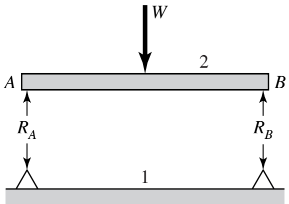
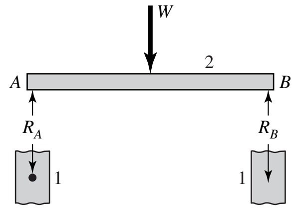
Chapter 3

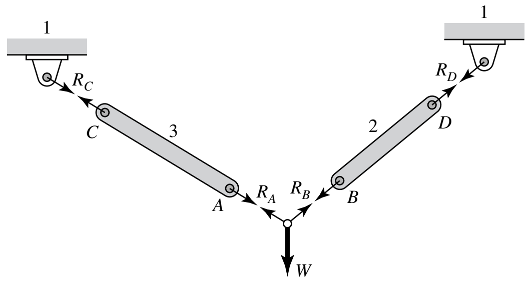
3-1



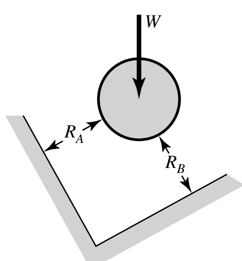
(a)



(b)



(c)



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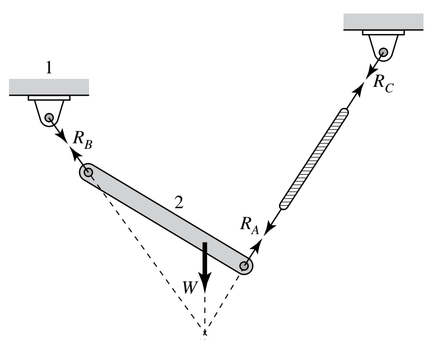
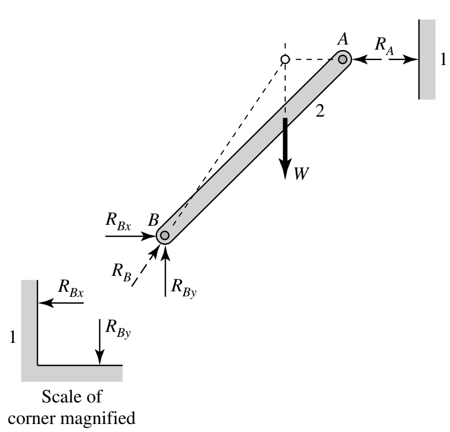


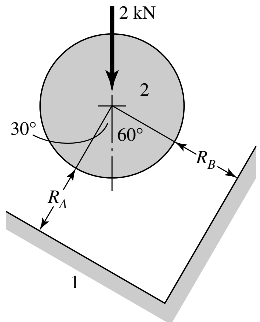
(e)

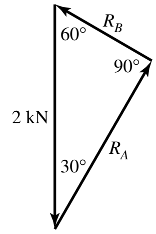


(f)

3-2

(a)

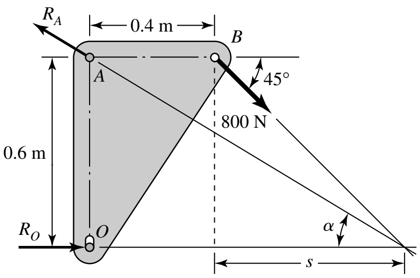




$$R_A = 2\sin 60 = 1.732 \text{ kN} \quad Ans.$$

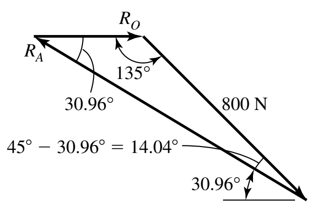
$$R_B = 2\sin 30 = 1 \text{ kN} \quad Ans.$$

(b)



$$S = 0.6 \text{ m}$$

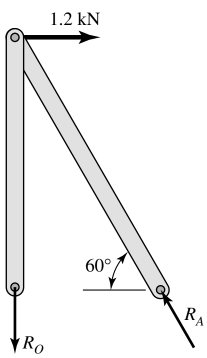
$$\alpha = \tan^{-1}\frac{0.6}{0.4+0.6} = 30.96^\circ$$

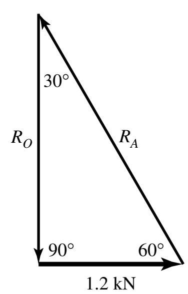


$$\frac{R_A}{\sin 135} = \frac{800}{\sin 30.96} \quad \Rightarrow \quad R_A = 1100 \text{ N} \quad Ans.$$

$$\frac{R_O}{\sin 14.04} = \frac{800}{\sin 30.96} \quad \Rightarrow \quad R_O = 377 \text{ N} \quad Ans.$$

