Chapter 5

5-1

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $n = \dfrac{S_y}{\sigma'}$

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

(a) MSS: $\sigma_1 = 12,\ \sigma_2 = 6,\ \sigma_3 = 0$ kpsi

$$n = \frac{50}{12} = 4.17 \quad \textit{Ans.}$$

DE: $\sigma' = (12^2 - 6(12) + 6^2)^{1/2} = 10.39$ kpsi, $\quad n = \dfrac{50}{10.39} = 4.81 \quad \textit{Ans.}$

(b) $\sigma_A, \sigma_B = \dfrac{12}{2} \pm \sqrt{\left(\dfrac{12}{2}\right)^2 + (-8)^2} = 16, -4$ kpsi

$\sigma_1 = 16,\ \sigma_2 = 0,\ \sigma_3 = -4$ kpsi

MSS: $n = \dfrac{50}{16 - (-4)} = 2.5 \quad \textit{Ans.}$

DE: $\sigma' = (12^2 + 3(-8^2))^{1/2} = 18.33$ kpsi, $\quad n = \dfrac{50}{18.33} = 2.73 \quad \textit{Ans.}$

(c) $\sigma_A, \sigma_B = \dfrac{-6-10}{2} \pm \sqrt{\left(\dfrac{-6+10}{2}\right)^2 + (-5)^2} = -2.615, -13.385$ kpsi

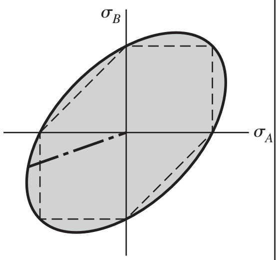
$\sigma_1 = 0,\ \sigma_2 = -2.615,\ \sigma_3 = -13.385$ kpsi

MSS: $n = \dfrac{50}{0 - (-13.385)} = 3.74 \quad \textit{Ans.}$

DE: $\sigma' = [(-6)^2 - (-6)(-10) + (-10)^2 + 3(-5)^2]^{1/2}$

$= 12.29$ kpsi

$$n = \frac{50}{12.29} = 4.07 \quad \textit{Ans.}$$



(d) $\sigma_A, \sigma_B = \dfrac{12+4}{2} \pm \sqrt{\left(\dfrac{12-4}{2}\right)^2 + 1^2} = 12.123, 3.877$ kpsi

$\sigma_1 = 12.123,\ \sigma_2 = 3.877,\ \sigma_3 = 0$ kpsi

MSS: $n = \dfrac{50}{12.123 - 0} = 4.12 \quad \textit{Ans.}$

DE: $\sigma' = [12^2 - 12(4) + 4^2 + 3(1^2)]^{1/2} = 10.72$ kpsi

$$n = \frac{50}{10.72} = 4.66 \quad \textit{Ans.}$$

5-2 $S_y = 50$ kpsi

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = 0$, $n = \dfrac{50}{12 - 0} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(12) + 12^2]^{1/2}} = 4.17$ *Ans.*

(b) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = 0$, $n = \dfrac{50}{12} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(6) + 6^2]^{1/2}} = 4.81$ *Ans.*

(c) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = -12$ kpsi, $n = \dfrac{50}{12 - (-12)} = 2.08$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(-12) + (-12)^2]^{1/3}} = 2.41$ *Ans.*

(d) MSS: $\sigma_1 = 0$, $\sigma_3 = -12$ kpsi, $n = \dfrac{50}{-(-12)} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[(-6)^2 - (-6)(-12) + (-12)^2]^{1/2}} = 4.81$

5-3 $S_y = 390$ MPa

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 180$ MPa, $\sigma_3 = 0$, $n = \dfrac{390}{180} = 2.17$ *Ans.*

DE: $n = \dfrac{390}{[180^2 - 180(100) + 100^2]^{1/2}} = 2.50$ *Ans.*

(b) $\sigma_A, \sigma_B = \dfrac{180}{2} \pm \sqrt{\left(\dfrac{180}{2}\right)^2 + 100^2} = 224.5, -44.5 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{224.5 - (-44.5)} = 1.45$ *Ans.*

DE: $n = \dfrac{390}{[180^2 + 3(100^2)]^{1/2}} = 1.56$ *Ans.*

(c) $\sigma_A, \sigma_B = -\frac{160}{2} \pm \sqrt{\left(-\frac{160}{2}\right)^2 + 100^2} = 48.06, -208.06 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \frac{390}{48.06 - (-208.06)} = 1.52$ *Ans.*

DE: $n = \frac{390}{[-160^2 + 3(100^2)]^{1/2}} = 1.65$ *Ans.*

(d) $\sigma_A, \sigma_B = 150, -150 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \frac{390}{150 - (-150)} = 1.30$ *Ans.*

