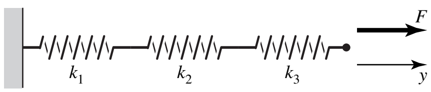
Chapter 4

4-1

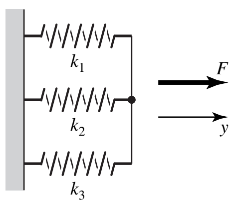
(a)



$$k = \frac{F}{y}; \quad y = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

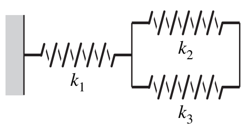
so
$$k = \frac{1}{(1/k_1) + (1/k_2) + (1/k_3)} \quad \textit{Ans.}$$

(b)



$$F = k_1 y + k_2 y + k_3 y$$
$$k = F/y = k_1 + k_2 + k_3 \quad \textit{Ans.}$$

(c)



$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2 + k_3} \qquad k = \left(\frac{1}{k_1} + \frac{1}{k_2 + k_3}\right)^{-1}$$

4-2 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_C = F/\delta$, $\delta = F/k_C$. For the assembly, $k = F/y$, $y = F/k = l\theta + \delta$

So
$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_C}$$

Or
$$k = \frac{1}{(l^2/k_T) + (1/k_C)} \quad \textit{Ans.}$$

4-3 For a torsion bar, $k = T/\theta = GJ/l$ where $J = \pi d^4/32$. So $k = \pi d^4 G/(32l) = Kd^4/l$. The springs, 1 and 2, are in parallel so

$$k = k_1 + k_2 = K\frac{d^4}{l_1} + K\frac{d^4}{l_2}$$
$$= Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)$$

And
$$\theta = \frac{T}{k} = \frac{T}{Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)}$$

Then
$$T = k\theta = \frac{Kd^4}{x}\theta + \frac{Kd^4\theta}{l - x}$$

Thus $$T_1 = \frac{Kd^4}{x}\theta; \qquad T_2 = \frac{Kd^4\theta}{l - x}$$

If $x = l/2$, then $T_1 = T_2$. If $x < l/2$, then $T_1 > T_2$

Using $\tau = 16T/\pi d^3$ and $\theta = 32Tl/(G\pi d^4)$ gives

$$T = \frac{\pi d^3 \tau}{16}$$

and so

$$\theta_{all} = \frac{32l}{G\pi d^4} \cdot \frac{\pi d^3 \tau}{16} = \frac{2l\tau_{all}}{Gd}$$

Thus, if $x < l/2$, the allowable twist is

$$\theta_{all} = \frac{2x\tau_{all}}{Gd} \quad \textit{Ans.}$$

Since $$k = Kd^4\left(\frac{1}{x} + \frac{1}{l - x}\right)$$

$$= \frac{\pi G d^4}{32}\left(\frac{1}{x} + \frac{1}{l - x}\right) \quad \textit{Ans.}$$

Then the maximum torque is found to be

$$T_{max} = \frac{\pi d^3 x \tau_{all}}{16}\left(\frac{1}{x} + \frac{1}{l - x}\right) \quad \textit{Ans.}$$

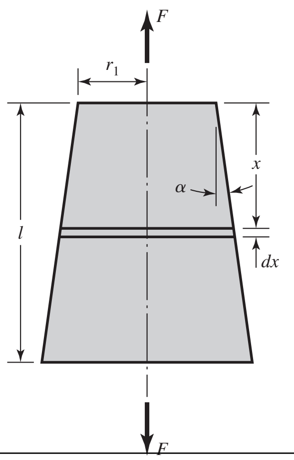
4-4 Both legs have the same twist angle. From Prob. 4-3, for equal shear, d is linear in x. Thus, $d_1 = 0.2d_2$ *Ans.*

$$k = \frac{\pi G}{32}\left[\frac{(0.2d_2)^4}{0.2l} + \frac{d_2^4}{0.8l}\right] = \frac{\pi G}{32l}\left(1.258d_2^4\right) \quad \textit{Ans.}$$

$$\theta_{all} = \frac{2(0.8l)\tau_{all}}{Gd_2} \quad \textit{Ans.}$$

$$T_{max} = k\theta_{all} = 0.198d_2^3\tau_{all} \quad \textit{Ans.}$$

4-5



$$A = \pi r^2 = \pi(r_1 + x\tan\alpha)^2$$

$$d\delta = \frac{F\,dx}{AE} = \frac{F\,dx}{E\pi(r_1 + x\tan\alpha)^2}$$

$$\delta = \frac{F}{\pi E}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^2}$$

$$= \frac{F}{\pi E}\left(-\frac{1}{\tan\alpha(r_1 + x\tan\alpha)}\right)_0^l$$

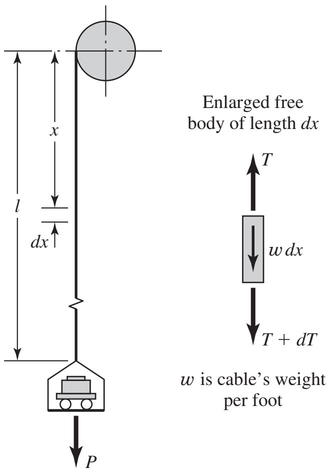
$$= \frac{F}{\pi E}\frac{1}{r_1(r_1 + l\tan\alpha)}$$

Then

$$k = \frac{F}{\delta} = \frac{\pi E r_1(r_1 + l\tan\alpha)}{l}$$

$$= \frac{EA_1}{l}\left(1 + \frac{2l}{d_1}\tan\alpha\right) \quad \textit{Ans.}$$

4-6



$$\sum F = (T + dT) + w\,dx - T = 0$$

$$\frac{dT}{dx} = -w$$

Solution is $T = -wx + c$

$$T|_{x=0} = P + wl = c$$

$$T = -wx + P + wl$$

$$T = P + w(l - x)$$

The infinitesmal stretch of the free body of original length dx is

$$d\delta = \frac{T\,dx}{AE}$$

$$= \frac{P + w(l - x)}{AE}\,dx$$

Integrating,

$$\delta = \int_0^l \frac{[P + w(l - x)]\,dx}{AE}$$

$$\delta = \frac{Pl}{AE} + \frac{wl^2}{2AE} \quad \textit{Ans.}$$

4-7

$$M = wlx - \frac{wl^2}{2} - \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{wlx^2}{2} - \frac{wl^2}{2}x - \frac{wx^3}{6} + C_1, \qquad \frac{dy}{dx} = 0 \text{ at } x = 0, \quad \therefore C_1 = 0$$

$$EIy = \frac{wlx^3}{6} - \frac{wl^2x^2}{4} - \frac{wx^4}{24} + C_2, \qquad y = 0 \text{ at } x = 0, \quad \therefore C_2 = 0$$

$$y = \frac{wx^2}{24EI}(4lx - 6l^2 - x^2) \quad \textit{Ans.}$$

4-8

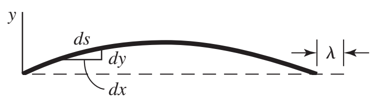
$$M = M_1 = M_B$$

$$EI\frac{dy}{dx} = M_B x + C_1, \qquad \frac{dy}{dx} = 0 \text{ at } x = 0, \quad \therefore C_1 = 0$$

$$EIy = \frac{M_B x^2}{2} + C_2, \qquad y = 0 \text{ at } x = 0, \quad \therefore C_2 = 0$$

$$y = \frac{M_B x^2}{2EI} \quad \textit{Ans.}$$

4-9



$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Expand right-hand term by Binomial theorem

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} = 1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 + \cdots$$

Since dy/dx is small compared to 1, use only the first two terms,

$$\begin{aligned} d\lambda &= ds - dx \\ &= dx\left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right] - dx \\ &= \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx \\ \therefore \lambda &= \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx \quad \textit{Ans.} \end{aligned}$$

This contraction becomes important in a nonlinear, non-breaking extension spring.

4-10 $y = Cx^2(4lx - x^2 - 6l^2) \qquad \text{where } C = \dfrac{w}{24EI}$

$$\frac{dy}{dx} = Cx(12lx - 4x^2 - 12l^2) = 4Cx(3lx - x^2 - 3l^2)$$

$$\left(\frac{dy}{dx}\right)^2 = 16C^2(15l^2x^4 - 6lx^5 - 18x^3l^3 + x^6 + 9l^4x^2)$$

$$\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx = 8C^2\int_0^l (15l^2x^4 - 6lx^5 - 18x^3l^3 + x^6 + 9l^4x^2)\, dx$$

$$= 8C^2\left(\frac{9}{14}l^7\right) = 8\left(\frac{w}{24EI}\right)^2\left(\frac{9}{14}l^7\right) = \frac{1}{112}\left(\frac{w}{EI}\right)^2 l^7 \qquad \textit{Ans.}$$

4-11 $y = Cx(2lx^2 - x^3 - l^3) \qquad \text{where } C = \dfrac{w}{24EI}$

$$\frac{dy}{dx} = C(6lx^2 - 4x^3 - l^3)$$

$$\left(\frac{dy}{dx}\right)^2 = C^2(36l^2x^4 - 48lx^5 - 12l^4x^2 + 16x^6 + 8x^3l^3 + l^6)$$

$$\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx = \frac{1}{2}C^2 \int_0^l (36l^2x^4 - 48lx^5 - 12l^4x^2 + 16x^6 + 8x^3l^3 + l^6)\, dx$$

$$= C^2\left(\frac{17}{70}l^7\right) = \left(\frac{w}{24EI}\right)^2\left(\frac{17}{70}l^7\right) = \frac{17}{40\,320}\left(\frac{w}{EI}\right)^2 l^7 \qquad \textit{Ans.}$$

4-12

$$I = 2(5.56) = 11.12 \text{ in}^4$$

$$y_{\max} = y_1 + y_2 = -\frac{wl^4}{8EI} + \frac{Fa^2}{6EI}(a - 3l)$$

Here $w = 50/12 = 4.167$ lbf/in, and $a = 7(12) = 84$ in, and $l = 10(12) = 120$ in.

$$y_1 = -\frac{4.167(120)^4}{8(30)(10^6)(11.12)} = -0.324 \text{ in}$$

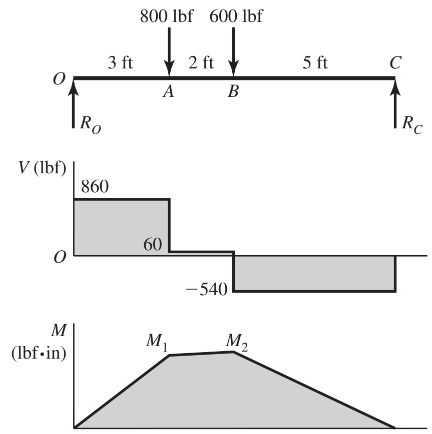
$$y_2 = -\frac{600(84)^2[3(120) - 84]}{6(30)(10^6)(11.12)} = -0.584 \text{ in}$$

So $$y_{\max} = -0.324 - 0.584 = -0.908 \text{ in} \quad \textit{Ans.}$$

$$M_0 = -Fa - (wl^2/2)$$
$$= -600(84) - [4.167(120)^2/2]$$
$$= -80\,400 \text{ lbf} \cdot \text{in}$$
$$c = 4 - 1.18 = 2.82 \text{ in}$$
$$\sigma_{\max} = \frac{-My}{I} = -\frac{(-80\,400)(-2.82)}{11.12}(10^{-3})$$
$$= -20.4 \text{ kpsi} \quad \textit{Ans.}$$

$\sigma_{\max}$ is at the bottom of the section.

4-13



$$R_O = \frac{7}{10}(800) + \frac{5}{10}(600) = 860\,\text{lbf}$$

$$R_C = \frac{3}{10}(800) + \frac{5}{10}(600) = 540\,\text{lbf}$$

$$M_1 = 860(3)(12) = 30.96(10^3)\,\text{lbf} \cdot \text{in}$$

$$M_2 = 30.96(10^3) + 60(2)(12)$$
$$= 32.40(10^3)\,\text{lbf} \cdot \text{in}$$

$$\sigma_{\max} = \frac{M_{\max}}{Z} \quad \Rightarrow \quad 6 = \frac{32.40}{Z} \qquad Z = 5.4\ \text{in}^3$$

$$y|_{x=5\text{ft}} = \frac{F_1 a[l - (l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] - \frac{F_2 l^3}{48EI}$$

$$-\frac{1}{16} = \frac{800(36)(60)}{6(30)(10^6)I(120)}[60^2 + 36^2 - 120^2] - \frac{600(120^3)}{48(30)(10^6)I}$$

$$I = 23.69\ \text{in}^4 \quad \Rightarrow \quad I/2 = 11.84\ \text{in}^4$$

Select two 6 in-8.2 lbf/ft channels; from Table A-7, $I = 2(13.1) = 26.2\ \text{in}^4$, $Z = 2(4.38)\ \text{in}^3$

$$y_{\max} = \frac{23.69}{26.2}\left(-\frac{1}{16}\right) = -0.0565\,\text{in}$$

$$\sigma_{\max} = \frac{32.40}{2(4.38)} = 3.70\,\text{kpsi}$$

4-14

$$I = \frac{\pi}{64}(40^4) = 125.66(10^3)\ \text{mm}^4$$

Superpose beams A-9-6 and A-9-7,

$$y_A = \frac{1500(600)400}{6(207)10^9(125.66)10^3(1000)}(400^2 + 600^2 - 1000^2)(10^3)^2$$
$$+ \frac{2000(400)}{24(207)10^9(125.66)10^3}[2(1000)400^2 - 400^3 - 1000^3]10^3$$

$$y_A = -2.061\ \text{mm} \quad \textit{Ans.}$$

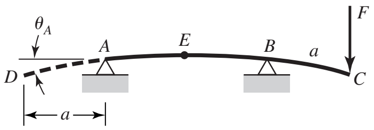
$$y|_{x=500} = \frac{1500(400)500}{24(207)10^9(125.66)10^3(1000)}[500^2 + 400^2 - 2(1000)500](10^3)^2$$
$$- \frac{5(2000)1000^4}{384(207)10^9(125.66)10^3}10^3 = -2.135\,\text{mm} \quad \textit{Ans.}$$

$$\%\ \text{difference} = \frac{2.135 - 2.061}{2.061}(100) = 3.59\% \quad \textit{Ans.}$$