(c)

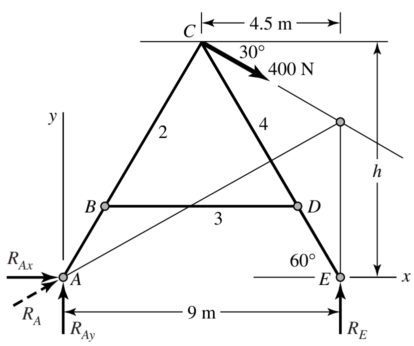




$$R_O = \frac{1.2}{\tan 30} = 2.078 \text{ kN} \quad Ans.$$

$$R_A = \frac{1.2}{\sin 30} = 2.4 \text{ kN} \quad Ans.$$

(d) Step 1: Find R_A and R_E



$$h = \frac{4.5}{\tan 30} = 7.794 \text{ m}$$

$$\circlearrowleft + \sum M_A = 0$$

$$9R_E - 7.794(400\cos 30) - 4.5(400\sin 30) = 0$$

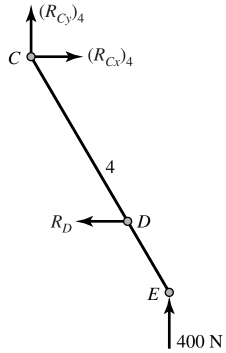
$$R_E = 400 \text{ N} \quad Ans.$$

$$\sum F_x = 0 \quad R_{Ax} + 400\cos 30 = 0 \quad \Rightarrow \quad R_{Ax} = -346.4 \text{ N}$$

$$\sum F_y = 0 \quad R_{Ay} + 400 - 400\sin 30 = 0 \quad \Rightarrow \quad R_{Ay} = -200 \text{ N}$$

$$R_A = \sqrt{346.4^2 + 200^2} = 400 \text{ N} \quad Ans.$$

Step 2: Find components of R_C on link 4 and R_D



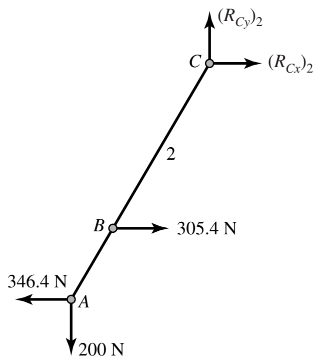
$$\circlearrowleft + \sum M_C = 0$$

$$400(4.5) - (7.794 - 1.9)R_D = 0 \quad \Rightarrow \quad R_D = 305.4\,\text{N} \quad \textit{Ans.}$$

$$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4\,\text{N}$$

$$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400\,\text{N}$$

Step 3: Find components of R_C on link 2

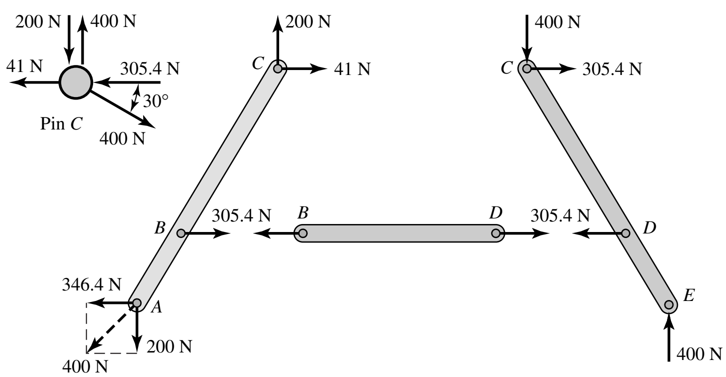


$$\sum F_x = 0$$

$$(R_{Cx})_2 + 305.4 - 346.4 = 0 \quad \Rightarrow \quad (R_{Cx})_2 = 41\,\text{N}$$

$$\sum F_y = 0$$

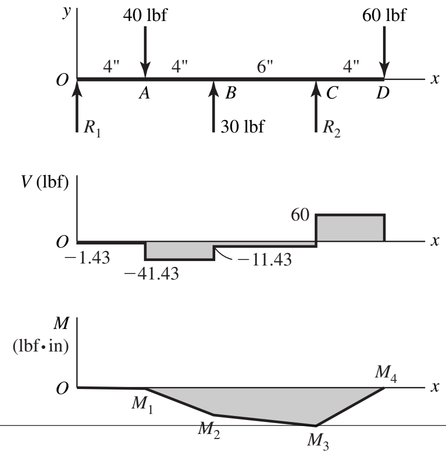
$$(R_{Cy})_2 = 200\,\text{N}$$



Ans.

