 = 1.561(10^{-10}), \quad \sum wy = 3.316(10^{-4}), \quad \sum wy^2 = 5.052(10^{-9})$$
$$\omega_1 = \sqrt{386\left[\frac{3.316(10^{-4})}{5.052(10^{-9})}\right]} = 5034 \text{ rad/s} \quad Ans.$$

A finite element model of the exact shaft gives $\omega_1 = 5340$ rad/s. The simple model is 5.7% low.

Combination Using Dunkerley's equation, Eq. (7-32):

$$\frac{1}{\omega_1^2} = \frac{1}{5860^2} + \frac{1}{5034^2} \quad \Rightarrow \quad 3819 \text{ rad/s} \quad Ans.$$

7-19 We must not let the basis of the stress concentration factor, as presented, impose a viewpoint on the designer. Table A-16 shows K_{ts} as a decreasing monotonic as a function of a/D. All is not what it seems.

Let us change the basis for data presentation to the full section rather than the net section.

$$\tau = K_{ts}\tau_0 = K'_{ts}\tau'_0$$
$$K_{ts} = \frac{32T}{\pi A D^3} = K'_{ts}\left(\frac{32T}{\pi D^3}\right)$$

Therefore

$$K'_{ts} = \frac{K_{ts}}{A}$$

Form a table:

(a/D)	A	K_{ts}	K'_{ts}	
0.050	0.95	1.77	1.86	
0.075	0.93	1.71	1.84	
0.100	0.92	1.68	1.83	← minimum
0.125	0.89	1.64	1.84	
0.150	0.87	1.62	1.86	
0.175	0.85	1.60	1.88	
0.200	0.83	1.58	1.90	

K'_{ts} has the following attributes:

- It exhibits a minimum;
- It changes little over a wide range;
- Its minimum is a stationary point minimum at $a/D \doteq 0.100$;
- Our knowledge of the minima location is

$$0.075 \le (a/D) \le 0.125$$

We can form a design rule: in torsion, the pin diameter should be about $1/10$ of the shaft diameter, for greatest shaft capacity. However, it is not catastrophic if one forgets the rule.

7-20 Choose 15 mm as basic size, D, d. Table 7-9: fit is designated as 15H7/h6. From Table A-11, the tolerance grades are $\Delta D = 0.018$ mm and $\Delta d = 0.011$ mm.

Hole: Eq. (7-36)

$$D_{\max} = D + \Delta D = 15 + 0.018 = 15.018 \text{ mm} \quad \textit{Ans.}$$
$$D_{\min} = D = 15.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12, fundamental deviation $\delta_F = 0$. From Eq. (2-39)

$$d_{\max} = d + \delta_F = 15.000 + 0 = 15.000 \text{ mm} \quad \textit{Ans.}$$
$$d_{\min} = d + \delta_R - \Delta d = 15.000 + 0 - 0.011 = 14.989 \text{ mm} \quad \textit{Ans.}$$

7-21 Choose 45 mm as basic size. Table 7-9 designates fit as 45H7/s6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm

Hole: Eq. (7-36)

$$D_{\max} = D + \Delta D = 45.000 + 0.025 = 45.025 \text{ mm} \quad \textit{Ans.}$$
$$D_{\min} = D = 45.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12, fundamental deviation $\delta_F = +0.043$ mm. From Eq. (7-38)

$$d_{\min} = d + \delta_F = 45.000 + 0.043 = 45.043 \text{ mm} \quad \textit{Ans.}$$
$$d_{\max} = d + \delta_F + \Delta d = 45.000 + 0.043 + 0.016 = 45.059 \text{ mm} \quad \textit{Ans.}$$

7-22 Choose 50 mm as basic size. From Table 7-9 fit is 50H7/g6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm.

Hole:

$$D_{\max} = D + \Delta D = 50 + 0.025 = 50.025 \text{ mm} \quad \textit{Ans.}$$
$$D_{\min} = D = 50.000 \text{ mm} \quad \textit{Ans.}$$

Shaft: From Table A-12 fundamental deviation $= -0.009$ mm

$$d_{\max} = d + \delta_F = 50.000 + (-0.009) = 49.991 \text{ mm} \quad \textit{Ans.}$$
$$\begin{aligned} d_{\min} &= d + \delta_F - \Delta d \\ &= 50.000 + (-0.009) - 0.016 \\ &= 49.975 \text{ mm} \end{aligned}$$

7-23 Choose the basic size as 1.000 in. From Table 7-9, for 1.0 in, the fit is H8/f7. From Table A-13, the tolerance grades are $\Delta D = 0.0013$ in and $\Delta d = 0.0008$ in.