4-15

$$I = \frac{1}{12}(9)(35^3) = 32.156(10^3)\ \text{mm}^4$$

From Table A-9-10



$$y_C = -\frac{Fa^2}{3EI}(l+a)$$

$$\frac{dy_{AB}}{dx} = \frac{Fa}{6EIl}(l^2 - 3x^2)$$

Thus,

$$\theta_A = \frac{Fal^2}{6EIl} = \frac{Fal}{6EI}$$

$$y_D = -\theta_A a = -\frac{Fa^2 l}{6EI}$$

With both loads,

$$y_D = -\frac{Fa^2 l}{6EI} - \frac{Fa^2}{3EI}(l+a)$$

$$= -\frac{Fa^2}{6EI}(3l + 2a) = -\frac{500(250^2)}{6(207)(10^9)(32.156)(10^3)}[3(500) + 2(250)](10^3)^2$$

$$= -1.565\ \text{mm} \quad \textit{Ans.}$$

$$y_E = \frac{2Fa(l/2)}{6EIl}\left[l^2 - \left(\frac{l}{2}\right)^2\right] = \frac{Fal^2}{8EI}$$

$$= \frac{500(250)(500^2)(10^3)^2}{8(207)(10^9)(32.156)(10^3)} = 0.587\ \text{mm} \quad \textit{Ans.}$$

4-16 $a = 36$ in, $l = 72$ in, $I = 13\ \text{in}^4$, $E = 30$ Mpsi

$$y = \frac{F_1 a^2}{6EI}(a - 3l) - \frac{F_2 l^3}{3EI}$$

$$= \frac{400(36)^2(36 - 216)}{6(30)(10^6)(13)} - \frac{400(72)^3}{3(30)(10^6)(13)}$$

$$= -0.1675\ \text{in} \quad \textit{Ans.}$$

4-17

$$I = 2(1.85) = 3.7\ \text{in}^4$$

Adding the weight of the channels, $2(5)/12 = 0.833$ lbf/in,

$$y_A = -\frac{wl^4}{8EI} - \frac{Fl^3}{3EI} = -\frac{10.833(48^4)}{8(30)(10^6)(3.7)} - \frac{220(48^3)}{3(30)(10^6)(3.7)}$$

$$= -0.1378\ \text{in} \quad \textit{Ans.}$$

4-18

$$I = \pi d^4/64 = \pi(2)^4/64 = 0.7854\ \text{in}^4$$

Tables A-9-5 and A-9-9

$$y = -\frac{F_2 l^3}{48EI} + \frac{F_1 a}{24EI}(4a^2 - 3l^2)$$

$$= -\frac{120(40)^3}{48(30)(10^6)(0.7854)} + \frac{85(10)(400 - 4800)}{24(30)(10^6)(0.7854)} = -0.0134\ \text{in} \quad \textit{Ans.}$$

4-19

(a) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{2400(48)^3}{48(30)10^6} = 0.1843\ \text{in}^4$$

From $I = bh^3/12$

$$h = \sqrt[3]{\frac{12(0.1843)}{b}}$$

Form a table. First, Table A-17 gives likely available fractional sizes for b:

$$8\tfrac{1}{2},\ 9,\ 9\tfrac{1}{2},\ 10\ \text{in}$$

For h:

$$\frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}$$

For available b what is necessary h for required I?

b	$\sqrt[3]{\frac{12(0.1843)}{b}}$	
8.5	0.638	
9.0	0.626 ←	choose 9" × $\frac{5"}{8}$ *Ans.*
9.5	0.615	
10.0	0.605	

(b)

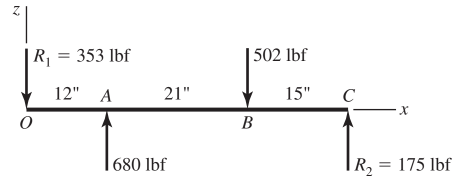
$$I = 9(0.625)^3/12 = 0.1831\ \text{in}^4$$

$$k = \frac{48EI}{l^3} = \frac{48(30)(10^6)(0.1831)}{48^3} = 2384\ \text{lbf/in}$$

$$F = \frac{4\sigma I}{cl} = \frac{4(90\,000)(0.1831)}{(0.625/2)(48)} = 4394\ \text{lbf}$$

$$y = \frac{F}{k} = \frac{4394}{2384} = 1.84\ \text{in} \quad \textit{Ans.}$$

4-20



$$\text{Torque} = (600 - 80)(9/2) = 2340\,\text{lbf} \cdot \text{in}$$

$$(T_2 - T_1)\frac{12}{2} = T_2(1 - 0.125)(6) = 2340$$

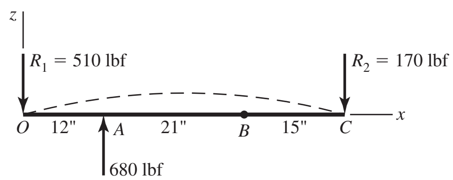
$$T_2 = \frac{2340}{6(0.875)} = 446\,\text{lbf}, \quad T_1 = 0.125(446) = 56\,\text{lbf}$$

$$\sum M_0 = 12(680) - 33(502) + 48R_2 = 0$$

$$R_2 = \frac{33(502) - 12(680)}{48} = 175\text{ lbf}$$

$$R_1 = 680 - 502 + 175 = 353\text{ lbf}$$

We will treat this as two separate problems and then sum the results.
First, consider the 680 lbf load as acting alone.



$$z_{OA} = -\frac{Fbx}{6EIl}(x^2 + b^2 - l^2); \quad \text{here } b = 36\text{"},$$

$$x = 12\text{"}, \quad l = 48\text{"}, \quad F = 680\text{ lbf}$$

Also,

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.5)^4}{64} = 0.2485\text{ in}^4$$

$$z_A = -\frac{680(36)(12)(144 + 1296 - 2304)}{6(30)(10^6)(0.2485)(48)}$$

$$= +0.1182\text{ in}$$

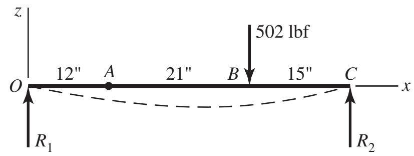
$$z_{AC} = -\frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

where $a = 12$" and $x = 21 + 12 = 33$"

$$z_B = -\frac{680(12)(15)(1089 + 144 - 3168)}{6(30)(10^6)(0.2485)(48)}$$

$$= +0.1103\text{ in}$$

Next, consider the 502 lbf load as acting alone.



$$z_{OB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2), \quad \text{where } b = 15",$$

$$x = 12", \quad l = 48", \quad I = 0.2485 \text{ in}^4$$

Then, $$z_A = \frac{502(15)(12)(144 + 225 - 2304)}{6(30)(10^6)(0.2485)(48)} = -0.081\,44 \text{ in}$$

For z_B use $x = 33"$

$$z_B = \frac{502(15)(33)(1089 + 225 - 2304)}{6(30)(10^6)(0.2485)(48)}$$
$$= -0.1146 \text{ in}$$

Therefore, by superposition

$$z_A = +0.1182 - 0.0814 = +0.0368 \text{ in} \quad \textit{Ans.}$$
$$z_B = +0.1103 - 0.1146 = -0.0043 \text{ in} \quad \textit{Ans.}$$

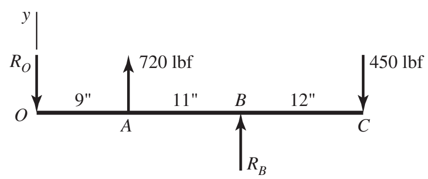
4-21

(a) Calculate torques and moment of inertia

$$T = (400 - 50)(16/2) = 2800 \text{ lbf} \cdot \text{in}$$

$$(8T_2 - T_2)(10/2) = 2800 \quad \Rightarrow \quad T_2 = 80 \text{ lbf}, \quad T_1 = 8(80) = 640 \text{ lbf}$$

$$I = \frac{\pi}{64}(1.25^4) = 0.1198 \text{ in}^4$$



Due to 720 lbf, flip beam A-9-6 such that $y_{AB} \rightarrow b = 9, x = 0, l = 20, F = -720$ lbf

$$\theta_B = \left.\frac{dy}{dx}\right|_{x=0} = -\frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$

$$= -\frac{-720(9)}{6(30)(10^6)(0.1198)(20)}(0 + 81 - 400) = -4.793(10^{-3}) \text{ rad}$$

$$y_C = -12\theta_B = -0.057\,52 \text{ in}$$

Due to 450 lbf, use beam A-9-10,

$$y_C = -\frac{Fa^2}{3EI}(l + a) = -\frac{450(144)(32)}{3(30)(10^6)(0.1198)} = -0.1923 \text{ in}$$

Adding the two deflections,

$$y_C = -0.057\,52 - 0.1923 = -0.2498 \text{ in} \quad \textit{Ans.}$$

(b) At O:
Due to 450 lbf:

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{Fa}{6EIl}(l^2 - 3x^2)\Big|_{x=0} = \frac{Fal}{6EI}$$

$$\theta_O = -\frac{720(11)(0 + 11^2 - 400)}{6(30)(10^6)(0.1198)(20)} + \frac{450(12)(20)}{6(30)(10^6)(0.1198)} = 0.010\,13 \text{ rad} = 0.5805^\circ$$

At B:

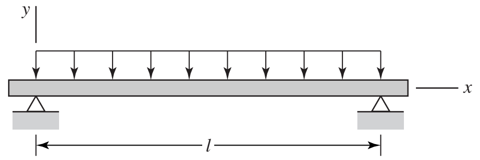
$$\theta_B = -4.793(10^{-3}) + \frac{450(12)}{6(30)(10^6)(0.1198)(20)}[20^2 - 3(20^2)]$$
$$= -0.014\,81 \text{ rad} = 0.8485^\circ$$
$$I = 0.1198\left(\frac{0.8485^\circ}{0.06^\circ}\right) = 1.694 \text{ in}^4$$
$$d = \left(\frac{64I}{\pi}\right)^{1/4} = \left[\frac{64(1.694)}{\pi}\right]^{1/4} = 2.424 \text{ in}$$

Use $d = 2.5$ in *Ans.*

$$I = \frac{\pi}{64}(2.5^4) = 1.917 \text{ in}^4$$
$$y_C = -0.2498\left(\frac{0.1198}{1.917}\right) = -0.015\,61 \text{ in} \quad \textit{Ans.}$$

4-22

(a) $l = 36(12) = 432$ in



$$y_{\max} = -\frac{5wl^4}{384EI} = -\frac{5(5000/12)(432)^4}{384(30)(10^6)(5450)}$$
$$= -1.16 \text{ in}$$

The frame is bowed up 1.16 in with respect to the bolsters. It is fabricated upside down and then inverted. *Ans.*

(b) The equation in xy-coordinates is for the center sill neutral surface

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) \quad \textit{Ans.}$$

Differentiating this equation and solving for the slope at the left bolster gives

$$\frac{dy}{dx} = \frac{w}{24EI}(6lx^2 - 4x^3 - l^3)$$

Thus,

$$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{wl^3}{24EI} = -\frac{(5000/12)(432)^3}{24(30)(10^6)(5450)}$$

$$= -0.008\,57$$

The slope at the right bolster is 0.008 57, so equation at left end is $y = -0.008\,57x$ and at the right end is $y = 0.008\,57(x - l)$. *Ans.*

4-23 From Table A-9-6,

$$y_L = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_L = \frac{Fb}{6EIl}(x^3 + b^2x - l^2x)$$

$$\frac{dy_L}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2)$$

$$\left.\frac{dy_L}{dx}\right|_{x=0} = \frac{Fb(b^2 - l^2)}{6EIl}$$

Let

$$\xi = \left|\frac{Fb(b^2 - l^2)}{6EIl}\right|$$

And set $I = \dfrac{\pi d_L^4}{64}$

And solve for d_L

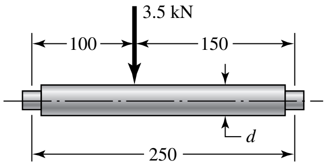
$$d_L = \left|\frac{32Fb(b^2 - l^2)}{3\pi El\xi}\right|^{1/4} \quad \textit{Ans.}$$

For the other end view, observe the figure of Table A-9-6 from the back of the page, noting that *a* and *b* interchange as do *x* and $-x$

$$d_R = \left|\frac{32Fa(l^2 - a^2)}{3\pi El\xi}\right|^{1/4} \quad \textit{Ans.}$$

For a uniform diameter shaft the necessary diameter is the larger of d_L and d_R.