DE: $n = \frac{390}{[3(150)^2]^{1/2}} = 1.50$ *Ans.*

5-4 $S_y = 220$ MPa

(a) $\sigma_1 = 100, \sigma_2 = 80, \sigma_3 = 0$ MPa

MSS: $n = \frac{220}{100 - 0} = 2.20$ *Ans.*

DET: $\sigma' = [100^2 - 100(80) + 80^2]^{1/2} = 91.65$ MPa

$n = \frac{220}{91.65} = 2.40$ *Ans.*

(b) $\sigma_1 = 100, \sigma_2 = 10, \sigma_3 = 0$ MPa

MSS: $n = \frac{220}{100} = 2.20$ *Ans.*

DET: $\sigma' = [100^2 - 100(10) + 10^2]^{1/2} = 95.39$ MPa

$n = \frac{220}{95.39} = 2.31$ *Ans.*

(c) $\sigma_1 = 100, \sigma_2 = 0, \sigma_3 = -80$ MPa

MSS: $n = \frac{220}{100 - (-80)} = 1.22$ *Ans.*

DE: $\sigma' = [100^2 - 100(-80) + (-80)^2]^{1/2} = 156.2$ MPa

$n = \frac{220}{156.2} = 1.41$ *Ans.*

(d) $\sigma_1 = 0, \sigma_2 = -80, \sigma_3 = -100$ MPa

MSS: $n = \frac{220}{0 - (-100)} = 2.20$ *Ans.*

DE: $\sigma' = [(-80)^2 - (-80)(-100) + (-100)^2] = 91.65$ MPa

$n = \frac{220}{91.65} = 2.40$ *Ans.*

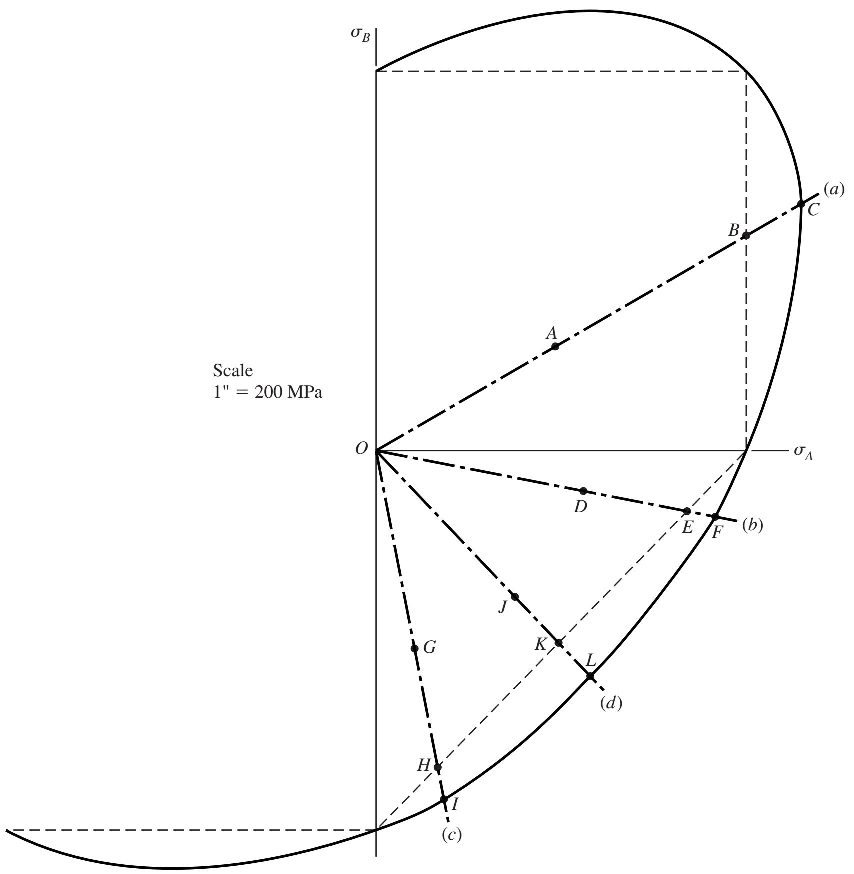
5-5

(a) MSS: $n = \frac{OB}{OA} = \frac{2.23}{1.08} = 2.1$

DE: $n = \frac{OC}{OA} = \frac{2.56}{1.08} = 2.4$

(b) MSS: $n = \frac{OE}{OD} = \frac{1.65}{1.10} = 1.5$

DE: $n = \frac{OF}{OD} = \frac{1.8}{1.1} = 1.6$



(c) MSS: $n = \frac{OH}{OG} = \frac{1.68}{1.05} = 1.6$

DE: $n = \frac{OI}{OG} = \frac{1.85}{1.05} = 1.8$

(d) MSS: $n = \frac{OK}{OJ} = \frac{1.38}{1.05} = 1.3$

DE: $n = \frac{OL}{OJ} = \frac{1.62}{1.05} = 1.5$

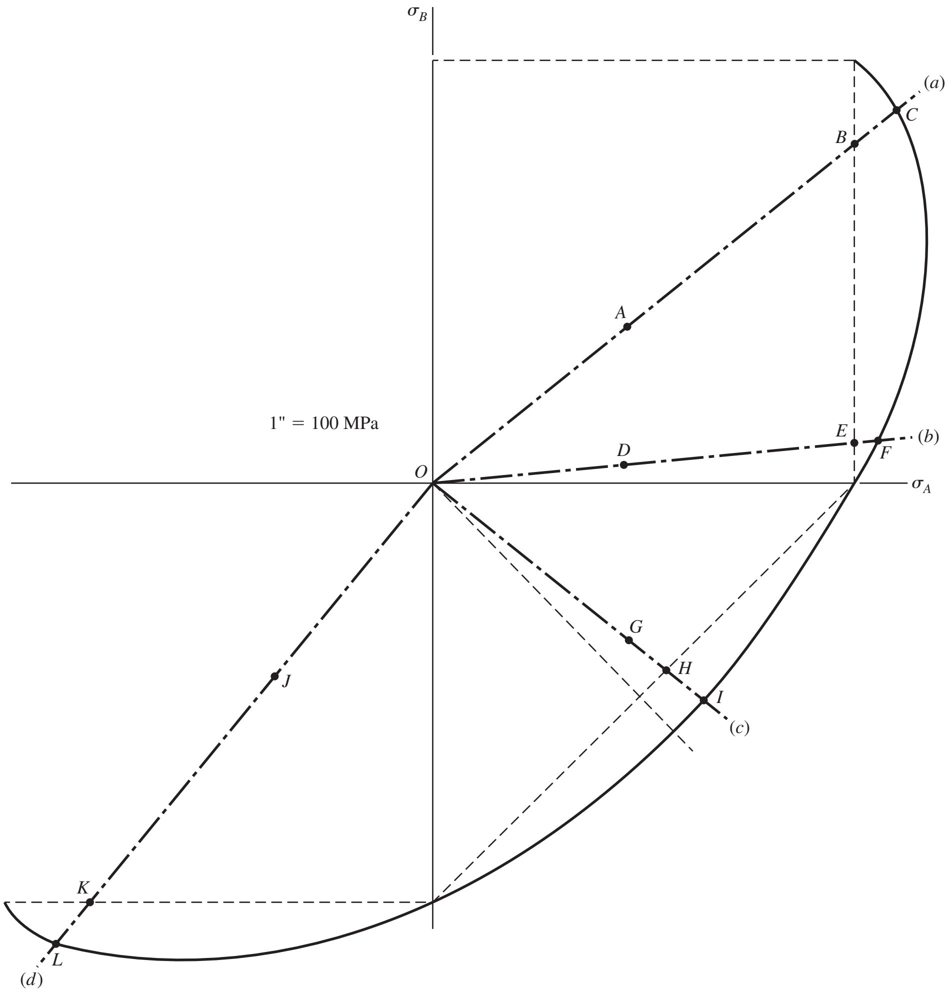
5-6 $S_y = 220$ MPa

(a) MSS: $n = \dfrac{OB}{OA} = \dfrac{2.82}{1.3} = 2.2$

DE: $n = \dfrac{OC}{OA} = \dfrac{3.1}{1.3} = 2.4$

(b) MSS: $n = \dfrac{OE}{OD} = \dfrac{2.2}{1} = 2.2$

DE: $n = \dfrac{OF}{OD} = \dfrac{2.33}{1} = 2.3$



(c) MSS: $n = \dfrac{OH}{OG} = \dfrac{1.55}{1.3} = 1.2$

DE: $n = \dfrac{OI}{OG} = \dfrac{1.8}{1.3} = 1.4$

(d) MSS: $n = \dfrac{OK}{OJ} = \dfrac{2.82}{1.3} = 2.2$

DE: $n = \dfrac{OL}{OJ} = \dfrac{3.1}{1.3} = 2.4$

5-7 $S_{ut} = 30$ kpsi, $S_{uc} = 100$ kpsi; $\sigma_A = 20$ kpsi, $\sigma_B = 6$ kpsi

(a) MNS: Eq. (5-30*a*) $$n = \frac{S_{ut}}{\sigma_x} = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

BCM: Eq. (5-31*a*) $$n = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

MM: Eq. (5-32*a*) $$n = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi

$$\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$$

MNS: Eq. (5-30*a*) $$n = \frac{30}{16} = 1.88 \quad \textit{Ans.}$$

BCM: Eq. (5-31*b*) $$\frac{1}{n} = \frac{16}{30} - \frac{(-4)}{100} \quad \Rightarrow \quad n = 1.74 \quad \textit{Ans.}$$

MM: Eq. (5-32*a*) $$n = \frac{30}{16} = 1.88 \quad \textit{Ans.}$$

(c) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi

$$\sigma_A, \sigma_B = \frac{-6-10}{2} \pm \sqrt{\left(\frac{-6+10}{2}\right)^2 + (-5)^2} = -2.61, -13.39 \text{ kpsi}$$

MNS: Eq. (5-30*b*) $$n = -\frac{100}{-13.39} = 7.47 \quad \textit{Ans.}$$

BCM: Eq. (5-31*c*) $$n = -\frac{100}{-13.39} = 7.47 \quad \textit{Ans.}$$

MM: Eq. (5-32*c*) $$n = -\frac{100}{-13.39} = 7.47 \quad \textit{Ans.}$$

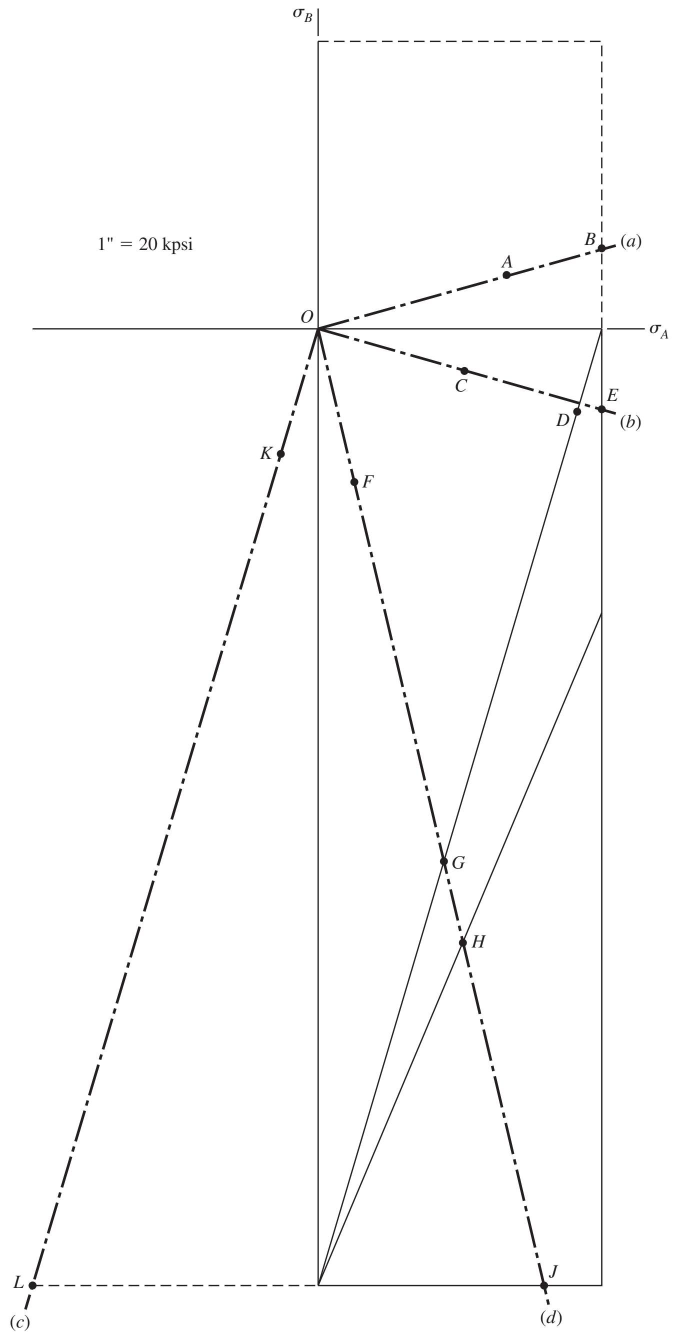
(d) $\sigma_x = -12$ kpsi, $\tau_{xy} = 8$ kpsi

$$\sigma_A, \sigma_B = -\frac{12}{2} \pm \sqrt{\left(-\frac{12}{2}\right)^2 + 8^2} = 4, -16 \text{ kpsi}$$

MNS: Eq. (5-30*b*) $$n = \frac{-100}{-16} = 6.25 \quad \textit{Ans.}$$

BCM: Eq. (5-31*b*) $\frac{1}{n} = \frac{4}{30} - \frac{(-16)}{100} \quad \Rightarrow \quad n = 3.41 \quad Ans.$

MM: Eq. (5-32*b*) $\frac{1}{n} = \frac{(100-30)4}{100(30)} - \frac{-16}{100} \quad \Rightarrow \quad n = 3.95 \quad Ans.$



5-8 See Prob. 5-7 for plot.

(a) For all methods: $$n = \frac{OB}{OA} = \frac{1.55}{1.03} = 1.5$$

(b) BCM: $$n = \frac{OD}{OC} = \frac{1.4}{0.8} = 1.75$$

All other methods: $$n = \frac{OE}{OC} = \frac{1.55}{0.8} = 1.9$$

(c) For all methods: $$n = \frac{OL}{OK} = \frac{5.2}{0.68} = 7.6$$

(d) MNS: $$n = \frac{OJ}{OF} = \frac{5.12}{0.82} = 6.2$$

BCM: $$n = \frac{OG}{OF} = \frac{2.85}{0.82} = 3.5$$

MM: $$n = \frac{OH}{OF} = \frac{3.3}{0.82} = 4.0$$

5-9 Given: $S_y = 42$ kpsi, $S_{ut} = 66.2$ kpsi, $\varepsilon_f = 0.90$. Since $\varepsilon_f > 0.05$, the material is ductile and thus we may follow convention by setting $S_{yc} = S_{yt}$.

Use DE theory for analytical solution. For σ', use Eq. (5-13) or (5-15) for plane stress and Eq. (5-12) or (5-14) for general 3-D.