3-3

(a)



$$\circlearrowleft + \sum M_0 = 0$$

$$-18(60) + 14R_2 + 8(30) - 4(40) = 0$$

$$R_2 = 71.43\text{ lbf}$$

$$\sum F_y = 0\text{: } R_1 - 40 + 30 + 71.43 - 60 = 0$$

$$R_1 = -1.43\text{ lbf}$$

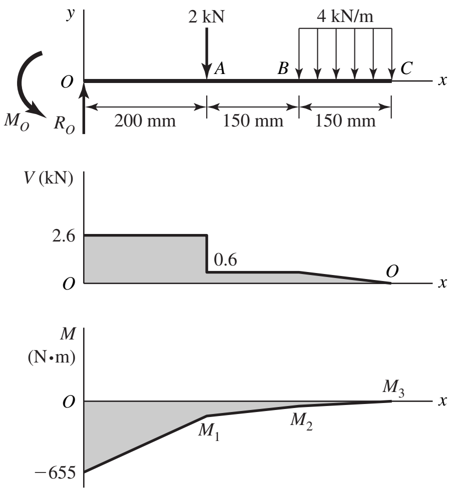
$$M_1 = -1.43(4) = -5.72\text{ lbf}\cdot\text{in}$$

$$M_2 = -5.72 - 41.43(4) = -171.44\text{ lbf}\cdot\text{in}$$

$$M_3 = -171.44 - 11.43(6) = -240\text{ lbf}\cdot\text{in}$$

$$M_4 = -240 + 60(4) = 0 \quad \text{checks!}$$

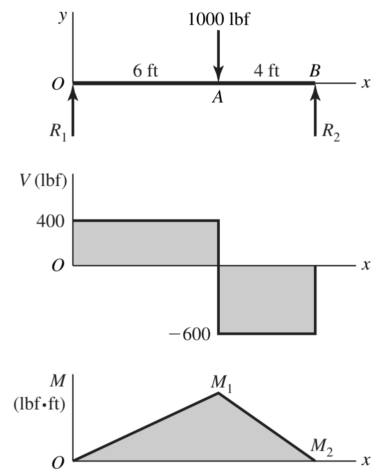
(b)



$$\sum F_y = 0$$
$$R_0 = 2 + 4(0.150) = 2.6\,\text{kN}$$
$$\sum M_0 = 0$$
$$M_0 = 2000(0.2) + 4000(0.150)(0.425)$$
$$= 655\ \text{N}\cdot\text{m}$$

$$M_1 = -655 + 2600(0.2) = -135\ \text{N}\cdot\text{m}$$
$$M_2 = -135 + 600(0.150) = -45\ \text{N}\cdot\text{m}$$
$$M_3 = -45 + \frac{1}{2}600(0.150) = 0 \quad \text{checks!}$$

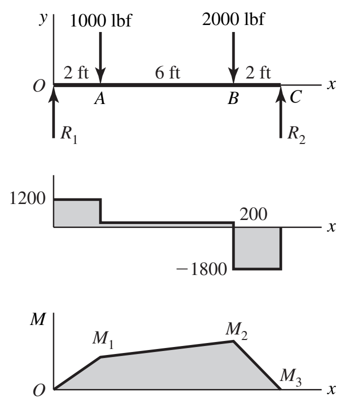
(c)



$$\sum M_0 = 0\text{: } 10R_2 - 6(1000) = 0 \quad \Rightarrow \quad R_2 = 600\ \text{lbf}$$
$$\sum F_y = 0\text{: } R_1 - 1000 + 600 = 0 \quad \Rightarrow \quad R_1 = 400\ \text{lbf}$$

$$M_1 = 400(6) = 2400\ \text{lbf}\cdot\text{ft}$$
$$M_2 = 2400 - 600(4) = 0 \quad \text{checks!}$$

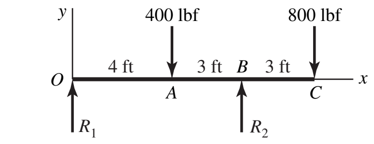
(d)



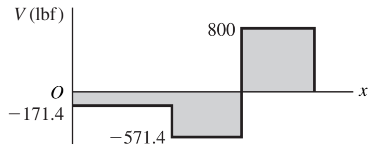
$$\circlearrowleft + \sum M_C = 0$$
$$-10R_1 + 2(2000) + 8(1000) = 0$$
$$R_1 = 1200\ \text{lbf}$$
$$\sum F_y = 0\text{: } 1200 - 1000 - 2000 + R_2 = 0$$
$$R_2 = 1800\ \text{lbf}$$

$$M_1 = 1200(2) = 2400\ \text{lbf}\cdot\text{ft}$$
$$M_2 = 2400 + 200(6) = 3600\ \text{lbf}\cdot\text{ft}$$
$$M_3 = 3600 - 1800(2) = 0 \quad \text{checks!}$$