4-24 Incorporating a design factor into the solution for d_L of Prob. 4-23,



$$d = \left[\frac{32n}{3\pi El\xi} Fb(l^2 - b^2)\right]^{1/4}$$

$$= \left|(\text{mm } 10^{-3}) \frac{\text{kN mm}^3}{\text{GPa mm}} \frac{10^3(10^{-9})}{10^9(10^{-3})}\right|^{1/4}$$

$$d = \sqrt[4]{\frac{32(1.28)(3.5)(150)|(250^2 - 150^2)|}{3\pi(207)(250)(0.001)} 10^{-12}}$$

$$= 36.4 \text{ mm} \quad \textit{Ans.}$$

4-25 The maximum occurs in the right section. Flip beam A-9-6 and use

$$y = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2) \quad \text{where } b = 100 \text{ mm}$$

$$\frac{dy}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = 0$$

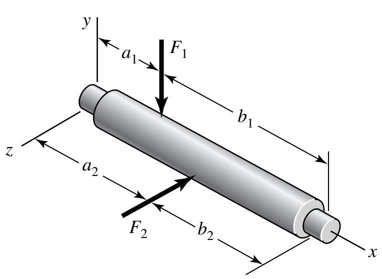
Solving for x,

$$x = \sqrt{\frac{l^2 - b^2}{3}} = \sqrt{\frac{250^2 - 100^2}{3}} = 132.29 \text{ mm} \quad \text{from right}$$

$$y = \frac{3.5(10^3)(0.1)(0.132\,29)}{6(207)(10^9)(\pi/64)(0.0364^4)(0.25)}[0.132\,29^2 + 0.1^2 - 0.25^2](10^3)$$

$$= -0.0606 \text{ mm} \quad \textit{Ans.}$$

4-26



The slope at $x = 0$ due to F_1 in the xy plane is

$$\theta_{xy} = \frac{F_1 b_1 (b_1^2 - l^2)}{6EIl}$$

and in the xz plane due to F_2 is

$$\theta_{xz} = \frac{F_2 b_2 (b_2^2 - l^2)}{6EIl}$$

For small angles, the slopes add as vectors. Thus

$$\theta_L = \left(\theta_{xy}^2 + \theta_{xz}^2\right)^{1/2}$$

$$= \left[\left(\frac{F_1 b_1 (b_1^2 - l^2)}{6EIl}\right)^2 + \left(\frac{F_2 b_2 (b_2^2 - l^2)}{6EIl}\right)^2\right]^{1/2}$$

Designating the slope constraint as ξ, we then have

$$\xi = |\theta_L| = \frac{1}{6EIl}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left|\frac{32}{3\pi El\xi}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

For the LH bearing, $E = 30$ Mpsi, $\xi = 0.001$, $b_1 = 12$, $b_2 = 6$, and $l = 16$. The result is $d_L = 1.31$ in. Using a similar flip beam procedure, we get $d_R = 1.36$ in for the RH bearing. So use $d = 1\ 3/8$ in   *Ans.*

4-27 $I = \frac{\pi}{64}(1.375^4) = 0.17546\text{ in}^4$. For the *xy* plane, use y_{BC} of Table A-9-6

$$y = \frac{100(4)(16-8)}{6(30)(10^6)(0.17546)(16)}[8^2 + 4^2 - 2(16)8] = -1.115(10^{-3})\text{ in}$$

For the *xz* plane use y_{AB}

$$z = \frac{300(6)(8)}{6(30)(10^6)(0.17546)(16)}[8^2 + 6^2 - 16^2] = -4.445(10^{-3})\text{ in}$$

$$\delta = (-1.115\mathbf{j} - 4.445\mathbf{k})(10^{-3})\text{ in}$$

$$|\delta| = 4.583(10^{-3})\text{ in} \quad \textit{Ans.}$$

4-28 $$d_L = \left|\frac{32n}{3\pi El\xi}\left\{\sum\left[F_i b_i\left(b_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

$$= \left|\frac{32(1.5)}{3\pi(207)(10^9)(250)0.001}\left\{[3.5(150)(150^2 - 250^2)]^2 + [2.7(75)(75^2 - 250^2)]^2\right\}^{1/2}(10^3)^3\right|^{1/4}$$

$$= 39.2\text{ mm}$$

$$d_R = \left|\frac{32(1.5)}{3\pi(207)10^9(250)0.001}\left\{[3.5(100)(100^2 - 250^2)]^2 + [2.7(175)(175^2 - 250^2)]^2\right\}^{1/2}(10^3)^3\right|^{1/4}$$

$$= 39.1\text{ mm}$$

Choose $d \ge 39.2$ mm   *Ans.*

4-29 From Table A-9-8 we have

$$y_L = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_L}{dx} = \frac{M_B}{6EIl}(3x^2 + 3a^2 - 6al + 2l^2)$$

At $x = 0$, the LH slope is

$$\theta_L = \frac{dy_L}{dx} = \frac{M_B}{6EIl}(3a^2 - 6al + 2l^2)$$

from which

$$\xi = |\theta_L| = \frac{M_B}{6EIl}(l^2 - 3b^2)$$

Setting $I = \pi d^4/64$ and solving for d

$$d = \left|\frac{32M_B(l^2 - 3b^2)}{3\pi El\xi}\right|^{1/4}$$

For a multiplicity of moments, the slopes add vectorially and

$$d_L = \left|\frac{32}{3\pi El\xi}\left\{\sum\left[M_i\left(l^2 - 3b_i^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

$$d_R = \left|\frac{32}{3\pi El\xi}\left\{\sum\left[M_i\left(3a_i^2 - l^2\right)\right]^2\right\}^{1/2}\right|^{1/4}$$

The greatest slope is at the LH bearing. So

$$d = \left|\frac{32(1200)[9^2 - 3(4^2)]}{3\pi(30)(10^6)(9)(0.002)}\right|^{1/4} = 0.706 \text{ in}$$

So use $d = 3/4$ in *Ans.*

4-30

$$6F_{AC} = 18(80)$$

$$F_{AC} = 240 \text{ lbf}$$

$$R_O = 160 \text{ lbf}$$

$$I = \frac{1}{12}(0.25)(2^3) = 0.1667 \text{ in}^4$$

Initially, ignore the stretch of AC. From Table A-9-10

$$y_{B1} = -\frac{Fa^2}{3EI}(l + a) = -\frac{80(12^2)}{3(10)(10^6)(0.1667)}(6 + 12) = -0.041\,47 \text{ in}$$

Stretch of AC: $\delta = \left(\frac{FL}{AE}\right)_{AC} = \frac{240(12)}{(\pi/4)(1/2)^2(10)(10^6)} = 1.4668(10^{-3})$ in

Due to stretch of AC

$$y_{B2} = -3\delta = -4.400(10^{-3}) \text{ in}$$

By superposition, $y_B = -0.041\,47 - 0.0044 = -0.045\,87$ in *Ans.*

4-31

$$\theta = \frac{TL}{JG} = \frac{(0.1F)(1.5)}{(\pi/32)(0.012^4)(79.3)(10^9)} = 9.292(10^{-4})F$$

Due to twist

$$\delta_{B1} = 0.1(\theta) = 9.292(10^{-5})F$$

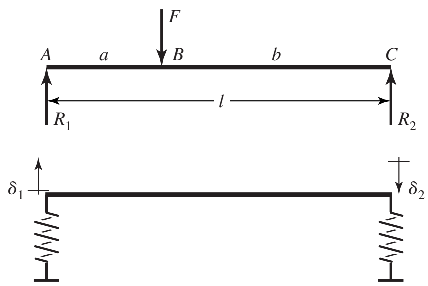
Due to bending

$$\delta_{B2} = \frac{FL^3}{3EI} = \frac{F(0.1^3)}{3(207)(10^9)(\pi/64)(0.012^4)} = 1.582(10^{-6})F$$

$$\delta_B = 1.582(10^{-6})F + 9.292(10^{-5})F = 9.450(10^{-5})F$$

$$k = \frac{1}{9.450(10^{-5})} = 10.58(10^3)\text{ N/m} = 10.58\text{ kN/m} \quad \textit{Ans.}$$

4-32



$$R_1 = \frac{Fb}{l} \quad R_2 = \frac{Fa}{l}$$

$$\delta_1 = \frac{R_1}{k_1} \quad \delta_2 = \frac{R_2}{k_2}$$

Spring deflection

$$y_S = -\delta_1 + \left(\frac{\delta_1 - \delta_2}{l}\right)x = -\frac{Fb}{k_1 l} + \left(\frac{Fb}{k_1 l^2} - \frac{Fa}{k_2 l^2}\right)x$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2) + \frac{Fx}{l^2}\left(\frac{b}{k_1} - \frac{a}{k_2}\right) - \frac{Fb}{k_1 l} \quad \textit{Ans.}$$

$$y_{BC} = \frac{Fa(l-x)}{6EIl}(x^2 + a^2 - 2lx) + \frac{Fx}{l^2}\left(\frac{b}{k_1} - \frac{a}{k_2}\right) - \frac{Fb}{k_1 l} \quad \textit{Ans.}$$

4-33 See Prob. 4-32 for deflection due to springs. Replace Fb/l and Fa/l with $wl/2$

$$y_S = -\frac{wl}{2k_1} + \left(\frac{wl}{2k_1 l} - \frac{wl}{2k_2 l}\right)x = \frac{wx}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{wl}{2k_1}$$

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) + \frac{wx}{2}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \frac{wl}{2k_1} \quad \textit{Ans.}$$

4-34 Let the load be at $x > l/2$. The maximum deflection will be in Section AB (Table A-9-10)

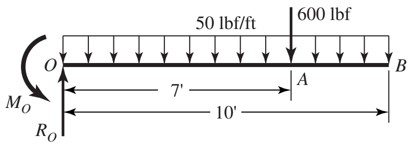
$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = 0 \quad \Rightarrow \quad 3x^2 + b^2 - l^2 = 0$$

$$x = \sqrt{\frac{l^2 - b^2}{3}}, \qquad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \quad Ans.$$

For $x < l/2 \quad x_{\min} = l - 0.577l = 0.423l \quad Ans.$

4-35



$$M_O = 50(10)(60) + 600(84)$$
$$= 80\,400 \text{ lbf} \cdot \text{in}$$
$$R_O = 50(10) + 600 = 1100 \text{ lbf}$$

$I = 11.12 \text{ in}^4$ from Prob. 4-12

$$M = -80\,400 + 1100x - \frac{4.167x^2}{2} - 600\langle x - 84\rangle^1$$

$$EI\frac{dy}{dx} = -80\,400x + 550x^2 - 0.6944x^3 - 300\langle x - 84\rangle^2 + C_1$$

$\dfrac{dy}{dx} = 0$ at $x = 0 \quad \therefore C_1 = 0$

$$EIy = -402\,00x^2 + 183.33x^3 - 0.1736x^4 - 100\langle x - 84\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \therefore C_2 = 0$

$$y_B = \frac{1}{30(10^6)(11.12)}[-40\,200(120^2) + 183.33(120^3) - 0.1736(120^4) - 100(120 - 84)^3]$$
$$= -0.9075 \text{ in} \quad Ans.$$

4-36 See Prob. 4-13 for reactions: $R_O = 860$ lbf, $R_C = 540$ lbf

$$M = 860x - 800\langle x - 36\rangle^1 - 600\langle x - 60\rangle^1$$

$$EI\frac{dy}{dx} = 430x^2 - 400\langle x - 36\rangle^2 - 300\langle x - 60\rangle^2 + C_1$$

$$EIy = 143.33x^3 - 133.33\langle x - 36\rangle^3 - 100\langle x - 60\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = 120$ in $\Rightarrow C_1 = -1.2254(10^6)$ lbf · in^2

Substituting C_1 and C_2 and evaluating at $x = 60$,

$$EIy = 30(10^6)I\left(-\frac{1}{16}\right) = 143.33(60^3) - 133.33(60 - 36)^3 - 1.2254(10^6)(60)$$
$$I = 23.68 \text{ in}^4$$

Agrees with Prob. 4-13. The rest of the solution is the same.

4-37

$$I = \frac{\pi}{64}(40^4) = 125.66(10^3)\ \text{mm}^4$$

$$R_O = 2(500) + \frac{600}{1000}1500 = 1900\ \text{N}$$

$$M = 1900x - \frac{2000}{2}x^2 - 1500\langle x - 0.4\rangle^1 \text{ where } x \text{ is in meters}$$

$$EI\frac{dy}{dx} = 950x^2 - \frac{1000}{3}x^3 - 750\langle x - 0.4\rangle^2 + C_1$$

$$EIy = \frac{900}{3}x^3 - \frac{250}{3}x^4 - 250\langle x - 0.4\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = 1$ m $\Rightarrow C_1 = -179.33\ \text{N}\cdot\text{m}^2$

Substituting C_1 and C_2 and evaluating y at $x = 0.4$ m,

$$y_A = \frac{1}{207(10^9)125.66(10^{-9})}\left[\frac{950}{3}(0.4^3) - \frac{250}{3}(0.4^4) - 179.33(0.4)\right]10^3$$

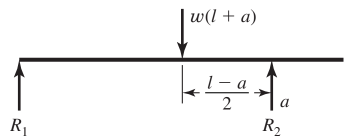
$= -2.061$ mm *Ans.*

$$y|_{x=500} = \frac{1}{207(10^9)125.66(10^{-9})}\left[\frac{950}{3}(0.5^3) - \frac{250}{3}(0.5^4) - 250(0.5-0.4)^3 - 179.33(0.5)\right]10^3$$

$= -2.135$ mm *Ans.*

$$\%\ \text{difference} = \frac{2.135 - 2.061}{2.061}(100) = 3.59\%$$ *Ans.*

4-38



$$R_1 = \frac{w(l + a)[(l - a)/2)]}{l} = \frac{w}{2l}(l^2 - a^2)$$

$$R_2 = w(l + a) - \frac{w}{2l}(l^2 - a^2) = \frac{w}{2l}(l + a)^2$$

$$M = \frac{w}{2l}(l^2 - a^2)x - \frac{wx^2}{2} + \frac{w}{2l}(l + a)^2\langle x - l\rangle^1$$

$$EI\frac{dy}{dx} = \frac{w}{4l}(l^2 - a^2)x^2 - \frac{w}{6}x^3 + \frac{w}{4l}(l + a)^2\langle x - l\rangle^2 + C_1$$

$$EIy = \frac{w}{12l}(l^2 - a^2)x^3 - \frac{w}{24}x^4 + \frac{w}{12l}(l + a)^2\langle x - l\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$y = 0$ at $x = l$

$$0 = \frac{w}{12l}(l^2 - a^2)l^3 - \frac{w}{24}l^4 + C_1 l \quad \Rightarrow \quad C_1 = \frac{wl}{24}(2a^2 - l^2)$$

$$y = \frac{w}{24EIl}[2(l^2 - a^2)x^3 - lx^4 + 2(l + a)^2\langle x - l\rangle^3 + l^2(2a^2 - l^2)x] \quad Ans.$$