(a) $\sigma' = [9^2 - 9(-5) + (-5)^2]^{1/2} = 12.29$ kpsi

$$n = \frac{42}{12.29} = 3.42 \quad \textit{Ans.}$$

(b) $\sigma' = [12^2 + 3(3^2)]^{1/2} = 13.08$ kpsi

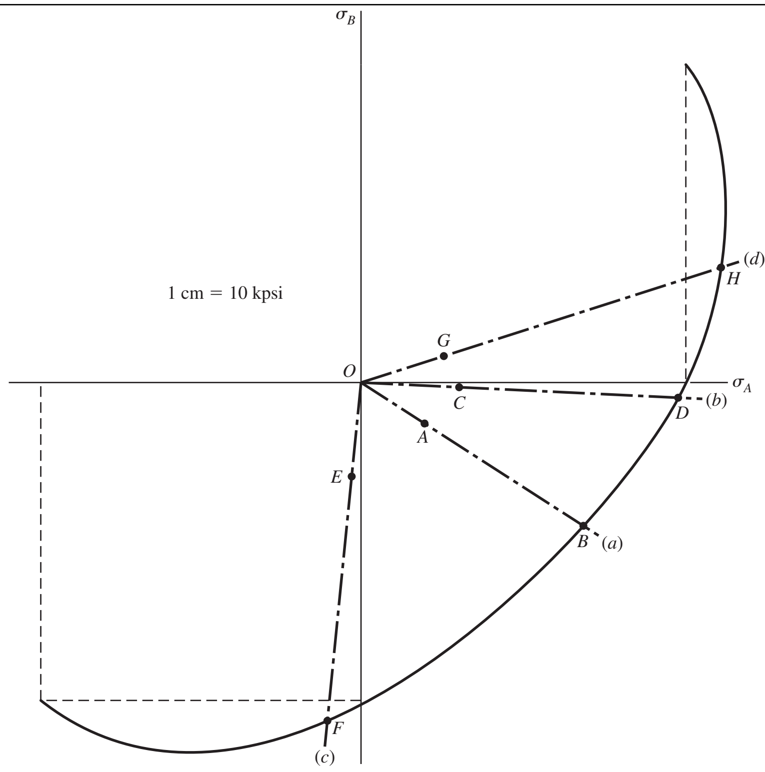
$$n = \frac{42}{13.08} = 3.21 \quad \textit{Ans.}$$

(c) $\sigma' = [(-4)^2 - (-4)(-9) + (-9)^2 + 3(5^2)]^{1/2} = 11.66$ kpsi

$$n = \frac{42}{11.66} = 3.60 \quad \textit{Ans.}$$

(d) $\sigma' = [11^2 - (11)(4) + 4^2 + 3(1^2)]^{1/2} = 9.798$

$$n = \frac{42}{9.798} = 4.29 \quad \textit{Ans.}$$



For graphical solution, plot load lines on DE envelope as shown.

(a) $\sigma_A = 9, \sigma_B = -5$ kpsi

$$n = \frac{OB}{OA} = \frac{3.5}{1} = 3.5 \quad Ans.$$

(b) $\sigma_A, \sigma_B = \dfrac{12}{2} \pm \sqrt{\left(\dfrac{12}{2}\right)^2 + 3^2} = 12.7, -0.708$ kpsi

$$n = \frac{OD}{OC} = \frac{4.2}{1.3} = 3.23$$

(c) $\sigma_A, \sigma_B = \dfrac{-4-9}{2} \pm \sqrt{\left(\dfrac{4-9}{2}\right)^2 + 5^2} = -0.910, -12.09$ kpsi

$$n = \frac{OF}{OE} = \frac{4.5}{1.25} = 3.6 \quad Ans.$$

(d) $\sigma_A, \sigma_B = \dfrac{11+4}{2} \pm \sqrt{\left(\dfrac{11-4}{2}\right)^2 + 1^2} = 11.14, 3.86$ kpsi

$$n = \frac{OH}{OG} = \frac{5.0}{1.15} = 4.35 \quad Ans.$$

5-10 This heat-treated steel exhibits $S_{yt} = 235$ kpsi, $S_{yc} = 275$ kpsi and $\varepsilon_f = 0.06$. The steel is ductile ($\varepsilon_f > 0.05$) but of unequal yield strengths. The Ductile Coulomb-Mohr hypothesis (DCM) of Fig. 5-19 applies — confine its use to first and fourth quadrants.

(a) $\sigma_x = 90$ kpsi, $\sigma_y = -50$ kpsi, $\sigma_z = 0$ $\therefore$ $\sigma_A = 90$ kpsi and $\sigma_B = -50$ kpsi. For the fourth quadrant, from Eq. (5-31b)

$$n = \frac{1}{(\sigma_A/S_{yt}) - (\sigma_B/S_{uc})} = \frac{1}{(90/235) - (-50/275)} = 1.77 \quad Ans.$$

(b) $\sigma_x = 120$ kpsi, $\tau_{xy} = -30$ kpsi ccw. $\sigma_A, \sigma_B = 127.1, -7.08$ kpsi. For the fourth quadrant

$$n = \frac{1}{(127.1/235) - (-7.08/275)} = 1.76 \quad Ans.$$

(c) $\sigma_x = -40$ kpsi, $\sigma_y = -90$ kpsi, $\tau_{xy} = 50$ kpsi. σ_A, $\sigma_B = -9.10$, -120.9 kpsi. Although no solution exists for the third quadrant, use

$$n = -\frac{S_{yc}}{\sigma_y} = -\frac{275}{-120.9} = 2.27 \quad Ans.$$

(d) $\sigma_x = 110$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 10$ kpsi cw. $\sigma_A, \sigma_B = 111.4, 38.6$ kpsi. For the first quadrant

$$n = \frac{S_{yt}}{\sigma_A} = \frac{235}{111.4} = 2.11 \quad Ans.$$

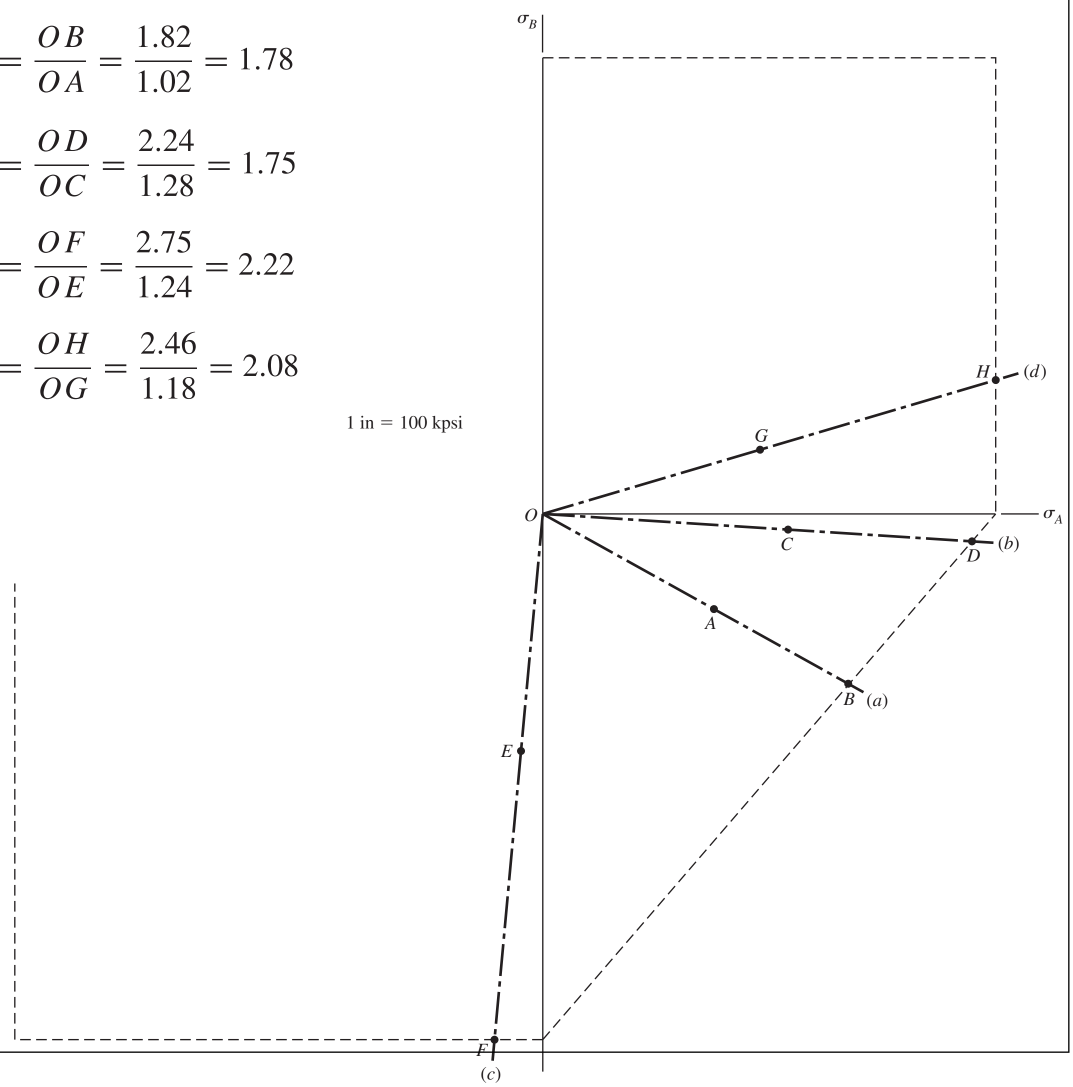
Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{1.82}{1.02} = 1.78$

(b) $n = \dfrac{OD}{OC} = \dfrac{2.24}{1.28} = 1.75$

(c) $n = \dfrac{OF}{OE} = \dfrac{2.75}{1.24} = 2.22$

(d) $n = \dfrac{OH}{OG} = \dfrac{2.46}{1.18} = 2.08$



5-11 The material is brittle and exhibits unequal tensile and compressive strengths. *Decision:* Use the Modified Mohr theory.