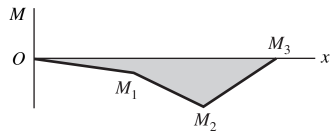
(e)



$$\circlearrowleft + \sum M_B = 0$$
$$-7R_1 + 3(400) - 3(800) = 0$$
$$R_1 = -171.4 \text{ lbf}$$

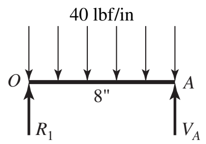


$$\sum F_y = 0: -171.4 - 400 + R_2 - 800 = 0$$
$$R_2 = 1371.4 \text{ lbf}$$

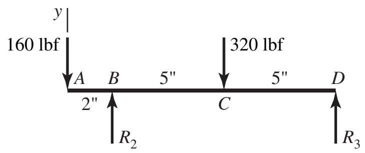


$$M_1 = -171.4(4) = -685.7 \text{ lbf} \cdot \text{ft}$$
$$M_2 = -685.7 - 571.4(3) = -2400 \text{ lbf} \cdot \text{ft}$$
$$M_3 = -2400 + 800(3) = 0 \quad \text{checks!}$$

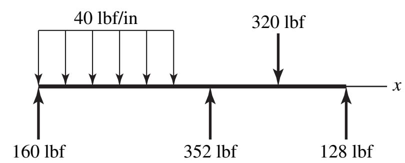
(f) Break at A



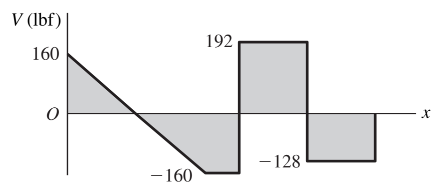
$$R_1 = V_A = \frac{1}{2}40(8) = 160 \text{ lbf}$$

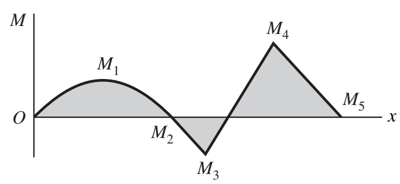


$$\circlearrowleft + \sum M_D = 0$$
$$12(160) - 10R_2 + 320(5) = 0$$
$$R_2 = 352 \text{ lbf}$$



$$\sum F_y = 0$$
$$-160 + 352 - 320 + R_3 = 0$$
$$R_3 = 128 \text{ lbf}$$





$$M_1 = \frac{1}{2}160(4) = 320 \text{ lbf} \cdot \text{in}$$
$$M_2 = 320 - \frac{1}{2}160(4) = 0 \quad \text{checks! (hinge)}$$
$$M_3 = 0 - 160(2) = -320 \text{ lbf} \cdot \text{in}$$
$$M_4 = -320 + 192(5) = 640 \text{ lbf} \cdot \text{in}$$
$$M_5 = 640 - 128(5) = 0 \quad \text{checks!}$$

3-4

(a) $q = R_1\langle x\rangle^{-1} - 40\langle x-4\rangle^{-1} + 30\langle x-8\rangle^{-1} + R_2\langle x-14\rangle^{-1} - 60\langle x-18\rangle^{-1}$

$$V = R_1 - 40\langle x-4\rangle^0 + 30\langle x-8\rangle^0 + R_2\langle x-14\rangle^0 - 60\langle x-18\rangle^0 \quad (1)$$

$$M = R_1 x - 40\langle x-4\rangle^1 + 30\langle x-8\rangle^1 + R_2\langle x-14\rangle^1 - 60\langle x-18\rangle^1 \quad (2)$$

for $x = 18^+$ $\quad V = 0 \quad$ and $\quad M = 0 \quad$ Eqs. (1) and (2) give

$$0 = R_1 - 40 + 30 + R_2 - 60 \quad \Rightarrow \quad R_1 + R_2 = 70 \quad (3)$$

$$0 = R_1(18) - 40(14) + 30(10) + 4R_2 \quad \Rightarrow \quad 9R_1 + 2R_2 = 130 \quad (4)$$

Solve (3) and (4) simultaneously to get $R_1 = -1.43$ lbf, $R_2 = 71.43$ lbf. *Ans.*

From Eqs. (1) and (2), at $x = 0^+$, $V = R_1 = -1.43$ lbf, $M = 0$

$x = 4^+$: $\quad V = -1.43 - 40 = -41.43,\ M = -1.43x$

$x = 8^+$: $\quad V = -1.43 - 40 + 30 = -11.43$

$\quad M = -1.43(8) - 40(8-4)^1 = -171.44$

$x = 14^+$: $\quad V = -1.43 - 40 + 30 + 71.43 = 60$

$\quad M = -1.43(14) - 40(14-4) + 30(14-8) = -240.$

$x = 18^+$: $\quad V = 0, M = 0 \quad$ See curves of V and M in Prob. 3-3 solution.