Hole:

$$D_{\text{max}} = D + (\Delta D)_{\text{hole}} = 1.000 + 0.0013 = 1.0013 \text{ in} \quad Ans.$$

$$D_{\text{min}} = D = 1.0000 \text{ in} \quad Ans.$$

Shaft: From Table A-14: Fundamental deviation $= -0.0008$ in

$$d_{\text{max}} = d + \delta_F = 1.0000 + (-0.0008) = 0.9992 \text{ in} \quad Ans.$$

$$d_{\text{min}} = d + \delta_F - \Delta d = 1.0000 + (-0.0008) - 0.0008 = 0.9984 \text{ in} \quad Ans.$$

Alternatively,

$$d_{\text{min}} = d_{\text{max}} - \Delta d = 0.9992 - 0.0008 = 0.9984 \text{ in.} \quad Ans.$$

7-24 **(a)** Basic size is $D = d = 1.5$ in.

Table 7-9: H7/s6 is specified for medium drive fit.

Table A-13: Tolerance grades are $\Delta D = 0.001$ in and $\Delta d = 0.0006$ in.

Table A-14: Fundamental deviation is $\delta_F = 0.0017$ in.

Eq. (7-36): $D_{\text{max}} = D + \Delta D = 1.501$ in *Ans.*

$D_{\text{min}} = D = 1.500$ in *Ans.*

Eq. (7-37): $d_{\text{max}} = d + \delta_F + \Delta d = 1.5 + 0.0017 + 0.0006 = 1.5023$ in *Ans.*

Eq. (7-38): $d_{\text{min}} = d + \delta_F = 1.5 + 0.0017 + 1.5017$ in *Ans.*

(b) Eq. (7-42): $\delta_{\text{min}} = d_{\text{min}} - D_{\text{max}} = 1.5017 - 1.501 = 0.0007$ in

Eq. (7-43): $\delta_{\text{max}} = d_{\text{max}} - D_{\text{min}} = 1.5023 - 1.500 = 0.0023$ in

Eq. (7-40):

$$p_{\text{max}} = \frac{E\delta_{\text{max}}}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right]$$

$$= \frac{(30)(10^6)(0.0023)}{2(1.5)^3}\left[\frac{(2.5^2 - 1.5^2)(1.5^2 - 0)}{2.5^2 - 0}\right] = 14\,720 \text{ psi} \quad Ans.$$

$$p_{\text{min}} = \frac{E\delta_{\text{min}}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$

$$= \frac{(30)(10^6)(0.0007)}{2(1.5)^3}\left[\frac{(2.5^2 - 1.5^2)(1.5^2 - 0)}{2.5^2 - 0}\right] = 4480 \text{ psi} \quad Ans.$$

(c) For the shaft:

Eq. (7-44): $\sigma_{t,\text{shaft}} = -p = -14\,720$ psi

Eq. (7-46): $\sigma_{r,\text{shaft}} = -p = -14\,720$ psi

Eq. (5-13):

$$\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$$

$$= [(-14\,720)^2 - (-14\,720)(-14\,720) + (-14\,720)^2]^{1/2}$$

$$= 14\,720 \text{ psi}$$

$$n = S_y/\sigma' = 57\,000/14\,720 = 3.9 \quad Ans.$$

For the hub:

Eq. (7-45): $\sigma_{t,\mathrm{hub}} = p\dfrac{d_o^2 + d^2}{d_o^2 - d^2} = (14\,720)\left(\dfrac{2.5^2 + 1.5^2}{2.5^2 - 1.5^2}\right) = 31\,280$ psi

Eq. (7-46): $\sigma_{r,\mathrm{hub}} = -p = -14\,720$ psi

Eq. (5-13): $\sigma' = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{1/2}$

$$= [(31\,280)^2 - (31\,280)(-14\,720) + (-14\,720)^2]^{1/2} = 40\,689 \text{ psi}$$

$$n = S_y/\sigma' = 85\,000/40\,689 = 2.1 \quad \textit{Ans.}$$

(d) Eq. (7-49) $T = (\pi/2) f p_{\min} l d^2$

$$= (\pi/2)(0.3)(4480)(2)(1.5)^2 = 9500 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$