$$S_{ut} = 22 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

(a) $\sigma_x = 9$ kpsi, $\sigma_y = -5$ kpsi. $\sigma_A, \sigma_B = 9, -5$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{5}{9} < 1$, use Eq. (5-32*a*)

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{9} = 2.44 \quad Ans.$$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -3$ kpsi ccw. $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{0.708}{12.7} < 1$,

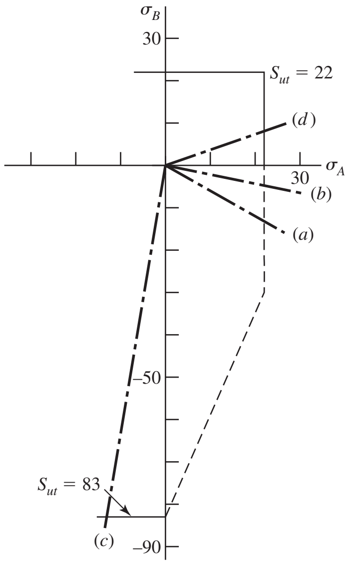
$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{12.7} = 1.73 \quad Ans.$$

(c) $\sigma_x = -4$ kpsi, $\sigma_y = -9$ kpsi, $\tau_{xy} = 5$ kpsi. $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi. For the third quadrant, no solution exists; however, use Eq. (6-32*c*)

$$n = \frac{-83}{-12.09} = 6.87 \quad Ans.$$

(d) $\sigma_x = 11$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi. $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi. For the first quadrant

$$n = \frac{S_A}{\sigma_A} = \frac{S_{yt}}{\sigma_A} = \frac{22}{11.14} = 1.97 \quad Ans.$$



5-12 Since $\varepsilon_f < 0.05$, the material is brittle. Thus, $S_{ut} \doteq S_{uc}$ and we may use MM which is basically the same as MNS.

(a) $\sigma_A, \sigma_B = 9, -5$ kpsi

$$n = \frac{35}{9} = 3.89 \quad Ans.$$

(b) $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi

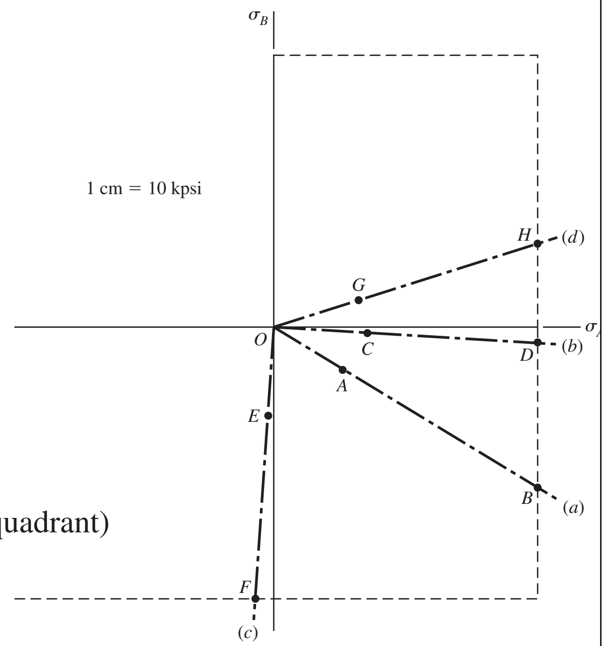
$$n = \frac{35}{12.7} = 2.76 \quad Ans.$$

(c) $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi (3rd quadrant)

$$n = \frac{36}{12.09} = 2.98 \quad Ans.$$

(d) $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi

$$n = \frac{35}{11.14} = 3.14 \quad Ans.$$



Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{4}{1} = 4.0 \quad Ans.$

(b) $n = \dfrac{OD}{OC} = \dfrac{3.45}{1.28} = 2.70 \quad Ans.$

(c) $n = \dfrac{OF}{OE} = \dfrac{3.7}{1.3} = 2.85 \quad Ans.$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{3.6}{1.15} = 3.13 \quad Ans.$

5-13 $S_{ut} = 30$ kpsi, $S_{uc} = 109$ kpsi

Use MM:

(a) $\sigma_A, \sigma_B = 20, 20$ kpsi

Eq. (5-32*a*): $\quad n = \dfrac{30}{20} = 1.5 \quad Ans.$

(b) $\sigma_A, \sigma_B = \pm\sqrt{(15)^2} = 15, -15$ kpsi

Eq. (5-32*a*) $\quad n = \dfrac{30}{15} = 2 \quad Ans.$

(c) $\sigma_A, \sigma_B = -80, -80$ kpsi

For the 3rd quadrant, there is no solution but use Eq. (5-32*c*).

Eq. (5-32*c*): $\quad n = -\dfrac{109}{-80} = 1.36 \quad Ans.$

(d) $\sigma_A, \sigma_B = 15, -25$ kpsi, $|\sigma_B|\sigma_A| = 25/15 > 1$,

Eq. (5-32*b*): $$\frac{(109-30)15}{109(30)} - \frac{-25}{109} = \frac{1}{n}$$

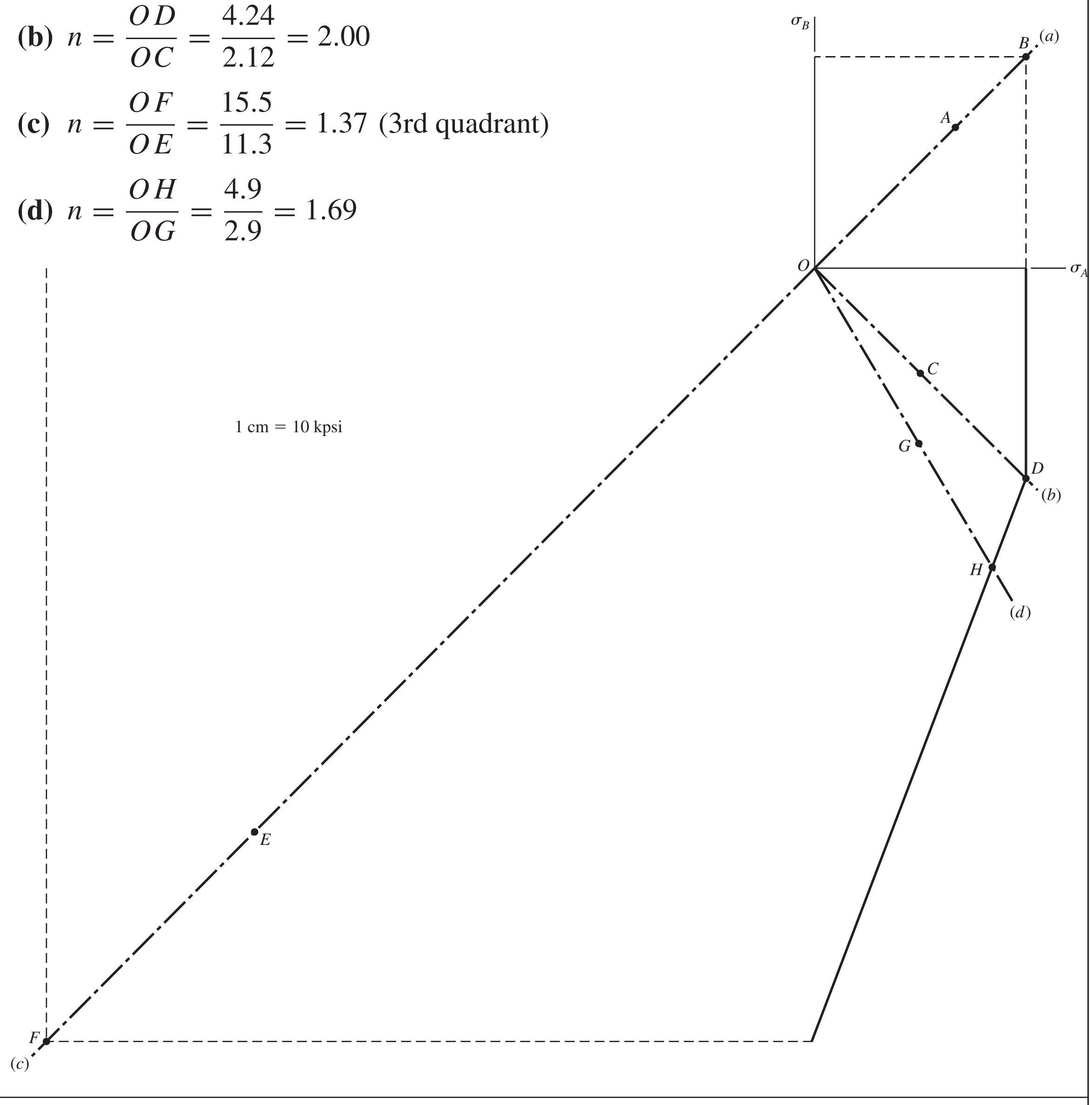
$$n = 1.69 \quad Ans.$$

(a) $n = \dfrac{OB}{OA} = \dfrac{4.25}{2.83} = 1.50$

(b) $n = \dfrac{OD}{OC} = \dfrac{4.24}{2.12} = 2.00$

(c) $n = \dfrac{OF}{OE} = \dfrac{15.5}{11.3} = 1.37$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{4.9}{2.9} = 1.69$



5-14 Given: AISI 1006 CD steel, $F = 0.55$ N, $P = 8.0$ kN, and $T = 30$ N · m, applying the DE theory to stress elements A and B with $S_y = 280$ MPa

A: $$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)(10^3)(0.1)}{\pi(0.020^3)} + \frac{4(8)(10^3)}{\pi(0.020^2)}$$

$$= 95.49(10^6)\text{ Pa} = 95.49\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020^3)} = 19.10(10^6)\text{ Pa} = 19.10\text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = [95.49^2 + 3(19.1)^2]^{1/2} = 101.1\text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{101.1} = 2.77 \quad Ans.$$

B:

$$\sigma_x = \frac{4P}{\pi d^3} = \frac{4(8)(10^3)}{\pi(0.020^2)} = 25.47(10^6)\text{ Pa} = 25.47\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(30)}{\pi(0.020^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)(0.020^2)}\right]$$

$$= 21.43(10^6)\text{ Pa} = 21.43\text{ MPa}$$

$$\sigma' = [25.47^2 + 3(21.43^2)]^{1/2} = 45.02\text{ MPa}$$

$$n = \frac{280}{45.02} = 6.22 \quad Ans.$$

5-15 $S_y = 32\text{ kpsi}$

At A, $M = 6(190) = 1\,140$ lbf·in, $T = 4(190) = 760$ lbf · in.