(b) $q = R_0\langle x\rangle^{-1} - M_0\langle x\rangle^{-2} - 2000\langle x-0.2\rangle^{-1} - 4000\langle x-0.35\rangle^0 + 4000\langle x-0.5\rangle^0$

$$V = R_0 - M_0\langle x\rangle^{-1} - 2000\langle x-0.2\rangle^0 - 4000\langle x-0.35\rangle^1 + 4000\langle x-0.5\rangle^1 \quad (1)$$

$$M = R_0 x - M_0 - 2000\langle x-0.2\rangle^1 - 2000\langle x-0.35\rangle^2 + 2000\langle x-0.5\rangle^2 \quad (2)$$

at $x = 0.5^+$ m, $V = M = 0$, Eqs. (1) and (2) give

$$R_0 - 2000 - 4000(0.5 - 0.35) = 0 \quad \Rightarrow \quad R_1 = 2600\,\text{N} = 2.6\ \text{kN} \quad \textit{Ans.}$$

$$R_0(0.5) - M_0 - 2000(0.5-0.2) - 2000(0.5-0.35)^2 = 0$$

with $R_0 = 2600$ N, $M_0 = 655$ N · m *Ans.*

With R_0 and M_0, Eqs. (1) and (2) give the same V and M curves as Prob. 3-3 (note for V, $M_0\langle x\rangle^{-1}$ has no physical meaning).

(c) $q = R_1\langle x\rangle^{-1} - 1000\langle x-6\rangle^{-1} + R_2\langle x-10\rangle^{-1}$

$$V = R_1 - 1000\langle x-6\rangle^0 + R_2\langle x-10\rangle^0 \quad (1)$$

$$M = R_1 x - 1000\langle x-6\rangle^1 + R_2\langle x-10\rangle^1 \quad (2)$$

at $x = 10^+$ ft, $\quad V = M = 0$, Eqs. (1) and (2) give

$$R_1 - 1000 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 1000$$

$$10R_1 - 1000(10-6) = 0 \quad \Rightarrow \quad R_1 = 400\,\text{lbf}, \quad R_2 = 1000 - 400 = 600\,\text{lbf}$$

$0 \le x \le 6$: $\quad V = 400$ lbf, $\quad M = 400x$

$6 \le x \le 10$: $\quad V = 400 - 1000(x-6)^0 = 600$ lbf

$\quad M = 400x - 1000(x-6) = 6000 - 600x$

See curves of Prob. 3-3 solution.

(d) $q = R_1\langle x\rangle^{-1} - 1000\langle x-2\rangle^{-1} - 2000\langle x-8\rangle^{-1} + R_2\langle x-10\rangle^{-1}$

$$V = R_1 - 1000\langle x-2\rangle^0 - 2000\langle x-8\rangle^0 + R_2\langle x-10\rangle^0 \quad (1)$$

$$M = R_1 x - 1000\langle x-2\rangle^1 - 2000\langle x-8\rangle^1 + R_2\langle x-10\rangle^1 \quad (2)$$

At $x = 10^+$, $V = M = 0$ from Eqs. (1) and (2)

$$R_1 - 1000 - 2000 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 3000$$

$$10R_1 - 1000(10-2) - 2000(10-8) = 0 \quad \Rightarrow \quad R_1 = 1200\,\text{lbf},$$

$$R_2 = 3000 - 1200 = 1800\,\text{lbf}$$

$0 \le x \le 2$: $V = 1200\,\text{lbf}$, $M = 1200x\ \text{lbf}\cdot\text{ft}$

$2 \le x \le 8$: $V = 1200 - 1000 = 200\,\text{lbf}$

$M = 1200x - 1000(x-2) = 200x + 2000\ \text{lbf}\cdot\text{ft}$

$8 \le x \le 10$: $V = 1200 - 1000 - 2000 = -1800\,\text{lbf}$

$M = 1200x - 1000(x-2) - 2000(x-8) = -1800x + 18\,000\ \text{lbf}\cdot\text{ft}$

Plots are the same as in Prob. 3-3.