4-39 $R_A = R_B = 500$ N, and $I = \frac{1}{12}(9)35^3 = 32.156(10^3)$ mm^4

For first half of beam, $M = -500x + 500\langle x - 0.25\rangle^1$ where x is in meters

$$EI\frac{dy}{dx} = -250x^2 + 250\langle x - 0.25\rangle^2 + C_1$$

At $x = 0.5$ m, $dy/dx = 0 \Rightarrow 0 = -250(0.5^2) + 250(0.5 - 0.250)^2 + C_1 \Rightarrow C_1 = 46.875\text{ N}\cdot\text{m}^2$

$$EIy = -\frac{250}{3}x^3 + \frac{250}{3}\langle x - 0.25\rangle^3 + 46.875x + C_2$$

$$y = 0 \text{ at } x = 0.25\text{ m} \Rightarrow 0 = -\frac{250}{3}0.25^3 + 46.875(0.25) + C_2 \Rightarrow C_2 = -10.417\text{ N}\cdot\text{m}^3$$

$$\therefore EIy = -\frac{250}{3}x^3 + \frac{250}{3}\langle x - 0.25\rangle^3 + 46.875x - 10.42$$

Evaluating y at A and the center,

$$y_A = \frac{1}{207(10^9)32.156(10^{-9})}\left[-\frac{250}{3}(0^3) + \frac{250}{3}(0)^3 + 46.875(0) - 10.417\right]10^3$$
$$= -1.565\text{ mm} \quad Ans.$$

$$y|_{x=0.5\text{m}} = \frac{1}{207(10^9)32.156(10^{-9})}\left[-\frac{250}{3}(0.5^3) + \frac{250}{3}(0.5 - 0.25)^3 + 46.875(0.5) - 10.417\right]10^3$$
$$= -2.135\text{ mm} \quad Ans.$$

4-40 From Prob. 4-30, $R_O = 160$ lbf $\downarrow$, $F_{AC} = 240$ lbf $\quad I = 0.1667$ in^4

$$M = -160x + 240\langle x - 6\rangle^1$$
$$EI\frac{dy}{dx} = -80x^2 + 120\langle x - 6\rangle^2 + C_1$$
$$EIy = -26.67x^3 + 40\langle x - 6\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0 \Rightarrow C_2 = 0$

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{240(12)}{(\pi/4)(1/2)^2(10)(10^6)} = -1.4668(10^{-3})\text{ in}$$

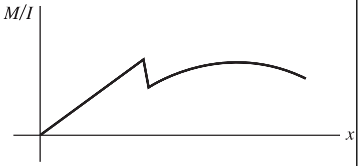
at $x = 6$

$$10(10^6)(0.1667)(-1.4668)(10^{-3}) = -26.67(6^3) + C_1(6)$$
$$C_1 = 552.58\text{ lbf}\cdot\text{in}^2$$

$$y_B = \frac{1}{10(10^6)(0.1667)}[-26.67(18^3) + 40(18-6)^3 + 552.58(18)]$$
$$= -0.045\,87 \text{ in} \quad Ans.$$

4-41

$$I_1 = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4 \qquad I_2 = \frac{\pi}{64}(2^4) = 0.7854 \text{ in}^4$$



$$R_1 = \frac{200}{2}(12) = 1200 \text{ lbf}$$

For $0 \le x \le 16$ in, $\quad M = 1200x - \dfrac{200}{2}\langle x-4\rangle^2$

$$\frac{M}{I} = \frac{1200x}{I_1} - 4800\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-4\rangle^0 - 1200\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-4\rangle^1 - \frac{100}{I_2}\langle x-4\rangle^2$$
$$= 4829x - 13\,204\langle x-4\rangle^0 - 3301.1\langle x-4\rangle^1 - 127.32\langle x-4\rangle^2$$

$$E\frac{dy}{dx} = 2414.5x^2 - 13\,204\langle x-4\rangle^1 - 1651\langle x-4\rangle^2 - 42.44\langle x-4\rangle^3 + C_1$$

Boundary Condition: $\dfrac{dy}{dx} = 0 \quad$ at $x = 10$ in

$$0 = 2414.5(10^2) - 13\,204(10-4)^1 - 1651(10-4)^2 - 42.44(10-4)^3 + C_1$$
$$C_1 = -9.362(10^4)$$
$$Ey = 804.83x^3 - 6602\langle x-4\rangle^2 - 550.3\langle x-4\rangle^3 - 10.61\langle x-4\rangle^4 - 9.362(10^4)x + C_2$$

$y = 0 \quad$ at $x = 0 \Rightarrow C_2 = 0$

For $0 \le x \le 16$ in

$$y = \frac{1}{30(10^6)}[804.83x^3 - 6602\langle x-4\rangle^2 - 550.3\langle x-4\rangle^3 - 10.61\langle x-4\rangle^4 - 9.362(10^4)x] \quad Ans.$$

at $x = 10$ in

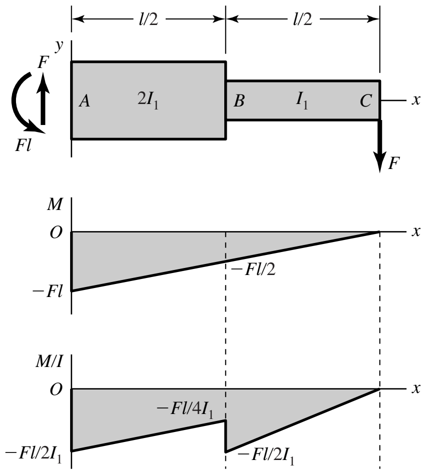
$$y|_{x=10} = \frac{1}{30(10^6)}[804.83(10^3) - 6602(10-4)^2 - 550.3(10-4)^3 - 10.61(10-4)^4 - 9.362(10^4)(10)]$$
$$= -0.016\,72 \text{ in} \quad Ans.$$

4-42 $\quad q = F\langle x\rangle^{-1} - Fl\langle x\rangle^{-2} - F\langle x-l\rangle^{-1}$

Integrations produce

$$V = F\langle x\rangle^0 - Fl\langle x\rangle^{-1} - F\langle x-l\rangle^0$$
$$M = F\langle x\rangle^1 - Fl\langle x\rangle^0 - F\langle x-l\rangle^1 = Fx - Fl$$

Plots for M and M/I are shown below



M/I can be expressed by singularity functions as

$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2}\right\rangle^1$$

where the step down and increase in slope at $x = l/2$ are given by the last two terms.

Since $E\,d^2y/dx^2 = M/I$, two integrations yield

$$E\frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^1 + \frac{F}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + C_1$$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x - \frac{l}{2}\right\rangle^2 + \frac{F}{12I_1}\left\langle x - \frac{l}{2}\right\rangle^3 + C_1x + C_2$$

At $x = 0$, $y = dy/dx = 0$. This gives $C_1 = C_2 = 0$, and

$$y = \frac{F}{24EI_1}\left(2x^3 - 6lx^2 - 3l\left\langle x - \frac{l}{2}\right\rangle^2 + 2\left\langle x - \frac{l}{2}\right\rangle^3\right)$$

At $x = l/2$ and l,

$$y|_{x=l/2} = \frac{F}{24EI_1}\left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0)\right] = -\frac{5Fl^3}{96EI_1} \qquad \textit{Ans.}$$

$$y|_{x=l} = \frac{F}{24EI_1}\left[2(l)^3 - 6l(l)^2 - 3l\left(l - \frac{l}{2}\right)^2 + 2\left(l - \frac{l}{2}\right)^3\right] = -\frac{3Fl^3}{16EI_1} \qquad \textit{Ans.}$$

The answers are identical to Ex. 4-11.

4-43 Define δ_{ij} as the deflection in the direction of the load at station i due to a unit load at station j. If U is the potential energy of strain for a body obeying Hooke's law, apply P_1 first. Then

$$U = \frac{1}{2}P_1(P_1\delta_{11})$$

When the second load is added, U becomes

$$U = \frac{1}{2}P_1(P_1\delta_{11}) + \frac{1}{2}P_2(P_2\,\delta_{22}) + P_1(P_2\,\delta_{12})$$

For loading in the reverse order

$$U' = \frac{1}{2}P_2(P_2\,\delta_{22}) + \frac{1}{2}P_1(P_1\delta_{11}) + P_2(P_1\,\delta_{21})$$

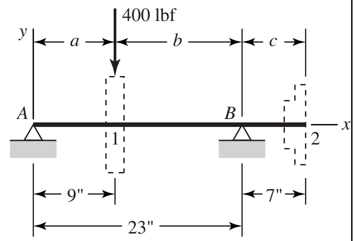
Since the order of loading is immaterial $U = U'$ and

$$P_1 P_2 \delta_{12} = P_2 P_1 \delta_{21} \qquad \text{when } P_1 = P_2,\ \delta_{12} = \delta_{21}$$

which states that the deflection at station 1 due to a unit load at station 2 is the same as the deflection at station 2 due to a unit load at 1. δ is sometimes called an *influence coefficient*.

4-44

(a) From Table A-9-10



$$y_{AB} = \frac{Fcx(l^2 - x^2)}{6EIl}$$

$$\delta_{12} = \frac{y}{F}\bigg|_{x=a} = \frac{ca(l^2 - a^2)}{6EIl}$$

$$y_2 = F\delta_{21} = F\delta_{12} = \frac{Fca(l^2 - a^2)}{6EIl}$$

Substituting $I = \dfrac{\pi d^4}{64}$

$$y_2 = \frac{400(7)(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)} = 0.00347 \text{ in} \quad \textit{Ans.}$$

(b) The slope of the shaft at *left* bearing at $x = 0$ is

$$\theta = \frac{Fb(b^2 - l^2)}{6EIl}$$

Viewing the illustration in Section 6 of Table A-9 from the back of the page provides the correct view of this problem. Noting that a is to be interchanged with b and $-x$ with x leads to

$$\theta = \frac{Fa(l^2 - a^2)}{6EIl} = \frac{Fa(l^2 - a^2)(64)}{6E\pi d^4 l}$$

$$\theta = \frac{400(9)(23^2 - 9^2)(64)}{6(30)(10^6)(\pi)(2)^4(23)} = 0.000\,496 \text{ in/in}$$

So $y_2 = 7\theta = 7(0.000\,496) = 0.00347$ in *Ans.*

4-45 Place a dummy load Q at the center. Then,

$$M = \frac{wx}{2}(l - x) + \frac{Qx}{2}$$

$$U = 2\int_0^{l/2} \frac{M^2 dx}{2EI}, \quad y_{\max} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

$$y_{\max} = 2\left[\int_0^{l/2} \frac{2M}{2EI}\left(\frac{\partial M}{\partial Q}\right) dx\right]_{Q=0}$$

$$y_{\max} = \frac{2}{EI}\left\{\int_0^{l/2}\left[\frac{wx}{2}(l - x) + \frac{Qx}{2}\right]\frac{x}{2}\,dx\right\}_{Q=0}$$

Set $Q = 0$ and integrate

$$y_{\max} = \frac{w}{2EI}\left(\frac{lx^3}{3} - \frac{x^4}{4}\right)_0^{l/2}$$

$$y_{\max} = \frac{5wl^4}{384EI} \quad Ans.$$

4-46

$$I = 2(1.85) = 3.7 \text{ in}^4$$

Adding weight of channels of 0.833 lbf · in,

$$M = -Fx - \frac{10.833}{2}x^2 = -Fx - 5.417x^2 \qquad \frac{\partial M}{\partial F} = -x$$

$$\delta_B = \frac{1}{EI}\int_0^{48} M\frac{\partial M}{\partial F}\,dx = \frac{1}{EI}\int_0^{48}(Fx + 5.417x^2)(x)\,dx$$

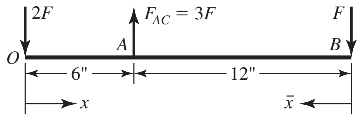
$$= \frac{(220/3)(48^3) + (5.417/4)(48^4)}{30(10^6)(3.7)} = 0.1378 \text{ in} \qquad \text{in direction of 220 lbf}$$

$$\therefore y_B = -0.1378 \text{ in} \quad Ans.$$

4-47

$$I_{OB} = \frac{1}{12}(0.25)(2^3) = 0.1667 \text{ in}^4, \quad A_{AC} = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.196\,35 \text{ in}^2$$

$$F_{AC} = 3F, \quad \frac{\partial F_{AC}}{\partial F} = 3$$



right	left
$M = -F\bar{x}$	$M = -2Fx$
$\frac{\partial M}{\partial F} = -\bar{x}$	$\frac{\partial M}{\partial F} = -2x$

$$U = \frac{1}{2EI}\int_0^l M^2 dx + \frac{F_{AC}^2 L_{AC}}{2A_{AC}E}$$

$$\delta_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial F}\,dx + \frac{F_{AC}(\partial F_{AC}/\partial F)L_{AC}}{A_{AC}E}$$

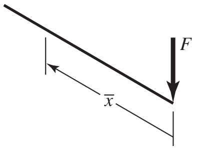
$$= \frac{1}{EI}\left[\int_0^{12} -F\bar{x}(-\bar{x})\,d\bar{x} + \int_0^6 (-2Fx)(-2x)\,dx\right] + \frac{3F(3)(12)}{A_{AC}E}$$

$$= \frac{1}{EI}\left[\frac{F}{3}(12^3) + 4F\left(\frac{6^3}{3}\right)\right] + \frac{108F}{A_{AC}E}$$

$$= \frac{864F}{EI} + \frac{108F}{A_{AC}E}$$

$$= \frac{864(80)}{10(10^6)(0.1667)} + \frac{108(80)}{0.196\,35(10)(10^6)} = 0.045\,86 \text{ in} \quad \textit{Ans.}$$