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(1140)}{\pi(3/4)^3} = 27\,520\text{ psi}$$

$$\tau_{zx} = \frac{16T}{\pi d^3} = \frac{16(760)}{\pi(3/4)^3} = 9175\text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{27\,520}{2}\right)^2 + 9175^2} = 16\,540\text{ psi}$$

$$n = \frac{S_y}{2\tau_{\max}} = \frac{32}{2(16.54)} = 0.967 \quad Ans.$$

MSS predicts yielding

5-16 From Prob. 4-15, $\sigma_x = 27.52$ kpsi, $\tau_{zx} = 9.175$ kpsi. For Eq. (5-15), adjusted for coordinates,

$$\sigma' = \left[27.52^2 + 3(9.175)^2\right]^{1/2} = 31.78\text{ kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{31.78} = 1.01 \quad Ans.$$

DE predicts no yielding, but it is extremely close. Shaft size should be increased.

5-17 Design decisions required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{1/3}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y =$ 81 000).

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(81\,000)}\right]^{1/3} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274$$

Set design factor to $n_d = 1.274$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.274} = 63\,580 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(63\,580)}\right]^{1/3} = 1.000 \text{ in} \quad (OK)$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274 \quad \text{(OK)}$$

5-18 For a thin walled cylinder made of AISI 1018 steel, $S_y = 54$ kpsi, $S_{ut} = 64$ kpsi.

The state of stress is

$$\sigma_t = \frac{pd}{4t} = \frac{p(8)}{4(0.05)} = 40p, \quad \sigma_l = \frac{pd}{8t} = 20p, \quad \sigma_r = -p$$

These three are all principal stresses. Therefore,

$$\sigma' = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$= \frac{1}{\sqrt{2}}[(40p - 20p)^2 + (20p + p)^2 + (-p - 40p)^2]$$

$$= 35.51p = 54 \quad \Rightarrow \quad p = 1.52 \text{ kpsi} \quad \text{(for yield)} \quad Ans.$$

For rupture, $35.51p \doteq 64 \Rightarrow p \doteq 1.80$ kpsi *Ans.*

5-19 For hot-forged AISI steel $w = 0.282$ lbf/in^3, $S_y = 30$ kpsi and $\nu = 0.292$. Then $\rho = w/g = 0.282/386$ lbf $\cdot$ s^2/in; $r_i = 3$ in; $r_o = 5$ in; $r_i^2 = 9$; $r_o^2 = 25$; $3 + \nu = 3.292$; $1 + 3\nu = 1.876$.

Eq. (3-55) for $r = r_i$ becomes

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left[2r_o^2 + r_i^2\left(1 - \frac{1+3\nu}{3+\nu}\right)\right]$$

Rearranging and substituting the above values:

$$\frac{S_y}{\omega^2} = \frac{0.282}{386}\left(\frac{3.292}{8}\right)\left[50 + 9\left(1 - \frac{1.876}{3.292}\right)\right]$$

$$= 0.016\,19$$

Setting the tangential stress equal to the yield stress,

$$\omega = \left(\frac{30\,000}{0.016\,19}\right)^{1/2} = 1361 \text{ rad/s}$$

or

$$n = 60\omega/2\pi = 60(1361)/(2\pi)$$

$$= 13\,000 \text{ rev/min}$$

Now check the stresses at $r = (r_o r_i)^{1/2}$, or $r = [5(3)]^{1/2} = 3.873$ in

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_o - r_i)^2$$

$$= \frac{0.282\omega^2}{386}\left(\frac{3.292}{8}\right)(5-3)^2$$

$$= 0.001\,203\omega^2$$

Applying Eq. (3-55) for σ_t

$$\sigma_t = \omega^2\left(\frac{0.282}{386}\right)\left(\frac{3.292}{8}\right)\left[9 + 25 + \frac{9(25)}{15} - \frac{1.876(15)}{3.292}\right]$$

$$= 0.012\,16\omega^2$$

Using the Distortion-Energy theory

$$\sigma' = \left(\sigma_t^2 - \sigma_r\sigma_t + \sigma_r^2\right)^{1/2} = 0.011\,61\omega^2$$

Solving $$\omega = \left(\frac{30\,000}{0.011\,61}\right)^{1/2} = 1607\text{ rad/s}$$

So the inner radius governs and $n = 13\,000$ rev/min *Ans.*

5-20 For a thin-walled pressure vessel,

$$d_i = 3.5 - 2(0.065) = 3.37\text{ in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212\text{ psi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481\text{ psi}$$

$$\sigma_r = -p_i = -500\text{ psi}$$

These are all principal stresses, thus,

$$\sigma' = \frac{1}{\sqrt{2}}\{(13\,212 - 6481)^2 + [6481 - (-500)]^2 + (-500 - 13\,212)^2\}^{1/2}$$

$$\sigma' = 11\,876\text{ psi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46\,000}{\sigma'} = \frac{46\,000}{11\,876}$$

$$= 3.87 \quad \textit{Ans.}$$

5-21 Table A-20 gives S_y as 320 MPa. The maximum significant stress condition occurs at r_i where $\sigma_1 = \sigma_r = 0$, $\sigma_2 = 0$, and $\sigma_3 = \sigma_t$. From Eq. (3-49) for $r = r_i$, $p_i = 0$,

$$\sigma_t = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} = -\frac{2(150^2)p_o}{150^2 - 100^2} = -3.6p_o$$

$$\sigma' = 3.6p_o = S_y = 320$$

$$p_o = \frac{320}{3.6} = 88.9\text{ MPa} \quad \textit{Ans.}$$

5-22 $S_{ut} = 30$ kpsi, $w = 0.260$ lbf/in^3, $\nu = 0.211$, $3 + \nu = 3.211$, $1 + 3\nu = 1.633$. At the inner radius, from Prob. 5-19

$$\frac{\sigma_t}{\omega^2} = \rho\left(\frac{3 + \nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1 + 3\nu}{3 + \nu}r_i^2\right)$$

Here $r_o^2 = 25$, $r_i^2 = 9$, and so

$$\frac{\sigma_t}{\omega^2} = \frac{0.260}{386}\left(\frac{3.211}{8}\right)\left(50 + 9 - \frac{1.633(9)}{3.211}\right) = 0.0147$$

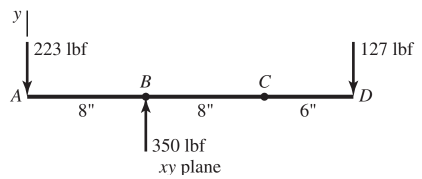
Since σ_r is of the same sign, we use M2M failure criteria in the first quadrant. From Table A-24, $S_{ut} = 31$ kpsi, thus,

$$\omega = \left(\frac{31\,000}{0.0147}\right)^{1/2} = 1452 \text{ rad/s}$$

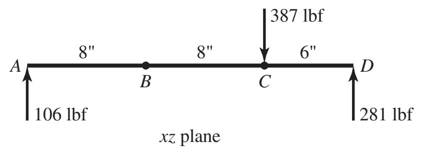
$$\text{rpm} = 60\omega/(2\pi) = 60(1452)/(2\pi)$$
$$= 13\,866 \text{ rev/min}$$

Using the grade number of 30 for $S_{ut} = 30\,000$ kpsi gives a bursting speed of 13640 rev/min.

5-23 $T_C = (360 - 27)(3) = 1000 \text{ lbf} \cdot \text{in}$, $T_B = (300 - 50)(4) = 1000 \text{ lbf} \cdot \text{in}$



In xy plane, $M_B = 223(8) = 1784 \text{ lbf} \cdot \text{in}$ and $M_C = 127(6) = 762 \text{ lbf} \cdot \text{in}$.



In the xz plane, $M_B = 848 \text{ lbf} \cdot \text{in}$ and $M_C = 1686 \text{ lbf} \cdot \text{in}$. The resultants are

$$M_B = [(1784)^2 + (848)^2]^{1/2} = 1975 \text{ lbf} \cdot \text{in}$$

$$M_C = [(1686)^2 + (762)^2]^{1/2} = 1850 \text{ lbf} \cdot \text{in}$$

So point B governs and the stresses are

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3} \text{ psi}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(1975)}{\pi d^3} = \frac{20\,120}{d^3} \text{ psi}$$

Then

$$\sigma_A, \sigma_B = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\frac{20.12}{2} \pm \left[\left(\frac{20.12}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$= \frac{(10.06 \pm 11.27)}{d^3} \text{ kpsi} \cdot \text{in}^3$$

Then

$$\sigma_A = \frac{10.06 + 11.27}{d^3} = \frac{21.33}{d^3}\text{ kpsi}$$

and

$$\sigma_B = \frac{10.06 - 11.27}{d^3} = -\frac{1.21}{d^3}\text{ kpsi}$$

For this state of stress, use the Brittle-Coulomb-Mohr theory for illustration. Here we use $S_{ut}(\text{min}) = 25$ kpsi, $S_{uc}(\text{min}) = 97$ kpsi, and Eq. (5-31*b*) to arrive at

$$\frac{21.33}{25d^3} - \frac{-1.21}{97d^3} = \frac{1}{2.8}$$

Solving gives $d = 1.34$ in. So use $d = 1\,3/8$ in *Ans.*

Note that this has been solved as a statics problem. Fatigue will be considered in the next chapter.