(e)

$$q = R_1\langle x\rangle^{-1} - 400\langle x-4\rangle^{-1} + R_2\langle x-7\rangle^{-1} - 800\langle x-10\rangle^{-1}$$

$$V = R_1 - 400\langle x-4\rangle^{0} + R_2\langle x-7\rangle^{0} - 800\langle x-10\rangle^{0} \quad (1)$$

$$M = R_1x - 400\langle x-4\rangle^{1} + R_2\langle x-7\rangle^{1} - 800\langle x-10\rangle^{1} \quad (2)$$

at $x = 10^+$, $V = M = 0$

$$R_1 - 400 + R_2 - 800 = 0 \quad \Rightarrow \quad R_1 + R_2 = 1200 \quad (3)$$

$$10R_1 - 400(6) + R_2(3) = 0 \quad \Rightarrow \quad 10R_1 + 3R_2 = 2400 \quad (4)$$

Solve Eqs. (3) and (4) simultaneously: $R_1 = -171.4$ lbf, $R_2 = 1371.4$ lbf

$0 \le x \le 4$: $V = -171.4\,\text{lbf}$, $M = -171.4x\ \text{lbf}\cdot\text{ft}$

$4 \le x \le 7$: $V = -171.4 - 400 = -571.4$ lbf

$M = -171.4x - 400(x-4)\ \text{lbf}\cdot\text{ft} = -571.4x + 1600$

$7 \le x \le 10$: $V = -171.4 - 400 + 1371.4 = 800\,\text{lbf}$

$M = -171.4x - 400(x-4) + 1371.4(x-7) = 800x - 8000\,\text{lbf}\cdot\text{ft}$

Plots are the same as in Prob. 3-3.

(f) $q = R_1\langle x\rangle^{-1} - 40\langle x\rangle^{0} + 40\langle x-8\rangle^{0} + R_2\langle x-10\rangle^{-1} - 320\langle x-15\rangle^{-1} + R_3\langle x-20\rangle$

$$V = R_1 - 40x + 40\langle x-8\rangle^{1} + R_2\langle x-10\rangle^{0} - 320\langle x-15\rangle^{0} + R_3\langle x-20\rangle^{0} \quad (1)$$

$$M = R_1x - 20x^2 + 20\langle x-8\rangle^{2} + R_2\langle x-10\rangle^{1} - 320\langle x-15\rangle^{1} + R_3\langle x-20\rangle^{1} \quad (2)$$

$M = 0$ at $x = 8$ in $\quad \therefore 8R_1 - 20(8)^2 = 0 \quad \Rightarrow \quad R_1 = 160$ lbf

at $x = 20^+$, V and $M = 0$

$$160 - 40(20) + 40(12) + R_2 - 320 + R_3 = 0 \quad \Rightarrow \quad R_2 + R_3 = 480$$

$$160(20) - 20(20)^2 + 20(12)^2 + 10R_2 - 320(5) = 0 \quad \Rightarrow \quad R_2 = 352\ \text{lbf}$$

$$R_3 = 480 - 352 = 128\ \text{lbf}$$

$0 \le x \le 8$: $V = 160 - 40x$ lbf, $M = 160x - 20x^2\ \text{lbf}\cdot\text{in}$

$8 \le x \le 10$: $V = 160 - 40x + 40(x-8) = -160\,\text{lbf}$,

$M = 160x - 20x^2 + 20(x-8)^2 = 1280 - 160x\ \text{lbf}\cdot\text{in}$

$10 \le x \le 15$: $V = 160 - 40x + 40(x-8) + 352 = 192\,\text{lbf}$

$$M = 160x - 20x^2 + 20(x-8) + 352(x-10) = 192x - 2240$$

$15 \le x \le 20$: $V = 160 - 40x + 40(x - 8) + 352 - 320 = -128$ lbf

$$M = 160x - 20x^2 - 20(x-8) + 352(x-10) - 320(x-15)$$
$$= -128x + 2560$$

Plots of V and M are the same as in Prob. 3-3.

3-5 Solution depends upon the beam selected.

3-6

(a) Moment at center, $x_c = (l - 2a)/2$

$$M_c = \frac{w}{2}\left[\frac{l}{2}(l - 2a) - \left(\frac{l}{2}\right)^2\right] = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

At reaction, $|M_r| = wa^2/2$

$a = 2.25$, $l = 10$ in, $w = 100$ lbf/in

$$M_c = \frac{100(10)}{2}\left(\frac{10}{4} - 2.25\right) = 125 \text{ lbf} \cdot \text{in}$$

$$M_r = \frac{100(2.25^2)}{2} = 253.1 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Minimum occurs when $M_c = |M_r|$

$$\frac{wl}{2}\left(\frac{l}{4} - a\right) = \frac{wa^2}{2} \quad \Rightarrow \quad a^2 + al - 0.25l^2 = 0$$