4-48



Torsion $\qquad T = 0.1F \qquad \dfrac{\partial T}{\partial F} = 0.1$

Bending $\qquad M = -F\bar{x} \qquad \dfrac{\partial M}{\partial F} = -\bar{x}$

$$U = \frac{1}{2EI}\int M^2\,dx + \frac{T^2 L}{2JG}$$

$$\delta_B = \frac{\partial U}{\partial F} = \frac{1}{EI}\int M\frac{\partial M}{\partial F}\,dx + \frac{T(\partial T/\partial F)L}{JG}$$

$$= \frac{1}{EI}\int_0^{0.1} -F\bar{x}(-\bar{x})\,d\bar{x} + \frac{0.1F(0.1)(1.5)}{JG}$$

$$= \frac{F}{3EI}(0.1^3) + \frac{0.015F}{JG}$$

Where

$$I = \frac{\pi}{64}(0.012)^4 = 1.0179(10^{-9})\text{ m}^4$$

$$J = 2I = 2.0358(10^{-9})\text{ m}^4$$

$$\delta_B = F\left[\frac{0.001}{3(207)(10^9)(1.0179)(10^{-9})} + \frac{0.015}{2.0358(10^{-9})(79.3)(10^9)}\right] = 9.45(10^{-5})F$$

$$k = \frac{1}{9.45(10^{-5})} = 10.58(10^3)\text{ N/m} = 10.58\text{ kN/m} \quad \textit{Ans.}$$

4-49 From Prob. 4-41, $I_1 = 0.2485 \text{ in}^4$, $I_2 = 0.7854 \text{ in}^4$

For a dummy load $\uparrow Q$ at the center

$$0 \le x \le 10 \text{ in} \qquad M = 1200x - \frac{Q}{2}x - \frac{200}{2}\langle x-4\rangle^2, \qquad \frac{\partial M}{\partial Q} = \frac{-x}{2}$$

$$y|_{x=10} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

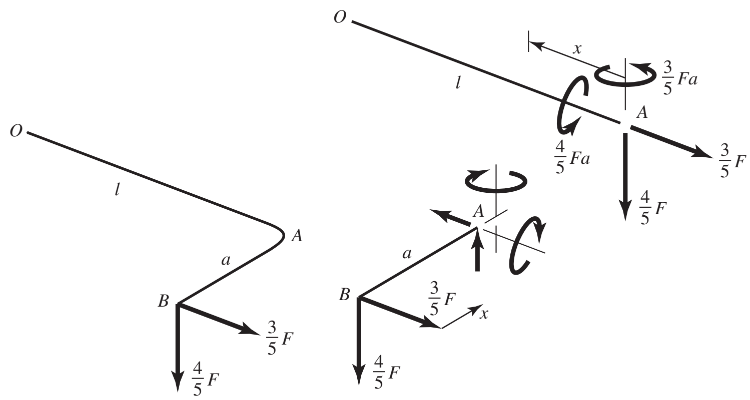
$$= \frac{2}{E}\left\{\frac{1}{I_1}\int_0^4 (1200x)\left(-\frac{x}{2}\right)dx + \frac{1}{I_2}\int_4^{10}[1200x - 100(x-4)^2]\left(-\frac{x}{2}\right)dx\right\}$$

$$= \frac{2}{E}\left[-\frac{200(4^3)}{I_1} - \frac{1.566(10^5)}{I_2}\right]$$

$$= -\frac{2}{30(10^6)}\left(\frac{1.28(10^4)}{0.2485} + \frac{1.566(10^5)}{0.7854}\right)$$

$$= -0.016\,73 \text{ in} \quad Ans.$$

4-50



AB

$$M = Fx \qquad \frac{\partial M}{\partial F} = x$$

OA

$$N = \frac{3}{5}F \qquad \frac{\partial N}{\partial F} = \frac{3}{5}$$

$$T = \frac{4}{5}Fa \qquad \frac{\partial T}{\partial F} = \frac{4}{5}a$$

$$M_1 = \frac{4}{5}F\bar{x} \qquad \frac{\partial M_1}{\partial F} = \frac{4}{5}\bar{x}$$

$$M_2 = \frac{3}{5}Fa \qquad \frac{\partial M_2}{\partial F} = \frac{3}{5}a$$

$$\delta_B = \frac{\partial u}{\partial F} = \frac{1}{EI}\int_0^a Fx(x)\,dx + \frac{(3/5)F(3/5)l}{AE} + \frac{(4/5)Fa(4a/5)l}{JG}$$

$$+ \frac{1}{EI}\int_0^l \frac{4}{5}F\bar{x}\left(\frac{4}{5}\bar{x}\right)d\bar{x} + \frac{1}{EI}\int_0^l \frac{3}{5}Fa\left(\frac{3}{5}a\right)d\bar{x}$$

$$= \frac{Fa^3}{3EI} + \frac{9}{25}\left(\frac{Fl}{AE}\right) + \frac{16}{25}\left(\frac{Fa^2l}{JG}\right) + \frac{16}{75}\left(\frac{Fl^3}{EI}\right) + \frac{9}{25}\left(\frac{Fa^2l}{EI}\right)$$

$$I = \frac{\pi}{64}d^4, \quad J = 2I, \quad A = \frac{\pi}{4}d^2$$

$$\delta_B = \frac{64Fa^3}{3E\pi d^4} + \frac{9}{25}\left(\frac{4Fl}{\pi d^2E}\right) + \frac{16}{25}\left(\frac{32Fa^2l}{\pi d^4G}\right) + \frac{16}{75}\left(\frac{64Fl^3}{E\pi d^4}\right) + \frac{9}{25}\left(\frac{64Fa^2l}{E\pi d^4}\right)$$

$$= \frac{4F}{75\pi Ed^4}\left(400a^3 + 27ld^2 + 384a^2l\frac{E}{G} + 256l^3 + 432a^2l\right) \quad \textit{Ans.}$$

4-51 The force applied to the copper and steel wire assembly is $F_c + F_s = 250$ lbf

Since $\delta_c = \delta_s$

$$\frac{F_cL}{3(\pi/4)(0.0801)^2(17.2)(10^6)} = \frac{F_sL}{(\pi/4)(0.0625)^2(30)(10^6)}$$

$$F_c = 2.825F_s$$

$$\therefore 3.825F_s = 250 \quad \Rightarrow \quad F_s = 65.36 \text{ lbf}, \quad F_c = 2.825F_s = 184.64 \text{ lbf}$$

$$\sigma_c = \frac{184.64}{3(\pi/4)(0.0801)^2} = 12\,200 \text{ psi} = 12.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_s = \frac{65.36}{(\pi/4)(0.0625^2)} = 21\,300 \text{ psi} = 21.3 \text{ kpsi} \quad \textit{Ans.}$$

4-52

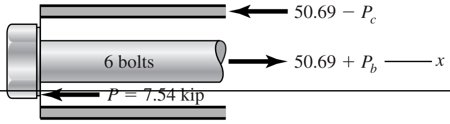
(a) Bolt stress $\sigma_b = 0.9(85) = 76.5$ kpsi *Ans.*

Bolt force $F_b = 6(76.5)\left(\frac{\pi}{4}\right)(0.375^2) = 50.69$ kips

Cylinder stress $\sigma_c = -\frac{F_b}{A_c} = -\frac{50.69}{(\pi/4)(4.5^2 - 4^2)} = -15.19$ kpsi *Ans.*

(b) Force from pressure

$$P = \frac{\pi D^2}{4}p = \frac{\pi(4^2)}{4}(600) = 7540 \text{ lbf} = 7.54 \text{ kip}$$



$$\sum F_x = 0$$

$$P_b + P_c = 7.54 \quad (1)$$

Since $\delta_c = \delta_b$, $$\frac{P_c L}{(\pi/4)(4.5^2 - 4^2)E} = \frac{P_b L}{6(\pi/4)(0.375^2)E}$$

$$P_c = 5.037 P_b \quad (2)$$

Substituting into Eq. (1)

$$6.037 P_b = 7.54 \quad \Rightarrow \quad P_b = 1.249 \text{ kip}; \quad \text{and from Eq. (2),} \quad P_c = 6.291 \text{ kip}$$

Using the results of (a) above, the total bolt and cylinder stresses are

$$\sigma_b = 76.5 + \frac{1.249}{6(\pi/4)(0.375^2)} = 78.4 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_c = -15.19 + \frac{6.291}{(\pi/4)(4.5^2 - 4^2)} = -13.3 \text{ kpsi} \quad \textit{Ans.}$$

4-53

$$T = T_c + T_s \quad \text{and} \quad \theta_c = \theta_s$$

Also, $$\frac{T_c L}{(GJ)_c} = \frac{T_s L}{(GJ)_s}$$

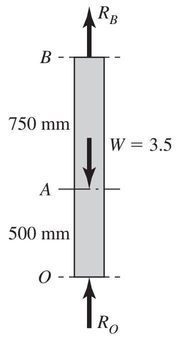
$$T_c = \frac{(GJ)_c}{(GJ)_s} T_s$$

Substituting into equation for T,

$$T = \left[1 + \frac{(GJ)_c}{(GJ)_s}\right] T_s$$

$$\%T_s = \frac{T_s}{T} = \frac{(GJ)_s}{(GJ)_s + (GJ)_c} \quad \textit{Ans.}$$

4-54



$$R_O + R_B = W \quad (1)$$

$$\delta_{OA} = \delta_{AB} \quad (2)$$

$$\frac{500 R_O}{AE} = \frac{750 R_B}{AE}, \quad R_O = \frac{3}{2} R_B$$

$$\frac{3}{2} R_B + R_B = 3.5$$

$$R_B = \frac{7}{5} = 1.4 \text{ kN} \quad \textit{Ans.}$$

$$R_O = 3.5 - 1.4 = 2.1 \text{ kN} \quad \textit{Ans.}$$

$$\sigma_O = -\frac{2100}{12(50)} = -3.50 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_B = \frac{1400}{12(50)} = 2.33 \text{ MPa} \quad \textit{Ans.}$$

4-55 Since $\theta_{OA} = \theta_{AB}$

$$\frac{T_{OA}(4)}{GJ} = \frac{T_{AB}(6)}{GJ}, \quad T_{OA} = \frac{3}{2}T_{AB}$$

Also $T_{OA} + T_{AB} = 50$

$$T_{AB}\left(\frac{3}{2}+1\right) = 50, \quad T_{AB} = \frac{50}{2.5} = 20\,\text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}(20) = 30\,\text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

4-56 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{G(\frac{\pi}{32}1.5^4)} = \frac{T_{AB}(6)}{G(\frac{\pi}{32}1.75^4)}, \quad T_{OA} = 0.80966T_{AB}$$

$$T_{OA} + T_{AB} = 50 \;\Rightarrow\; 0.80966\,T_{AB} + T_{AB} = 50 \;\Rightarrow\; T_{AB} = 27.63\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{OA} = 0.80966T_{AB} = 0.80966(27.63) = 22.37\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

4-57

$$F_1 = F_2 \quad\Rightarrow\quad \frac{T_1}{r_1} = \frac{T_2}{r_2} \quad\Rightarrow\quad \frac{T_1}{1.25} = \frac{T_2}{3}$$

$$T_2 = \frac{3}{1.25}T_1$$

$$\therefore \theta_1 + \frac{3}{1.25}\theta_2 = \frac{4\pi}{180}\text{ rad}$$

$$\frac{T_1(48)}{(\pi/32)(7/8)^4(11.5)(10^6)} + \frac{3}{1.25}\left[\frac{(3/1.25)T_1(48)}{(\pi/32)(1.25)^4(11.5)(10^6)}\right] = \frac{4\pi}{180}$$

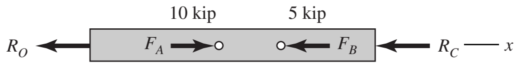
$$T_1 = 403.9\text{ lbf}\cdot\text{in}$$

$$T_2 = \frac{3}{1.25}T_1 = 969.4\text{ lbf}\cdot\text{in}$$

$$\tau_1 = \frac{16T_1}{\pi d^3} = \frac{16(403.9)}{\pi(7/8)^3} = 3071\text{ psi} \quad \textit{Ans.}$$

$$\tau_2 = \frac{16(969.4)}{\pi(1.25)^3} = 2528\text{ psi} \quad \textit{Ans.}$$

4-58



(1) Arbitrarily, choose R_C as redundant reaction

(2) $$\sum F_x = 0, \quad 10(10^3) - 5(10^3) - R_O - R_C = 0$$

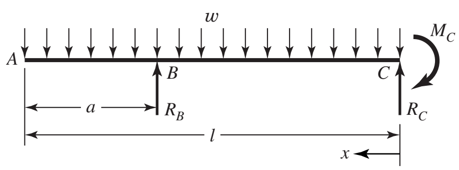
$$R_O + R_C = 5(10^3)\ \text{lbf}$$

(3) $$\delta_C = \frac{[10(10^3) - 5(10^3) - R_C]20}{AE} - \frac{[5(10^3) + R_C]}{AE}(10) - \frac{R_C(15)}{AE} = 0$$

$$-45R_C + 5(10^4) = 0 \quad \Rightarrow \quad R_C = 1111\ \text{lbf} \quad \textit{Ans.}$$

$$R_O = 5000 - 1111 = 3889\ \text{lbf} \quad \textit{Ans.}$$

4-59



(1) Choose R_B as redundant reaction

(2) $$R_B + R_C = wl \quad (a) \qquad R_B(l-a) - \frac{wl^2}{2} + M_C = 0 \quad (b)$$

(3) $$y_B = \frac{R_B(l-a)^3}{3EI} + \frac{w(l-a)^2}{24EI}[4l(l-a) - (l-a)^2 - 6l^2] = 0$$

$$R_B = \frac{w}{8(l-a)}[6l^2 - 4l(l-a) + (l-a)^2]$$

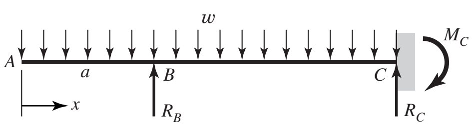
$$= \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad \textit{Ans.}$$