5-24 As in Prob. 5-23, we will assume this to be statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-23. Thus

xy plane: $M_B = 223(4) = 892\text{ lbf}\cdot\text{in}$

xz plane: $M_B = 106(4) = 424\text{ lbf}\cdot\text{in}$

So

$$M_{\max} = [(892)^2 + (424)^2]^{1/2} = 988\text{ lbf}\cdot\text{in}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(988)}{\pi d^3} = \frac{10\,060}{d^3}\text{ psi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = 5.09/d^3\text{ kpsi}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\left(\frac{10.06}{2}\right) \pm \left[\left(\frac{10.06}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$\sigma_A = 12.19/d^3 \quad\text{and}\quad \sigma_B = -2.13/d^3$$

Using the Brittle-Coulomb-Mohr, as was used in Prob. 5-23, gives

$$\frac{12.19}{25d^3} - \frac{-2.13}{97d^3} = \frac{1}{2.8}$$

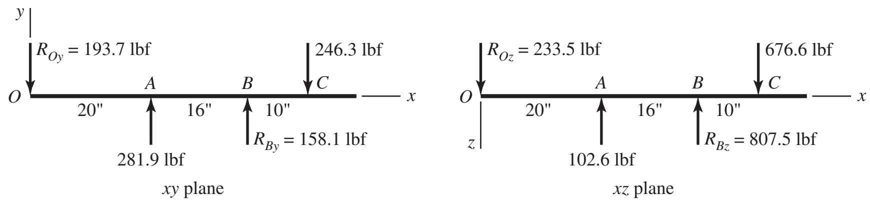
Solving gives $d = 1\,1/8$ in. *Ans.*

5-25

$$(F_A)_t = 300\cos 20 = 281.9\text{ lbf}, \quad (F_A)_r = 300\sin 20 = 102.6\text{ lbf}$$

$$T = 281.9(12) = 3383\text{ lbf}\cdot\text{in}, \quad (F_C)_t = \frac{3383}{5} = 676.6\text{ lbf}$$

$$(F_C)_r = 676.6\tan 20 = 246.3\text{ lbf}$$



$$M_A = 20\sqrt{193.7^2 + 233.5^2} = 6068 \text{ lbf} \cdot \text{in}$$

$$M_B = 10\sqrt{246.3^2 + 676.6^2} = 7200 \text{ lbf} \cdot \text{in} \quad \text{(maximum)}$$

$$\sigma_x = \frac{32(7200)}{\pi d^3} = \frac{73\,340}{d^3}$$

$$\tau_{xy} = \frac{16(3383)}{\pi d^3} = \frac{17\,230}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \frac{S_y}{n}$$

$$\left[\left(\frac{73\,340}{d^3}\right)^2 + 3\left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{79\,180}{d^3} = \frac{60\,000}{3.5}$$

$d = 1.665$ in so use a standard diameter size of 1.75 in *Ans.*

5-26 From Prob. 5-25,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

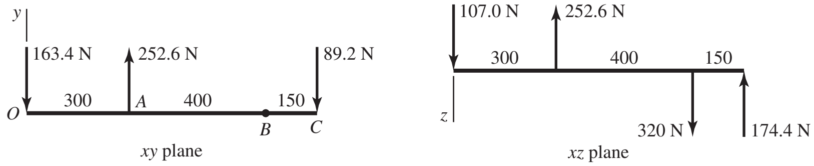
$$\left[\left(\frac{73\,340}{2d^3}\right)^2 + \left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{40\,516}{d^3} = \frac{60\,000}{2(3.5)}$$

$d = 1.678$ in so use 1.75 in *Ans.*

5-27 $T = (270 - 50)(0.150) = 33 \text{ N} \cdot \text{m}, S_y = 370 \text{ MPa}$

$$(T_1 - 0.15T_1)(0.125) = 33 \quad \Rightarrow \quad T_1 = 310.6 \text{ N}, \quad T_2 = 0.15(310.6) = 46.6 \text{ N}$$

$$(T_1 + T_2)\cos 45 = 252.6 \text{ N}$$



$$M_A = 0.3\sqrt{163.4^2 + 107^2} = 58.59\ \text{N}\cdot\text{m} \qquad \text{(maximum)}$$

$$M_B = 0.15\sqrt{89.2^2 + 174.4^2} = 29.38\ \text{N}\cdot\text{m}$$

$$\sigma_x = \frac{32(58.59)}{\pi d^3} = \frac{596.8}{d^3}$$

$$\tau_{xy} = \frac{16(33)}{\pi d^3} = \frac{168.1}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[\left(\frac{596.8}{d^3}\right)^2 + 3\left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{664.0}{d^3} = \frac{370(10^6)}{3.0}$$

$d = 17.5(10^{-3})$ m $= 17.5$ mm, so use 18 mm *Ans.*

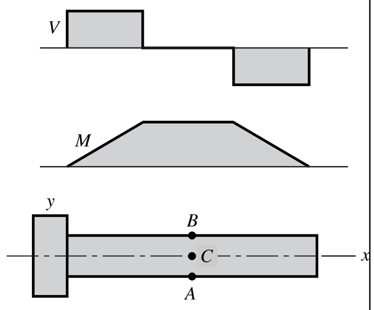
5-28 From Prob. 5-27,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

$$\left[\left(\frac{596.8}{2d^3}\right)^2 + \left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{342.5}{d^3} = \frac{370(10^6)}{2(3.0)}$$

$d = 17.7(10^{-3})$ m $= 17.7$ mm, so use 18 mm *Ans.*

5-29 For the loading scheme shown in Figure (c),



$$M_{\max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = \frac{4.4}{2}(6 + 4.5)$$
$$= 23.1\ \text{N}\cdot\text{m}$$

For a stress element at A:

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(23.1)(10^3)}{\pi(12)^3} = 136.2\ \text{MPa}$$

The shear at C is

$$\tau_{xy} = \frac{4(F/2)}{3\pi d^2/4} = \frac{4(4.4/2)(10^3)}{3\pi(12)^2/4} = 25.94\ \text{MPa}$$

$$\tau_{\max} = \left[\left(\frac{136.2}{2}\right)^2\right]^{1/2} = 68.1\ \text{MPa}$$

Since $S_y = 220$ MPa, $S_{sy} = 220/2 = 110$ MPa, and

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{110}{68.1} = 1.62 \quad \textit{Ans.}$$

For the loading scheme depicted in Figure (*d*)

$$M_{\max} = \frac{F}{2}\left(\frac{a+b}{2}\right) - \frac{F}{2}\left(\frac{1}{2}\right)\left(\frac{b}{2}\right)^2 = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right)$$

This result is the same as that obtained for Figure (*c*). At point *B*, we also have a surface compression of

$$\sigma_y = \frac{-F}{A} = \frac{-F}{bd} - \frac{-4.4(10^3)}{18(12)} = -20.4 \text{ MPa}$$

With $\sigma_x = -136.2$ MPa. From a Mohrs circle diagram, $\tau_{\max} = 136.2/2 = 68.1$ MPa.

$$n = \frac{110}{68.1} = 1.62 \text{ MPa} \quad Ans.$$

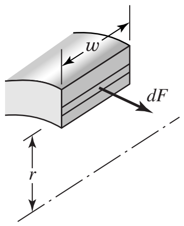
5-30 Based on Figure (*c*) and using Eq. (5-15)

$$\sigma' = \left(\sigma_x^2\right)^{1/2}$$
$$= (136.2^2)^{1/2} = 136.2 \text{ MPa}$$
$$n = \frac{S_y}{\sigma'} = \frac{220}{136.2} = 1.62 \quad Ans.$$

Based on Figure (*d*) and using Eq. (5-15) and the solution of Prob. 5-29,

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2\right)^{1/2}$$
$$= [(-136.2)^2 - (-136.2)(-20.4) + (-20.4)^2]^{1/2}$$
$$= 127.2 \text{ MPa}$$
$$n = \frac{S_y}{\sigma'} = \frac{220}{127.2} = 1.73 \quad Ans.$$

5-31



When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

We have the hoop tension at any radius. The differential hoop tension dF is

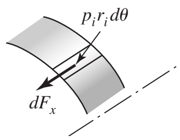
$$dF = w\sigma_t \, dr$$

$$F = \int_{r_i}^{r_o} w\sigma_t \, dr = \frac{w r_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right) dr = w r_i p_i \qquad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \qquad (2)$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2dwr_i}$$

$$dF_x = f p_i r_i \, d\theta$$

$$F_x = \int_o^{2\pi} f p_i w r_i \, d\theta = \frac{fTw}{0.2dwr_i} r_i \int_o^{2\pi} d\theta$$

$$= \frac{2\pi f T}{0.2d} \quad Ans.$$

5-32

(a) From Prob. 5-31, $T = 0.2F_i d$

$$F_i = \frac{T}{0.2d} = \frac{190}{0.2(0.25)} = 3800 \text{ lbf} \quad Ans.$$

(b) From Prob. 5-31, $F = wr_i p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{3800}{0.5(0.5)} = 15\,200 \text{ psi} \quad Ans.$$

(c)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$= \frac{15\,200(0.5^2 + 1^2)}{1^2 - 0.5^2} = 25\,333 \text{ psi} \quad Ans.$$

$$\sigma_r = -p_i = -15\,200 \text{ psi}$$

(d)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$

$$= \frac{25\,333 - (-15\,200)}{2} = 20\,267 \text{ psi} \quad Ans.$$

$$\sigma' = \left(\sigma_A^2 + \sigma_B^2 - \sigma_A\sigma_B\right)^{1/2}$$

$$= [25\,333^2 + (-15\,200)^2 - 25\,333(-15\,200)]^{1/2}$$

$$= 35\,466 \text{ psi} \quad Ans.$$

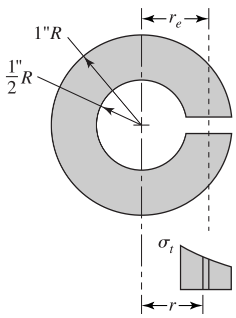
(e) Maximum Shear hypothesis

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.5S_y}{\tau_{\max}} = \frac{0.5(63)}{20.267} = 1.55 \quad Ans.$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{63}{35\,466} = 1.78 \quad Ans.$$