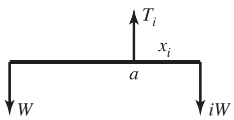
Taking the positive root

$$a = \frac{1}{2}\left[-l + \sqrt{l^2 + 4(0.25l^2)}\right] = \frac{l}{2}\left(\sqrt{2} - 1\right) = 0.2071l \quad \textit{Ans.}$$

for $l = 10$ in and $w = 100$ lbf, $M_{\min} = (100/2)[(0.2071)(10)]^2 = 214$ lbf · in

3-7 For the ith wire from bottom, from summing forces vertically

(a)



$$T_i = (i + 1)W$$

From summing moments about point a,

$$\sum M_a = W(l - x_i) - iWx_i = 0$$

Giving,

$$x_i = \frac{l}{i + 1}$$

So

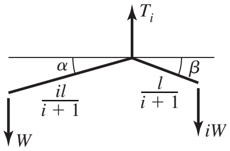
$$W = \frac{l}{1+1} = \frac{l}{2}$$

$$x = \frac{l}{2+1} = \frac{l}{3}$$

$$y = \frac{l}{3+1} = \frac{l}{4}$$

$$z = \frac{l}{4+1} = \frac{l}{5}$$

(b) With straight rigid wires, the mobile is not stable. Any perturbation can lead to all wires becoming collinear. Consider a wire of length l bent at its string support:



$$\sum M_a = 0$$

$$\sum M_a = \frac{iWl}{i+1}\cos\alpha - \frac{ilW}{i+1}\cos\beta = 0$$

$$\frac{iWl}{i+1}(\cos\alpha - \cos\beta) = 0$$

Moment vanishes when $\alpha = \beta$ for any wire. Consider a ccw rotation angle β, which makes $\alpha \rightarrow \alpha + \beta$ and $\beta \rightarrow \alpha - \beta$

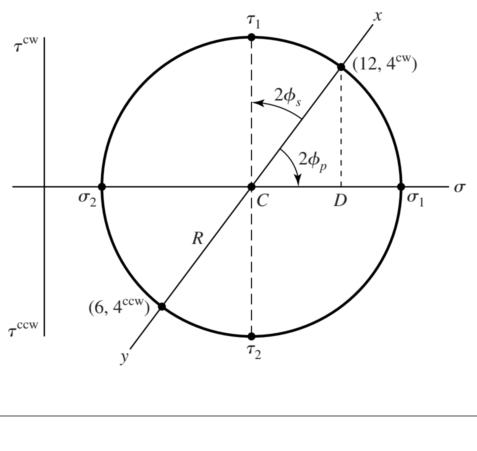
$$M_a = \frac{iWl}{i+1}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]$$

$$= \frac{2iWl}{i+1}\sin\alpha\sin\beta \doteq \frac{2iWl\beta}{i+1}\sin\alpha$$

There exists a correcting moment of opposite sense to arbitrary rotation β. An equation for an upward bend can be found by changing the sign of W. The moment will no longer be correcting. A curved, convex-upward bend of wire will produce stable equilibrium too, but the equation would change somewhat.

3-8

(a)



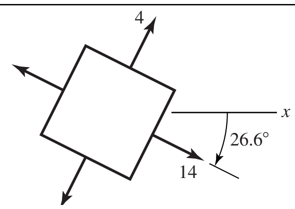
$$C = \frac{12+6}{2} = 9$$

$$CD = \frac{12-6}{2} = 3$$

$$R = \sqrt{3^2 + 4^2} = 5$$

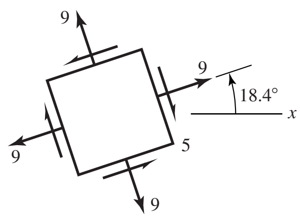
$$\sigma_1 = 5 + 9 = 14$$

$$\sigma_2 = 9 - 5 = 4$$

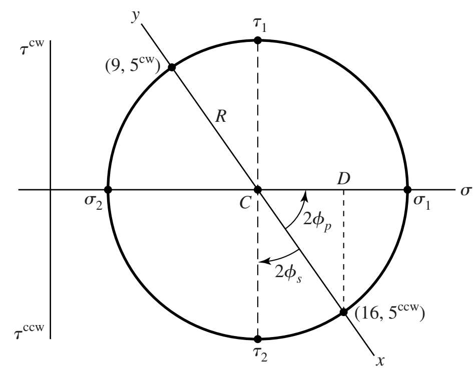


$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = 26.6^\circ \text{ cw}$$

$$\tau_1 = R = 5, \quad \phi_s = 45^\circ - 26.6^\circ = 18.4^\circ \text{ ccw}$$