Substituting,

Eq. (a) $$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad \textit{Ans.}$$

Eq. (b) $$M_C = \frac{wl^2}{2} - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad \textit{Ans.}$$

4-60



$$M = -\frac{wx^2}{2} + R_B\langle x-a\rangle^1, \qquad \frac{\partial M}{\partial R_B} = \langle x-a\rangle^1$$

$$\frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R_B}\,dx$$

$$= \frac{1}{EI}\int_0^a \frac{-wx^2}{2}(0)\,dx + \frac{1}{EI}\int_a^l \left[\frac{-wx^2}{2} + R_B(x-a)\right](x-a)\,dx = 0$$

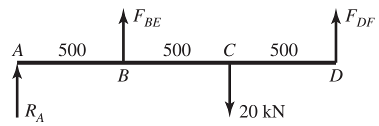
$$-\frac{w}{2}\left[\frac{1}{4}(l^4 - a^4) - \frac{a}{3}(l^3 - a^3)\right] + \frac{R_B}{3}\left[(l-a)^3 - (a-a)^3\right] = 0$$

$$R_B = \frac{w}{(l-a)^3}[3(L^4 - a^4) - 4a(l^3 - a^3)] = \frac{w}{8(l-a)}(3l^2 + 2al + a^2) \quad Ans.$$

$$R_C = wl - R_B = \frac{w}{8(l-a)}(5l^2 - 10al - a^2) \quad Ans.$$

$$M_C = \frac{wl^2}{2} - R_B(l-a) = \frac{w}{8}(l^2 - 2al - a^2) \quad Ans.$$

4-61



$$A = \frac{\pi}{4}(0.012^2) = 1.131(10^{-4})\ \text{m}^2$$

(1) $$R_A + F_{BE} + F_{DF} = 20\ \text{kN} \qquad (a)$$

$$\sum M_A = 3F_{DF} - 2(20) + F_{BE} = 0$$

$$F_{BE} + 3F_{DF} = 40\ \text{kN} \qquad (b)$$

(2) $$M = R_A x + F_{BE}\langle x - 0.5\rangle^1 - 20(10^3)\langle x - 1\rangle^1$$

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} + \frac{F_{BE}}{2}\langle x - 0.5\rangle^2 - 10(10^3)\langle x - 1\rangle^2 + C_1$$

$$EIy = R_A\frac{x^3}{6} + \frac{F_{BE}}{6}\langle x - 0.5\rangle^3 - \frac{10}{3}(10^3)\langle x - 1\rangle^3 + C_1 x + C_2$$

(3) $y = 0$ at $x = 0 \quad \therefore C_2 = 0$

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(1)}{1.131(10^{-4})209(10^9)} = -4.2305(10^{-8})F_{BE}$$

Substituting and evaluating at $x = 0.5$ m

$$EIy_B = 209(10^9)(8)(10^{-7})(-4.2305)(10^{-8})F_{BE} = R_A\frac{0.5^3}{6} + C_1(0.5)$$

$$2.0833(10^{-2})R_A + 7.0734(10^{-3})F_{BE} + 0.5C_1 = 0 \qquad (c)$$

$$y_D = -\left(\frac{Fl}{AE}\right)_{DF} = -\frac{F_{DF}(1)}{1.131(10^{-4})(209)(10^9)} = -4.2305(10^{-8})F_{DF}$$

Substituting and evaluating at $x = 1.5$ m

$$EIy_D = -7.0734(10^{-3})F_{DF} = R_A\frac{1.5^3}{6} + \frac{F_{BE}}{6}(1.5 - 0.5)^3 - \frac{10}{3}(10^3)(1.5 - 1)^3 + 1.5C_1$$

$$0.5625R_A + 0.166\,67F_{BE} + 7.0734(10^{-3})F_{DF} + 1.5C_1 = 416.67 \qquad (d)$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 2.0833(10^{-2}) & 7.0734(10^{-3}) & 0 & 0.5 \\ 0.5625 & 0.166\,67 & 7.0734(10^{-3}) & 1.5 \end{bmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 20\,000 \\ 40\,000 \\ 0 \\ 416.67 \end{Bmatrix}$$

Solve simultaneously or use software

$$R_A = -3885 \text{ N}, \quad F_{BE} = 15\,830 \text{ N}, \quad F_{DF} = 8058 \text{ N}, \quad C_1 = -62.045 \text{ N}\cdot\text{m}^2$$

$$\sigma_{BE} = \frac{15\,830}{(\pi/4)(12^2)} = 140 \text{ MPa} \quad \textit{Ans.}, \qquad \sigma_{DF} = \frac{8058}{(\pi/4)(12^2)} = 71.2 \text{ MPa} \quad \textit{Ans.}$$

$$EI = 209(10^9)(8)(10^{-7}) = 167.2(10^3) \text{ N}\cdot\text{m}^2$$

$$y = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}x^3 + \frac{15\,830}{6}\langle x-0.5\rangle^3 - \frac{10}{3}(10^3)\langle x-1\rangle^3 - 62.045x\right]$$

B: $x = 0.5$ m, $\quad y_B = -6.70(10^{-4})$ m $= -0.670$ mm *Ans.*

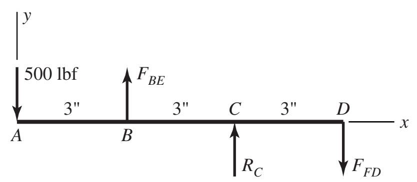
C: $x = 1$ m, $\quad y_C = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}(1^3) + \frac{15\,830}{6}(1-0.5)^3 - 62.045(1)\right]$

$$= -2.27(10^{-3}) \text{ m} = -2.27 \text{ mm} \quad \textit{Ans.}$$

D: $x = 1.5$, $\quad y_D = \frac{1}{167.2(10^3)}\left[-\frac{3885}{6}(1.5^3) + \frac{15\,830}{6}(1.5-0.5)^3 - \frac{10}{3}(10^3)(1.5-1)^3 - 62.045(1.5)\right]$

$$= -3.39(10^{-4}) \text{ m} = -0.339 \text{ mm} \quad \textit{Ans.}$$

4-62



$$EI = 30(10^6)(0.050) = 1.5(10^6) \text{ lbf}\cdot\text{in}^2$$

(1)
$$R_C + F_{BE} - F_{FD} = 500 \qquad (a)$$
$$3R_C + 6F_{BE} = 9(500) = 4500 \qquad (b)$$

(2) $\quad M = -500x + F_{BE}\langle x-3\rangle^1 + R_C\langle x-6\rangle^1$

$$EI\frac{dy}{dx} = -250x^2 + \frac{F_{BE}}{2}\langle x-3\rangle^2 + \frac{R_C}{2}\langle x-6\rangle^2 + C_1$$

$$EIy = -\frac{250}{3}x^3 + \frac{F_{BE}}{6}\langle x-3\rangle^3 + \frac{R_C}{6}\langle x-6\rangle^3 + C_1x + C_2$$

$$y_B = \left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(2)}{(\pi/4)(5/16)^2(30)(10^6)} = -8.692(10^{-7})F_{BE}$$

Substituting and evaluating at $x = 3$ in

$$EIy_B = 1.5(10^6)[-8.692(10^{-7})F_{BE}] = -\frac{250}{3}(3^3) + 3C_1 + C_2$$

$$1.3038F_{BE} + 3C_1 + C_2 = 2250 \qquad (c)$$

Since $y = 0$ at $x = 6$ in

$$EIy|_{=0} = -\frac{250}{3}(6^3) + \frac{F_{BE}}{6}(6-3)^3 + 6C_1 + C_2$$

$$4.5F_{BE} + 6C_1 + C_2 = 1.8(10^4) \qquad (d)$$

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(2.5)}{(\pi/4)(5/16)^2(30)(10^6)} = 1.0865(10^{-6})F_{DF}$$

Substituting and evaluating at $x = 9$ in

$$EIy_D = 1.5(10^6)[1.0865(10^{-6})F_{DF}] = -\frac{250}{3}(9^3) + \frac{F_{BE}}{6}(9-3)^3 + \frac{R_C}{6}(9-6)^3 + 9C_1 + C_2$$

$$4.5R_C + 36F_{BE} - 1.6297F_{DF} + 9C_1 + C_2 = 6.075(10^4) \qquad (e)$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 3 & 6 & 0 & 0 & 0 \\ 0 & 1.3038 & 0 & 3 & 1 \\ 0 & 4.5 & 0 & 6 & 1 \\ 4.5 & 36 & -1.6297 & 9 & 1 \end{bmatrix} \begin{Bmatrix} R_C \\ F_{BE} \\ F_{DF} \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 500 \\ 4500 \\ 2250 \\ 1.8(10^4) \\ 6.075(10^4) \end{Bmatrix}$$

$$R_C = -590.4 \text{ lbf}, \quad F_{BE} = 1045.2 \text{ lbf}, \quad F_{DF} = -45.2 \text{ lbf}$$

$$C_1 = 4136.4 \text{ lbf}\cdot\text{in}^2, \quad C_2 = -11\,522 \text{ lbf}\cdot\text{in}^3$$

$$\sigma_{BE} = \frac{1045.2}{(\pi/4)(5/16)^2} = 13\,627 \text{ psi} = 13.6 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_{DF} = -\frac{45.2}{(\pi/4)(5/16)^2} = -589 \text{ psi} \quad \textit{Ans.}$$

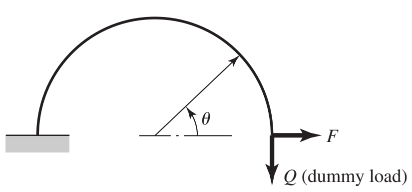
$$y_A = \frac{1}{1.5(10^6)}(-11\,522) = -0.007\,68 \text{ in} \quad \textit{Ans.}$$

$$y_B = \frac{1}{1.5(10^6)}\left[-\frac{250}{3}(3^3) + 4136.4(3) - 11\,522\right] = -0.000\,909 \text{ in} \quad \textit{Ans.}$$

$$y_D = \frac{1}{1.5(10^6)}\left[-\frac{250}{3}(9^3) + \frac{1045.2}{6}(9-3)^3 + \frac{-590.4}{6}(9-6)^3 + 4136.4(9) - 11\,522\right]$$

$$= -4.93(10^{-5}) \text{ in} \quad \textit{Ans.}$$

4-63



$$M = -PR\sin\theta + QR(1-\cos\theta) \qquad \frac{\partial M}{\partial Q} = R(1-\cos\theta)$$

$$\delta_Q = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{\pi} (-PR\sin\theta)R(1-\cos\theta)R\,d\theta = -2\frac{PR^3}{EI}$$

Deflection is upward and equals $2(PR^3/EI)$ *Ans.*

4-64 Equation (4-28) becomes

$$U = 2\int_0^{\pi} \frac{M^2R\,d\theta}{2EI} \qquad R/h > 10$$

where $M = FR(1-\cos\theta)$ and $\dfrac{\partial M}{\partial F} = R(1-\cos\theta)$

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi} M\frac{\partial M}{\partial F}R\,d\theta$$

$$= \frac{2}{EI}\int_0^{\pi} FR^3(1-\cos\theta)^2\,d\theta$$

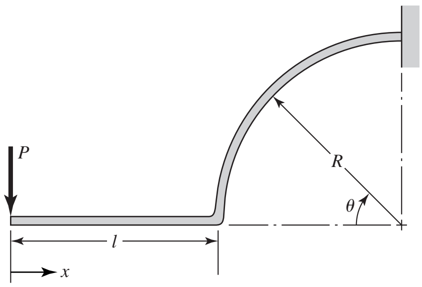
$$= \frac{3\pi FR^3}{EI}$$

Since $I = bh^3/12 = 4(6)^3/12 = 72\text{ mm}^4$ and $R = 81/2 = 40.5$ mm, we have

$$\delta = \frac{3\pi(40.5)^3F}{131(72)} = 66.4F\text{ mm} \quad \textit{Ans.}$$

where F is in kN.