5-33



The moment about the center caused by force F is Fr_e where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress.

$$Fr_e = \int_{r_i}^{r_o} r\sigma_t w\,dr$$

$$= \frac{wp_i r_i^2}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(r + \frac{r_o^2}{r}\right)dr$$

$$r_e = \frac{wp_i r_i^2}{F\left(r_o^2 - r_i^2\right)}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

From Prob. 5-31, $F = wr_i p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2}\left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln\frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-31, $r_i = 0.5$ and $r_o = 1$ in

$$r_e = \frac{0.5}{1^2 - 0.5^2}\left(\frac{1^2 - 0.5^2}{2} + 1^2 \ln\frac{1}{0.5}\right) = 0.712 \text{ in}$$

5-34 $\delta_{\text{nom}} = 0.0005$ in

(a) From Eq. (3-57)

$$p = \frac{30(10^6)(0.0005)}{(1^3)}\left[\frac{(1.5^2 - 1^2)(1^2 - 0.5^2)}{2(1.5^2 - 0.5^2)}\right] = 3516 \text{ psi} \quad \textit{Ans.}$$

Inner member:

Eq. (3-58) $$(\sigma_t)_i = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3516\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2}\right) = -5860 \text{ psi}$$

$$(\sigma_r)_i = -p = -3516 \text{ psi}$$

Eq. (5-13) $$\sigma_i' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

$$= [(-5860)^2 - (-5860)(-3516) + (-3516)^2]^{1/2}$$

$$= 5110 \text{ psi} \quad \textit{Ans.}$$

Outer member:

Eq. (3-59) $$(\sigma_t)_o = 3516\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 9142 \text{ psi}$$

$$(\sigma_r)_o = -p = -3516 \text{ psi}$$

Eq. (5-13) $$\sigma_o' = [9142^2 - 9142(-3516) + (-3516)^2]^{1/2}$$

$$= 11\,320 \text{ psi} \quad \textit{Ans.}$$

(b) For a solid inner tube,

$$p = \frac{30(10^6)(0.0005)}{1}\left[\frac{(1.5^2 - 1^2)(1^2)}{2(1^2)(1.5^2)}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_i = -p = -4167 \text{ psi}, \quad (\sigma_r)_i = -4167 \text{ psi}$$

$$\sigma_i' = [(-4167)^2 - (-4167)(-4167) + (-4167)^2]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10\,830 \text{ psi}, \quad (\sigma_r)_o = -4167 \text{ psi}$$

$$\sigma_o' = [10\,830^2 - 10\,830(-4167) + (-4167)^2]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-35 Using Eq. (3-57) with diametral values,

$$p = \frac{207(10^3)(0.02)}{(50^3)}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 19.41 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-58) $\quad (\sigma_t)_i = -19.41\left(\dfrac{50^2 + 25^2}{50^2 - 25^2}\right) = -32.35 \text{ MPa}$

$$(\sigma_r)_i = -19.41 \text{ MPa}$$

Eq. (5-13) $\quad \sigma_i' = [(-32.35)^2 - (-32.35)(-19.41) + (-19.41)^2]^{1/2}$

$$= 28.20 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-59) $\quad (\sigma_t)_o = 19.41\left(\dfrac{75^2 + 50^2}{75^2 - 50^2}\right) = 50.47 \text{ MPa},$

$$(\sigma_r)_o = -19.41 \text{ MPa}$$

$$\sigma_o' = [50.47^2 - 50.47(-19.41) + (-19.41)^2]^{1/2} = 62.48 \text{ MPa} \quad \textit{Ans.}$$

5-36 Max. shrink-fit conditions: Diametral interference $\delta_d = 50.01 - 49.97 = 0.04$ mm. Equation (3-57) using diametral values:

$$p = \frac{207(10^3)0.04}{50^3}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 38.81 \text{ MPa} \qquad \textit{Ans.}$$

Eq. (3-58): $\quad (\sigma_t)_i = -38.81\left(\dfrac{50^2 + 25^2}{50^2 - 25^2}\right) = -64.68 \text{ MPa}$

$$(\sigma_r)_i = -38.81 \text{ MPa}$$

Eq. (5-13):

$$\sigma_i' = \left[(-64.68)^2 - (-64.68)(-38.81) + (-38.81)^2\right]^{1/2}$$

$$= 56.39 \text{ MPa} \qquad \textit{Ans.}$$

5-37

$$\delta = \frac{1.9998}{2} - \frac{1.999}{2} = 0.0004 \text{ in}$$

Eq. (3-56)

$$0.0004 = \frac{p(1)}{14.5(10^6)}\left[\frac{2^2+1^2}{2^2-1^2} + 0.211\right] + \frac{p(1)}{30(10^6)}\left[\frac{1^2+0}{1^2-0} - 0.292\right]$$

$$p = 2613 \text{ psi}$$

Applying Eq. (4-58) at R,

$$(\sigma_t)_o = 2613\left(\frac{2^2+1^2}{2^2-1^2}\right) = 4355 \text{ psi}$$

$$(\sigma_r)_o = -2613 \text{ psi}, \quad S_{ut} = 20 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

$$\left|\frac{\sigma_o}{\sigma_A}\right| = \frac{2613}{4355} < 1, \ \therefore \text{ use Eq. (5-32a)}$$

$$h = S_{ut}/\sigma_A = 20/4.355 = 4.59 \quad \textit{Ans.}$$

5-38 $E = 30(10^6)$ psi, $\nu = 0.292$, $I = (\pi/64)(2^4 - 1.5^4) = 0.5369 \text{ in}^4$

Eq. (3-57) can be written in terms of diameters,

$$p = \frac{E\delta_d}{D}\left[\frac{\left(d_o^2 - D^2\right)\left(D^2 - d_i^2\right)}{2D^2\left(d_o^2 - d_i^2\right)}\right] = \frac{30(10^6)}{1.75}(0.002\,46)\left[\frac{(2^2-1.75^2)(1.75^2-1.5^2)}{2(1.75^2)(2^2-1.5^2)}\right]$$

$$= 2997 \text{ psi} = 2.997 \text{ kpsi}$$

Outer member:

Outer radius: $$(\sigma_t)_o = \frac{1.75^2(2.997)}{2^2-1.75^2}(2) = 19.58 \text{ kpsi}, (\sigma_r)_o = 0$$

Inner radius: $$(\sigma_t)_i = \frac{1.75^2(2.997)}{2^2-1.75^2}\left(1 + \frac{2^2}{1.75^2}\right) = 22.58 \text{ kpsi}, (\sigma_r)_i = -2.997 \text{ kpsi}$$

Bending:

r_o: $$(\sigma_x)_o = \frac{6.000(2/2)}{0.5369} = 11.18 \text{ kpsi}$$

r_i: $$(\sigma_x)_i = \frac{6.000(1.75/2)}{0.5369} = 9.78 \text{ kpsi}$$

Torsion: $$J = 2I = 1.0738 \text{ in}^4$$

r_o: $$(\tau_{xy})_o = \frac{8.000(2/2)}{1.0738} = 7.45 \text{ kpsi}$$

r_i: $$(\tau_{xy})_i = \frac{8.000(1.75/2)}{1.0738} = 6.52 \text{ kpsi}$$

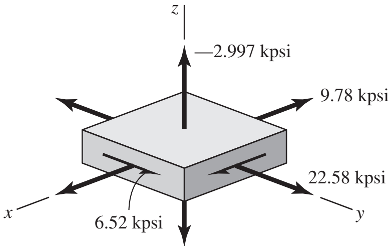
Outer radius is plane stress

$$\sigma_x = 11.18 \text{ kpsi}, \quad \sigma_y = 19.58 \text{ kpsi}, \quad \tau_{xy} = 7.45 \text{ kpsi}$$

Eq. (5-15) $\sigma' = [11.18^2 - (11.18)(19.58) + 19.58^2 + 3(7.45^2)]^{1/2} = \dfrac{S_y}{n_o} = \dfrac{60}{n_o}$

$$21.35 = \frac{60}{n_o} \quad \Rightarrow \quad n_o = 2.81 \quad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$

$$\sigma' = \frac{1}{\sqrt{2}}[(9.78 - 22.58)^2 + (22.58 + 2.997)^2 + (-2.997 - 9.78)^2 + 6(6.52)^2]^{1/2} = \frac{60}{n_i}$$

$$24.86 = \frac{60}{n_i} \quad \Rightarrow \quad n_i = 2.41 \quad \textit{Ans.}$$

5-39 From Prob. 5-38: $p = 2.997$ kpsi, $I = 0.5369$ in^4, $J = 1.0738$ in^4

Inner member:

Outer radius: $(\sigma_t)_o = -2.997\left[\dfrac{(0.875^2 + 0.75^2)}{(0.875^2 - 0.75^2)}\right] = -19.60$ kpsi

$(\sigma_r)_o = -2.997$ kpsi

Inner radius: $(\sigma_t)_i = -\dfrac{2(2.997)(0.875^2)}{0.875^2 - 0.75^2} = -22.59$ kpsi