4-65



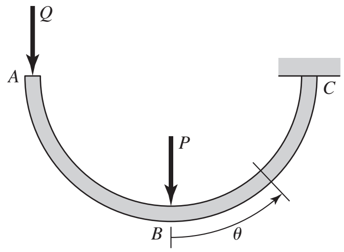
$$M = -Px, \quad \frac{\partial M}{\partial P} = -x \quad 0 \le x \le l$$

$$M = Pl + PR(1-\cos\theta), \quad \frac{\partial M}{\partial P} = l + R(1-\cos\theta) \quad 0 \le \theta \le l$$

$$\delta_P = \frac{1}{EI}\left\{\int_0^{l} -Px(-x)\,dx + \int_0^{\pi/2} P[l + R(1-\cos\theta)]^2R\,d\theta\right\}$$

$$= \frac{P}{12EI}\{4l^3 + 3R[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2]\} \quad \textit{Ans.}$$

4-66 *A*: Dummy load *Q* is applied at *A*. Bending in *AB* due only to *Q* which is zero.



$$M = PR\sin\theta + QR(1+\sin\theta), \quad \frac{\partial M}{\partial Q} = R(1+\sin\theta), \quad 0 \le \theta \le \frac{\pi}{2}$$

$$(\delta_A)_V = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} (PR\sin\theta)[R(1+\sin\theta)]R\,d\theta$$

$$= \frac{PR^3}{EI}\left(-\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\Bigg|_0^{\pi/2} = \frac{PR^3}{EI}\left(1+\frac{\pi}{4}\right)$$

$$= \frac{\pi+4}{4}\frac{PR^3}{EI} \quad \textit{Ans.}$$

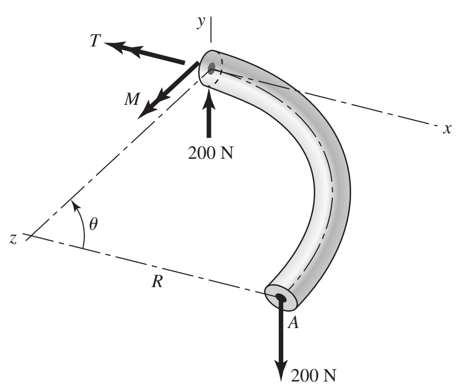
B:

$$M = PR\sin\theta, \quad \frac{\partial M}{\partial P} = R\sin\theta$$

$$(\delta_B)_V = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi/2}(PR\sin\theta)(R\sin\theta)R\,d\theta$$

$$= \frac{\pi}{4}\frac{PR^3}{EI} \quad \textit{Ans.}$$

4-67



$$M = PR\sin\theta, \qquad \frac{\partial M}{\partial P} = R\sin\theta \quad 0 < \theta < \frac{\pi}{2}$$

$$T = PR(1-\cos\theta), \quad \frac{\partial T}{\partial P} = R(1-\cos\theta)$$

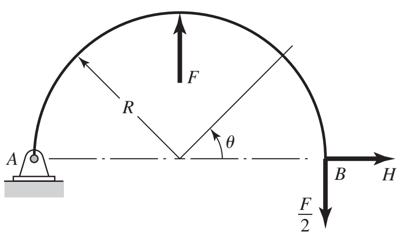
$$(\delta_A)_y = -\frac{\partial U}{\partial P} = -\left\{\frac{1}{EI}\int_0^{\pi/2} P(R\sin\theta)^2 R\,d\theta + \frac{1}{GJ}\int_0^{\pi/2} P[R(1-\cos\theta)]^2 R\,d\theta\right\}$$

Integrating and substituting $J = 2I$ and $G = E/[2(1+\nu)]$

$$(\delta_A)_y = -\frac{PR^3}{EI}\left[\frac{\pi}{4} + (1+\nu)\left(\frac{3\pi}{4} - 2\right)\right] = -[4\pi - 8 + (3\pi - 8)\nu]\frac{PR^3}{4EI}$$

$$= -[4\pi - 8 + (3\pi - 8)(0.29)]\frac{(200)(100)^3}{4(200)(10^3)(\pi/64)(5)^4} = -40.6 \text{ mm}$$

4-68 Consider the horizontal reaction, to be applied at B, subject to the constraint $(\delta_B)_H = 0$.



(a) $(\delta_B)_H = \dfrac{\partial U}{\partial H} = 0$

Due to symmetry, consider half of the structure. F does not deflect horizontally.

$$M = \frac{FR}{2}(1-\cos\theta) - HR\sin\theta, \qquad \frac{\partial M}{\partial H} = -R\sin\theta, \quad 0 < \theta < \frac{\pi}{2}$$

$$\frac{\partial U}{\partial H} = \frac{1}{EI}\int_0^{\pi/2}\left[\frac{FR}{2}(1-\cos\theta) - HR\sin\theta\right](-R\sin\theta)R\,d\theta = 0$$

$$-\frac{F}{2} + \frac{F}{4} + H\frac{\pi}{4} = 0 \quad \Rightarrow \quad H = \frac{F}{\pi} \quad \textit{Ans.}$$

Reaction at A is the same where H goes to the left

(b) For $0 < \theta < \dfrac{\pi}{2}$, $\quad M = \dfrac{FR}{2}(1-\cos\theta) - \dfrac{FR}{\pi}\sin\theta$

$$M = \frac{FR}{2\pi}[\pi(1-\cos\theta) - 2\sin\theta] \quad \textit{Ans.}$$

Due to symmetry, the solution for the left side is identical.

(c)
$$\frac{\partial M}{\partial F} = \frac{R}{2\pi}[\pi(1-\cos\theta) - 2\sin\theta]$$

$$\delta_F = \frac{\partial U}{\partial F} = \frac{2}{EI}\int_0^{\pi/2}\frac{FR^2}{4\pi^2}[\pi(1-\cos\theta) - 2\sin\theta]^2 R\,d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\int_0^{\pi/2}(\pi^2 + \pi^2\cos^2\theta + 4\sin^2\theta - 2\pi^2\cos\theta - 4\pi\sin\theta + 4\pi\sin\theta\cos\theta)\,d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\left[\pi^2\left(\frac{\pi}{2}\right) + \pi^2\left(\frac{\pi}{4}\right) + 4\left(\frac{\pi}{4}\right) - 2\pi^2 - 4\pi + 2\pi\right]$$

$$= \frac{(3\pi^2 - 8\pi - 4)}{8\pi}\,\frac{FR^3}{EI} \quad \textit{Ans.}$$

4-69 Must use Eq. (4-33)

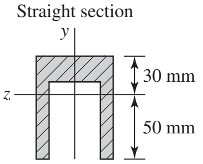
$$A = 80(60) - 40(60) = 2400 \text{ mm}^2$$

$$R = \frac{(25+40)(80)(60) - (25+20+30)(40)(60)}{2400} = 55 \text{ mm}$$

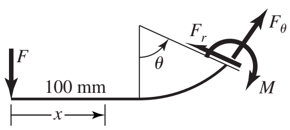
Section is equivalent to the "T" section of Table 3-4

$$r_n = \frac{60(20) + 20(60)}{60\ln[(25+20)/25] + 20\ln[(80+25)/(25+20)]} = 45.9654 \text{ mm}$$

$$e = R - r_n = 9.035 \text{ mm}$$



$$I_z = \frac{1}{12}(60)(20^3) + 60(20)(30-10)^2 + 2\left[\frac{1}{12}(10)(60^3) + 10(60)(50-30)^2\right] = 1.36(10^6) \text{ mm}^4$$



For $0 \le x \le 100$ mm

$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x; \quad V = F, \quad \frac{\partial V}{\partial F} = 1$$

For $\theta \le \pi/2$

$$F_r = F\cos\theta, \quad \frac{\partial F_r}{\partial F} = \cos\theta; \quad F_\theta = F\sin\theta, \quad \frac{\partial F_\theta}{\partial F} = \sin\theta$$

$$M = F(100 + 55\sin\theta), \quad \frac{\partial M}{\partial F} = (100 + 55\sin\theta)$$

Use Eq. (5-34), integrate from 0 to $\pi/2$, double the results and add straight part

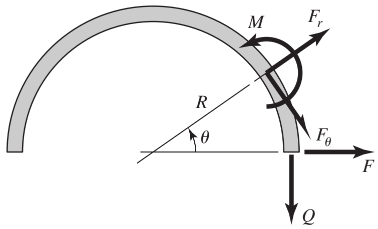
$$\delta = \frac{2}{E}\left\{\frac{1}{I}\int_0^{100} Fx^2 dx + \int_0^{100} \frac{(1)F(1)\,dx}{2400(G/E)} + \int_0^{\pi/2} F\frac{(100+55\sin\theta)^2}{2400(9.035)}d\theta + \int_0^{\pi/2} \frac{F\sin^2\theta(55)}{2400}d\theta - \int_0^{\pi/2} \frac{F(100+55\sin\theta)}{2400}\sin\theta d\theta - \int_0^{\pi/2} \frac{F\sin\theta(100+55\sin\theta)}{2400}d\theta + \int_0^{\pi/2} \frac{(1)F\cos^2\theta(55)}{2400(G/E)}d\theta\right\}$$

Substitute

$I = 1.36(10^3)$ mm^2, $F = 30(10^3)$ N, $E = 207(10^3)$ N/mm^2, $G = 79(10^3)$ N/mm^2

$$\delta = \frac{2}{207(10^3)}30(10^3)\left\{\frac{100^3}{3(1.36)(10^6)} + \frac{207}{79}\left(\frac{100}{2400}\right) + \frac{2.908(10^4)}{2400(9.035)} + \frac{55}{2400}\left(\frac{\pi}{4}\right) - \frac{2}{2400}(143.197) + \frac{207}{79}\left(\frac{55}{2400}\right)\left(\frac{\pi}{4}\right)\right\} = 0.476 \text{ mm} \quad \textit{Ans.}$$

4-70



$$M = FR\sin\theta - QR(1-\cos\theta), \quad \frac{\partial M}{\partial Q} = -R(1-\cos\theta)$$

$$F_\theta = Q\cos\theta + F\sin\theta, \quad \frac{\partial F_\theta}{\partial Q} = \cos\theta$$

$$\frac{\partial}{\partial Q}(MF_\theta) = [FR\sin\theta - QR(1-\cos\theta)]\cos\theta + [-R(1-\cos\theta)][Q\cos\theta + F\sin\theta]$$

$$F_r = F\cos\theta - Q\sin\theta, \quad \frac{\partial F_r}{\partial Q} = -\sin\theta$$

From Eq. (4-33)

$$\delta = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \frac{1}{AeE}\int_0^\pi (FR\sin\theta)[-R(1-\cos\theta)]\,d\theta + \frac{R}{AE}\int_0^\pi F\sin\theta\cos\theta d\theta$$

$$- \frac{1}{AE}\int_0^\pi [FR\sin\theta\cos\theta - FR\sin\theta(1-\cos\theta)]\,d\theta$$

$$+ \frac{CR}{AG}\int_0^\pi -F\cos\theta\sin\theta d\theta$$

$$= -\frac{2FR^2}{AeE} + 0 + \frac{2FR}{AE} + 0 = -\left(\frac{R}{e} - 1\right)\frac{2FR}{AE} \quad \textit{Ans.}$$

4-71 The cross section at A does not rotate, thus for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle θ to the x axis is

$$M = M_A - \frac{F}{2}(R - x) = M_A - \frac{FR}{2}(1-\cos\theta) \tag{1}$$

because $x = R\cos\theta$. Next,

$$U = \int \frac{M^2}{2EI}ds = \int_0^{\pi/2} \frac{M^2}{2EI}R\,d\theta$$

since $ds = R\,d\theta$. Then

$$\frac{\partial U}{\partial M_A} = \frac{R}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial M_A}\,d\theta = 0$$

But $\partial M/\partial M_A = 1$. Therefore

$$\int_0^{\pi/2} M\,d\theta = \int_0^{\pi/2} \left[M_A - \frac{FR}{2}(1-\cos\theta)\right] d\theta = 0$$

Since this term is zero, we have

$$M_A = \frac{FR}{2}\left(1 - \frac{2}{\pi}\right)$$

Substituting into Eq. (1)

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

The maximum occurs at B where $\theta = \pi/2$. It is

$$M_B = -\frac{FR}{\pi} \quad \textit{Ans.}$$

4-72 For one quadrant

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right); \quad \frac{\partial M}{\partial F} = \frac{R}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

$$\delta = \frac{\partial U}{\partial F} = 4\int_0^{\pi/2} \frac{M}{EI}\frac{\partial M}{\partial F} R\,d\theta$$

$$= \frac{FR^3}{EI}\int_0^{\pi/2}\left(\cos\theta - \frac{2}{\pi}\right)^2 d\theta$$

$$= \frac{FR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right) \quad \textit{Ans.}$$

4-73

$$P_{cr} = \frac{C\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi D^4}{64}(1 - K^4)$$

$$P_{cr} = \frac{C\pi^2 E}{l^2}\left[\frac{\pi D^4}{64}(1 - K^4)\right]$$

$$D = \left[\frac{64 P_{cr} l^2}{\pi^3 CE(1 - K^4)}\right]^{1/4} \quad \textit{Ans.}$$

4-74

$$A = \frac{\pi}{4}D^2(1-K^2), \quad I = \frac{\pi}{64}D^4(1-K^4) = \frac{\pi}{64}D^4(1-K^2)(1+K^2),$$

$$k^2 = \frac{I}{A} = \frac{D^2}{16}(1+K^2)$$

From Eq. (4-43)

$$\frac{P_{cr}}{(\pi/4)D^2(1-K^2)} = S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 CE} = S_y - \frac{S_y^2 l^2}{4\pi^2(D^2/16)(1+K^2)CE}$$

$$4P_{cr} = \pi D^2(1-K^2)S_y - \frac{4S_y^2 l^2 \pi D^2(1-K^2)}{\pi^2 D^2(1+K^2)CE}$$

$$\pi D^2(1-K^2)S_y = 4P_{cr} + \frac{4S_y^2 l^2(1-K^2)}{\pi(1+K^2)CE}$$

$$D = \left[\frac{4P_{cr}}{\pi S_y(1-K^2)} + \frac{4S_y^2 l^2(1-K^2)}{\pi(1+K^2)CE\pi(1-K^2)S_y}\right]^{1/2}$$

$$= 2\left[\frac{P_{cr}}{\pi S_y(1-K^2)} + \frac{S_y l^2}{\pi^2 CE(1+K^2)}\right]^{1/2} \quad Ans.$$

4-75 (a)

$$\overset{+}{\curvearrowleft}\sum M_A = 0, \qquad 2.5(180) - \frac{3}{\sqrt{3^2+1.75^2}}F_{BO}(1.75) = 0 \quad \Rightarrow \quad F_{BO} = 297.7 \text{ lbf}$$

Using $n_d = 5$, design for $F_{cr} = n_d F_{BO} = 5(297.7) = 1488$ lbf, $l = \sqrt{3^2+1.75^2} =$ 3.473 ft, $S_y = 24$ kpsi

In plane: $k = 0.2887h = 0.2887"$, $C = 1.0$

Try 1" × 1/2" section

$$\frac{l}{k} = \frac{3.473(12)}{0.2887} = 144.4$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(1)(30)(10^6)}{24(10^3)}\right)^{1/2} = 157.1$$

Since $(l/k)_1 > (l/k)$ use Johnson formula

$$P_{cr} = (1)\left(\frac{1}{2}\right)\left[24(10^3) - \left(\frac{24(10^3)}{2\pi}144.4\right)^2\left(\frac{1}{1(30)(10^6)}\right)\right] = 6930 \text{ lbf}$$

Try 1" × 1/4": $P_{cr} = 3465$ lbf

Out of plane: $k = 0.2887(0.5) = 0.1444$ in, $C = 1.2$

$$\frac{l}{k} = \frac{3.473(12)}{0.1444} = 289$$

Since $(l/k)_1 < (l/k)$ use Euler equation

$$P_{cr} = 1(0.5)\frac{1.2(\pi^2)(30)(10^6)}{289^2} = 2127 \text{ lbf}$$

1/4" increases l/k by 2, $\left(\frac{l}{k}\right)^2$ by 4, and A by 1/2

Try 1" × 3/8": $k = 0.2887(0.375) = 0.1083$ in

$$\frac{l}{k} = 385, \quad P_{cr} = 1(0.375)\frac{1.2(\pi^2)(30)(10^6)}{385^2} = 899 \text{ lbf} \quad \text{(too low)}$$

Use 1" × 1/2" *Ans.*

(b) $\sigma_b = -\frac{P}{\pi dl} = -\frac{298}{\pi(0.5)(0.5)} = -379$ psi *No,* bearing stress is not significant.