$(\sigma_r)_i = 0$

Bending:

r_o: $(\sigma_x)_o = \dfrac{6(0.875)}{0.5369} = 9.78$ kpsi

r_i: $(\sigma_x)_i = \dfrac{6(0.75)}{0.5369} = 8.38$ kpsi

Torsion:

r_o: $(\tau_{xy})_o = \dfrac{8(0.875)}{1.0738} = 6.52$ kpsi

r_i: $(\tau_{xy})_i = \dfrac{8(0.75)}{1.0738} = 5.59$ kpsi

The inner radius is in plane stress: $\sigma_x = 8.38$ kpsi, $\sigma_y = -22.59$ kpsi, $\tau_{xy} = 5.59$ kpsi

$$\sigma_i' = [8.38^2 - (8.38)(-22.59) + (-22.59)^2 + 3(5.59^2)]^{1/2} = 29.4 \text{ kpsi}$$

$$n_i = \frac{S_y}{\sigma_i'} = \frac{60}{29.4} = 2.04 \quad Ans.$$

Outer radius experiences a radial stress, σ_r

$$\sigma_o' = \frac{1}{\sqrt{2}}\left[(-19.60 + 2.997)^2 + (-2.997 - 9.78)^2 + (9.78 + 19.60)^2 + 6(6.52)^2\right]^{1/2}$$
$$= 27.9 \text{ kpsi}$$

$$n_o = \frac{60}{27.9} = 2.15 \quad Ans.$$

5-40

$$\sigma_p = \frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right), \quad \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right), \quad \sigma_3 = 0 \quad Ans.$$

Plane strain: σ_1 and σ_2 equations still valid however,

$$\sigma_3 = \nu(\sigma_x + \sigma_y) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \quad Ans.$$

5-41 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-40 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}, \quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

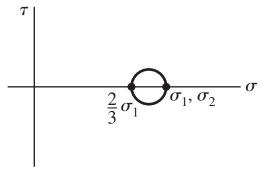
(a) DE: $$\frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (2\nu\sigma_1 - \sigma_1)^2]^{1/2} = S_y$$
$$\sigma_1 - 2\nu\sigma_1 = S_y$$

For $\nu = \frac{1}{3}$, $$\left[1 - 2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad Ans.$$

(b) MSS: $\sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$$

$$\sigma_3 = \frac{2}{3}\sigma_1$$



Radius of largest circle

$$R = \frac{1}{2}\left[\sigma_1 - \frac{2}{3}\sigma_1\right] = \frac{\sigma_1}{6}$$

5-42 **(a)** Ignoring stress concentration

$$F = S_y A = 160(4)(0.5) = 320 \text{ kips} \quad \textit{Ans.}$$

(b) From Fig. 6-36: $h/b = 1$, $a/b = 0.625/4 = 0.1563$, $\beta = 1.3$

Eq. (6-51) $$70 = 1.3\frac{F}{4(0.5)}\sqrt{\pi(0.625)}$$

$$F = 76.9 \text{ kips} \quad \textit{Ans.}$$

5-43 Given: $a = 12.5$ mm, $K_{Ic} = 80 \text{ MPa} \cdot \sqrt{m}$, $S_y = 1200$ MPa, $S_{ut} = 1350$ MPa

$$r_o = \frac{350}{2} = 175 \text{ mm}, \quad r_i = \frac{350 - 50}{2} = 150 \text{ mm}$$

$$a/(r_o - r_i) = \frac{12.5}{175 - 150} = 0.5$$

$$r_i/r_o = \frac{150}{175} = 0.857$$

Fig. 5-30: $\beta \doteq 2.5$

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a}$

$$80 = 2.5\sigma\sqrt{\pi(0.0125)}$$

$$\sigma = 161.5 \text{ MPa}$$

Eq. (3-50) at $r = r_o$:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2)$$

$$161.5 = \frac{150^2 p_i(2)}{175^2 - 150^2}$$

$$p_i = 29.2 \text{ MPa} \quad \textit{Ans.}$$

5-44

(a) First convert the data to radial dimensions to agree with the formulations of Fig. 3-33. Thus

$$r_o = 0.5625 \pm 0.001 \text{ in}$$
$$r_i = 0.1875 \pm 0.001 \text{ in}$$
$$R_o = 0.375 \pm 0.0002 \text{ in}$$
$$R_i = 0.376 \pm 0.0002 \text{ in}$$

The stochastic nature of the dimensions affects the $\boldsymbol{\delta} = |\mathbf{R}_i| - |\mathbf{R}_o|$ relation in Eq. (3-57) but not the others. Set $R = (1/2)(R_i + R_o) = 0.3755$. From Eq. (3-57)

$$\mathbf{p} = \frac{E\boldsymbol{\delta}}{R}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2R^2(r_o^2 - r_i^2)}\right]$$

Substituting and solving with $E = 30$ Mpsi gives

$$\mathbf{p} = 18.70(10^6)\,\boldsymbol{\delta}$$

Since $\boldsymbol{\delta} = \mathbf{R}_i - \mathbf{R}_o$

$$\bar{\delta} = \bar{R}_i - \bar{R}_o = 0.376 - 0.375 = 0.001 \text{ in}$$

and

$$\hat{\sigma}_\delta = \left[\left(\frac{0.0002}{4}\right)^2 + \left(\frac{0.0002}{4}\right)^2\right]^{1/2}$$
$$= 0.000\,070\,7 \text{ in}$$

Then

$$C_\delta = \frac{\hat{\sigma}_\delta}{\bar{\delta}} = \frac{0.000\,070\,7}{0.001} = 0.0707$$

The tangential inner-cylinder stress at the shrink-fit surface is given by

$$\boldsymbol{\sigma}_{it} = -\mathbf{p}\frac{\bar{R}^2 + \bar{r}_i^2}{\bar{R}^2 - \bar{r}_i^2}$$
$$= -18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.3755^2 + 0.1875^2}{0.3755^2 - 0.1875^2}\right)$$
$$= -31.1(10^6)\,\boldsymbol{\delta}$$
$$\bar{\sigma}_{it} = -31.1(10^6)\,\bar{\delta} = -31.1(10^6)(0.001)$$
$$= -31.1(10^3) \text{ psi}$$

Also

$$\hat{\sigma}_{\sigma_{it}} = |C_\delta \bar{\sigma}_{it}| = 0.0707(-31.1)10^3$$
$$= 2899 \text{ psi}$$
$$\boldsymbol{\sigma}_{it} = \mathbf{N}(-31\,100,\ 2899) \text{ psi} \quad \textit{Ans.}$$

(b) The tangential stress for the outer cylinder at the shrink-fit surface is given by

$$\boldsymbol{\sigma}_{ot} = \mathbf{p}\left(\frac{\bar{r}_o^2 + \bar{R}^2}{\bar{r}_o^2 - \bar{R}^2}\right)$$

$$= 18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.5625^2 + 0.3755^2}{0.5625^2 - 0.3755^2}\right)$$

$$= 48.76(10^6)\,\boldsymbol{\delta}\ \text{psi}$$

$$\bar{\sigma}_{ot} = 48.76(10^6)(0.001) = 48.76(10^3)\ \text{psi}$$

$$\hat{\sigma}_{\sigma_{ot}} = C_\delta \bar{\sigma}_{ot} = 0.0707(48.76)(10^3) = 34.45\ \text{psi}$$

$$\therefore \boldsymbol{\sigma}_{ot} = \mathbf{N}(48\,760,\ 3445)\ \text{psi} \quad \textit{Ans.}$$

5-45 From Prob. 5-44, at the fit surface $\boldsymbol{\sigma}_{ot} = \mathbf{N}(48.8, 3.45)$ kpsi. The radial stress is the fit pressure which was found to be

$$\mathbf{p} = 18.70(10^6)\,\boldsymbol{\delta}$$

$$\bar{p} = 18.70(10^6)(0.001) = 18.7(10^3)\ \text{psi}$$

$$\hat{\sigma}_p = C_\delta \bar{p} = 0.0707(18.70)(10^3)$$

$$= 1322\ \text{psi}$$

and so

$$\mathbf{p} = \mathbf{N}(18.7,\ 1.32)\ \text{kpsi}$$

and

$$\boldsymbol{\sigma}_{or} = -\mathbf{N}(18.7,\ 1.32)\ \text{kpsi}$$

These represent the principal stresses. The von Mises stress is next assessed.

$$\bar{\sigma}_A = 48.8\ \text{kpsi}, \quad \bar{\sigma}_B = -18.7\ \text{kpsi}$$

$$k = \bar{\sigma}_B/\bar{\sigma}_A = -18.7/48.8 = -0.383$$

$$\bar{\sigma}' = \bar{\sigma}_A(1 - k + k^2)^{1/2}$$

$$= 48.8[1 - (-0.383) + (-0.383)^2]^{1/2}$$

$$= 60.4\ \text{kpsi}$$

$$\hat{\sigma}_{\sigma'} = C_p \bar{\sigma}' = 0.0707(60.4) = 4.27\ \text{kpsi}$$

Using the interference equation

$$z = -\frac{\bar{S} - \bar{\sigma}'}{\left(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2\right)^{1/2}}$$

$$= -\frac{95.5 - 60.4}{[(6.59)^2 + (4.27)^2]^{1/2}} = -4.5$$

$$p_f = \alpha = 0.000\,003\,40,$$

or about 3 chances in a million. *Ans.*

5-46

$$\boldsymbol{\sigma}_t = \frac{\mathbf{p}d}{2t} = \frac{6000\mathbf{N}(1, 0.083\,33)(0.75)}{2(0.125)}$$
$$= 18\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

$$\boldsymbol{\sigma}_l = \frac{\mathbf{p}d}{4t} = \frac{6000\mathbf{N}(1, 0.083\,33)(0.75)}{4(0.125)}$$
$$= 9\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

$$\boldsymbol{\sigma}_r = -\mathbf{p} = -6000\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

These three stresses are principal stresses whose variability is due to the loading. From Eq. (5-12), we find the von Mises stress to be

$$\sigma' = \left\{\frac{(18-9)^2 + [9-(-6)]^2 + (-6-18)^2}{2}\right\}^{1/2}$$
$$= 21.0 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma'} = C_p\bar{\sigma}' = 0.083\,33(21.0) = 1.75 \text{ kpsi}$$

$$z = -\frac{\bar{S} - \bar{\sigma}'}{\left(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2\right)^{1/2}}$$
$$= \frac{50 - 21.0}{(4.1^2 + 1.75^2)^{1/2}} = -6.5$$

The reliability is very high

$$R = 1 - \Phi(6.5) = 1 - 4.02(10^{-11}) \doteq 1 \quad \textit{Ans.}$$