4-76 This is a design problem with no one distinct solution.

4-77

$$F = 800\left(\frac{\pi}{4}\right)(3^2) = 5655 \text{ lbf}, \quad S_y = 37.5 \text{ kpsi}$$

$$P_{cr} = n_d F = 3(5655) = 17\,000 \text{ lbf}$$

(a) Assume Euler with $C = 1$

$$I = \frac{\pi}{64}d^4 = \frac{P_{cr}l^2}{C\pi^2 E} \quad \Rightarrow \quad d = \left[\frac{64 P_{cr} l^2}{\pi^3 C E}\right]^{1/4} = \left[\frac{64(17)(10^3)(60^2)}{\pi^3(1)(30)(10^6)}\right]^{1/4} = 1.433 \text{ in}$$

Use $d = 1.5$ in; $k = d/4 = 0.375$

$$\frac{l}{k} = \frac{60}{0.375} = 160$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2(1)(30)(10^6)}{37.5(10^3)}\right)^{1/2} = 126 \quad \therefore \text{use Euler}$$

$$P_{cr} = \frac{\pi^2(30)(10^6)(\pi/64)(1.5^4)}{60^2} = 20\,440 \text{ lbf}$$

$d = 1.5$ in is satisfactory. *Ans.*

(b) $$d = \left[\frac{64(17)(10^3)(18^2)}{\pi^3(1)(30)(10^6)}\right]^{1/4} = 0.785 \text{ in}, \quad \text{so use } 0.875 \text{ in}$$

$$k = \frac{0.875}{4} = 0.2188 \text{ in}$$

$$l/k = \frac{18}{0.2188} = 82.3 \quad \text{try Johnson}$$

$$P_{cr} = \frac{\pi}{4}(0.875^2)\left[37.5(10^3) - \left(\frac{37.5(10^3)}{2\pi}82.3\right)^2 \frac{1}{1(30)(10^6)}\right] = 17\,714 \text{ lbf}$$

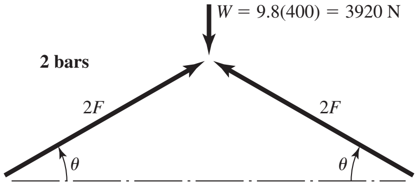
Use $d = 0.875$ in *Ans.*

(c)

$$n_{(a)} = \frac{20\,440}{5655} = 3.61 \quad \textit{Ans.}$$

$$n_{(b)} = \frac{17\,714}{5655} = 3.13 \quad \textit{Ans.}$$

4-78



$$4F \sin\theta = 3920$$

$$F = \frac{3920}{4\sin\theta}$$

In range of operation, F is maximum when $\theta = 15^\circ$

$$F_{max} = \frac{3920}{4\sin 15} = 3786 \text{ N per bar}$$

$$P_{cr} = n_d F_{max} = 2.5(3786) = 9465 \text{ N}$$

$l = 300$ mm, $h = 25$ mm

Try $b = 5$ mm: out of plane $k = (5/\sqrt{12}) = 1.443$ mm

$$\frac{l}{k} = \frac{300}{1.443} = 207.8$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{(2\pi^2)(1.4)(207)(10^9)}{380(10^6)}\right]^{1/2} = 123 \quad \therefore \text{use Euler}$$

$$P_{cr} = (25)(5)\frac{(1.4\pi^2)(207)(10^3)}{(207.8)^2} = 8280 \text{ N}$$

Try: 5.5 mm: $k = 5.5/\sqrt{12} = 1.588$ mm

$$\frac{l}{k} = \frac{300}{1.588} = 189$$

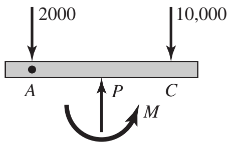
$$P_{cr} = 25(5.5)\frac{(1.4\pi^2)(207)(10^3)}{189^2} = 11\,010 \text{ N}$$

Use 25 × 5.5 mm bars *Ans.* The factor of safety is thus

$$n = \frac{11\,010}{3786} = 2.91 \quad Ans.$$

4-79
$$\sum F = 0 = 2000 + 10\,000 - P \quad \Rightarrow \quad P = 12\,000 \text{ lbf} \quad Ans.$$

$$\sum M_A = 12\,000\left(\frac{5.68}{2}\right) - 10\,000(5.68) + M = 0$$



$$M = 22\,720 \text{ lbf} \cdot \text{in}$$

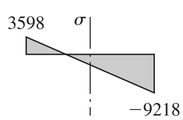
$$e = \frac{M}{P} = \frac{22}{12}\left(\frac{720}{000}\right) = 1.893 \text{ in} \quad Ans.$$

From Table A-8, $A = 4.271\ \text{in}^2$, $I = 7.090\ \text{in}^4$

$$k^2 = \frac{I}{A} = \frac{7.090}{4.271} = 1.66\ \text{in}^2$$

$$\sigma_c = -\frac{12\,000}{4.271}\left[1 + \frac{1.893(2)}{1.66}\right] = -9218 \text{ psi} \quad Ans.$$

$$\sigma_t = -\frac{12\,000}{4.271}\left[1 - \frac{1.893(2)}{1.66}\right] = 3598 \text{ psi}$$

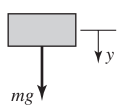


4-80 This is a design problem so the solutions will differ.

4-81 For free fall with $y \le h$

$$\sum F_y - m\ddot{y} = 0$$

$$mg - m\ddot{y} = 0, \quad \text{so } \ddot{y} = g$$

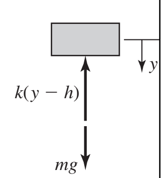


Using $y = a + bt + ct^2$, we have at $t = 0$, $y = 0$, and $\dot{y} = 0$, and so $a = 0$, $b = 0$, and $c = g/2$. Thus

$$y = \frac{1}{2}gt^2 \quad \text{and} \quad \dot{y} = gt \quad \text{for } y \le h$$

At impact, $y = h$, $t = (2h/g)^{1/2}$, and $v_0 = (2gh)^{1/2}$

After contact, the differential equatioin (D.E.) is



$$mg - k(y - h) - m\ddot{y} = 0 \qquad \text{for } y > h$$

Now let $x = y - h$; then $\dot{x} = \dot{y}$ and $\ddot{x} = \ddot{y}$. So the D.E. is $\ddot{x} + (k/m)x = g$ with solution $\omega = (k/m)^{1/2}$ and

$$x = A\cos\omega t' + B\sin\omega t' + \frac{mg}{k}$$

At contact, $t' = 0$, $x = 0$, and $\dot{x} = v_0$. Evaluating A and B then yields

$$x = -\frac{mg}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{mg}{k}$$

or

$$y = -\frac{W}{k}\cos\omega t' + \frac{v_0}{\omega}\sin\omega t' + \frac{W}{k} + h$$

and

$$\dot{y} = \frac{W\omega}{k}\sin\omega t' + v_0\cos\omega t'$$

To find $y_{\max}$ set $\dot{y} = 0$. Solving gives

$$\tan\omega t' = -\frac{v_0 k}{W\omega}$$

or

$$(\omega t')^* = \tan^{-1}\left(-\frac{v_0 k}{W\omega}\right)$$

The first value of $(\omega t')^*$ is a minimum and negative. So add π radians to it to find the maximum.

Numerical example: $h = 1$ in, $W = 30$ lbf, $k = 100$ lbf/in. Then

$$\omega = (k/m)^{1/2} = [100(386)/30]^{1/2} = 35.87 \text{ rad/s}$$
$$W/k = 30/100 = 0.3$$
$$v_0 = (2gh)^{1/2} = [2(386)(1)]^{1/2} = 27.78 \text{ in/s}$$

Then

$$y = -0.3\cos 35.87t' + \frac{27.78}{35.87}\sin 35.87t' + 0.3 + 1$$

For $y_{\max}$

$$\tan\omega t' = -\frac{v_0 k}{W\omega} = -\frac{27.78(100)}{30(35.87)} = -2.58$$
$$(\omega t')^* = -1.20 \text{ rad (minimum)}$$
$$(\omega t')^* = -1.20 + \pi = 1.940 \text{ (maximum)}$$

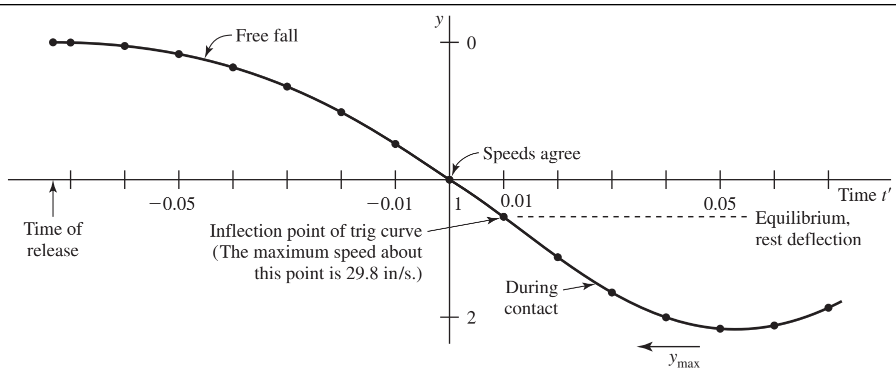
Then $t'^* = 1.940/35.87 = 0.0541$ s. This means that the spring bottoms out at t'^* seconds. Then $(\omega t')^* = 35.87(0.0541) = 1.94$ rad

So

$$y_{\max} = -0.3\cos 1.94 + \frac{27.78}{35.87}\sin 1.94 + 0.3 + 1 = 2.130 \text{ in} \quad \textit{Ans.}$$

The maximum spring force is $F_{\max} = k(y_{\max} - h) = 100(2.130 - 1) = 113$ lbf *Ans.*

The action is illustrated by the graph below. *Applications:* Impact, such as a dropped package or a pogo stick with a passive rider. The idea has also been used for a one-legged robotic walking machine.

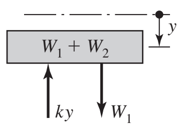


4-82 Choose $t' = 0$ at the instant of impact. At this instant, $v_1 = (2gh)^{1/2}$. Using momentum, $m_1 v_1 = m_2 v_2$. Thus

$$\frac{W_1}{g}(2gh)^{1/2} = \frac{W_1 + W_2}{g} v_2$$

$$v_2 = \frac{W_1 (2gh)^{1/2}}{W_1 + W_2}$$

Therefore at $t' = 0$, $y = 0$, and $\dot{y} = v_2$



Let $W = W_1 + W_2$

Because the spring force at $y = 0$ includes a reaction to W_2, the D.E. is

$$\frac{W}{g}\ddot{y} = -ky + W_1$$

With $\omega = (kg/W)^{1/2}$ the solution is

$$y = A\cos\omega t' + B\sin\omega t' + W_1/k$$

$$\dot{y} = -A\omega\sin\omega t' + B\omega\cos\omega t'$$

At $t' = 0$, $y = 0 \Rightarrow A = -W_1/k$

At $t' = 0$, $\dot{y} = v_2 \Rightarrow v_2 = B\omega$

Then

$$B = \frac{v_2}{\omega} = \frac{W_1(2gh)^{1/2}}{(W_1 + W_2)[kg/(W_1 + W_2)]^{1/2}}$$

We now have

$$y = -\frac{W_1}{k}\cos\omega t' + W_1\left[\frac{2h}{k(W_1 + W_2)}\right]^{1/2}\sin\omega t' + \frac{W_1}{k}$$

Transforming gives

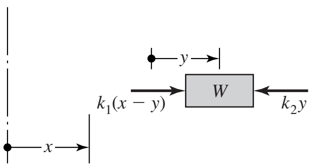
$$y = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2}\cos(\omega t' - \phi) + \frac{W_1}{k}$$

where ϕ is a phase angle. The maximum deflection of W_2 and the maximum spring force are thus

$$y_{max} = \frac{W_1}{k}\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2} + \frac{W_1}{k} \quad Ans.$$

$$F_{max} = ky_{max} + W_2 = W_1\left(\frac{2hk}{W_1 + W_2} + 1\right)^{1/2} + W_1 + W_2 \quad Ans.$$

4-83 Assume $x > y$ to get a free-body diagram.



Then

$$\frac{W}{g}\ddot{y} = k_1(x - y) - k_2 y$$

A particular solution for $x = a$ is

$$y = \frac{k_1 a}{k_1 + k_2}$$

Then the complementary plus the particular solution is

$$y = A\cos\omega t + B\sin\omega t + \frac{k_1 a}{k_1 + k_2}$$

where $$\omega = \left[\frac{(k_1 + k_2)g}{W}\right]^{1/2}$$

At $t = 0$, $y = 0$, and $\dot{y} = 0$. Therefore $B = 0$ and

$$A = -\frac{k_1 a}{k_1 + k_2}$$

Substituting,

$$y = \frac{k_1 a}{k_1 + k_2}(1 - \cos\omega t)$$

Since y is maximum when the cosine is -1

$$y_{max} = \frac{2k_1 a}{k_1 + k_2} \quad Ans.$$