(b)



$$C = \frac{9+16}{2} = 12.5$$

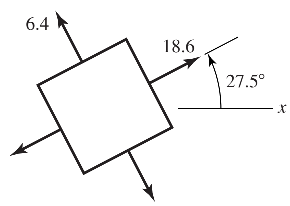
$$CD = \frac{16-9}{2} = 3.5$$

$$R = \sqrt{5^2 + 3.5^2} = 6.10$$

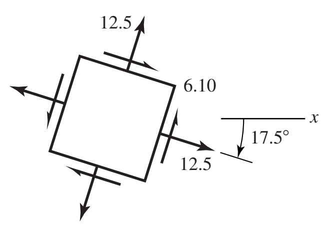
$$\sigma_1 = 6.1 + 12.5 = 18.6$$

$$\phi_p = \frac{1}{2}\tan^{-1}\frac{5}{3.5} = 27.5^\circ \text{ ccw}$$

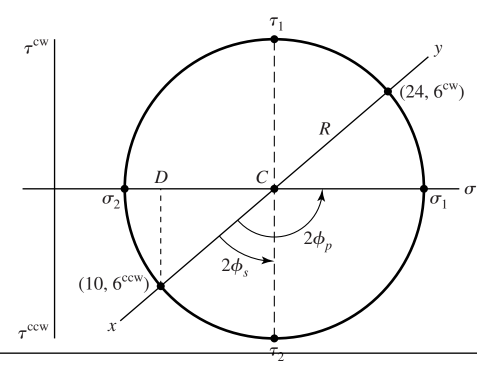
$$\sigma_2 = 12.5 - 6.1 = 6.4$$



$$\tau_1 = R = 6.10, \quad \phi_s = 45^\circ - 27.5^\circ = 17.5^\circ \text{ cw}$$



(c)



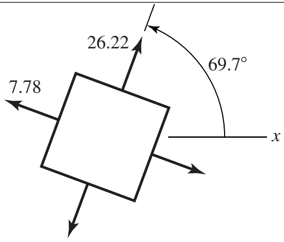
$$C = \frac{24+10}{2} = 17$$

$$CD = \frac{24-10}{2} = 7$$

$$R = \sqrt{7^2 + 6^2} = 9.22$$

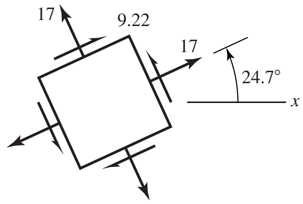
$$\sigma_1 = 17 + 9.22 = 26.22$$

$$\sigma_2 = 17 - 9.22 = 7.78$$

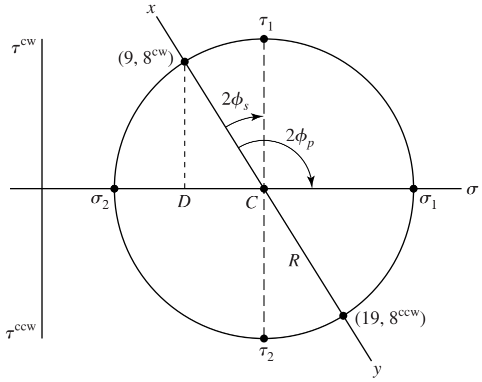


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{7}{6}\right] = 69.7^\circ \text{ ccw}$$

$$\tau_1 = R = 9.22, \quad \phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$



(d)



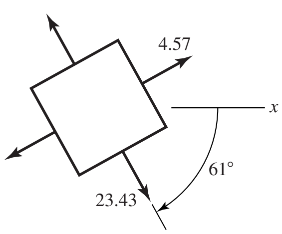
$$C = \frac{9 + 19}{2} = 14$$

$$CD = \frac{19 - 9}{2} = 5$$

$$R = \sqrt{5^2 + 8^2} = 9.434$$

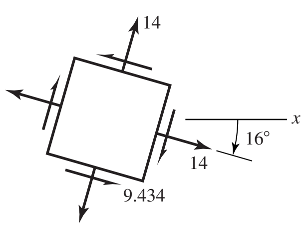
$$\sigma_1 = 14 + 9.43 = 23.43$$

$$\sigma_2 = 14 - 9.43 = 4.57$$



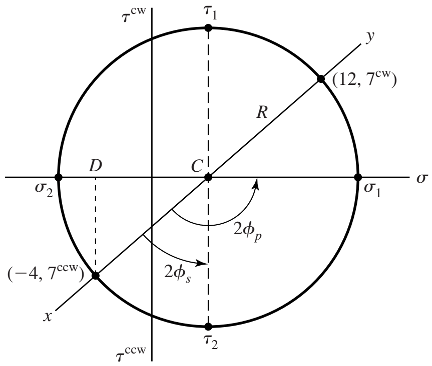
$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{5}{8}\right] = 61.0^\circ \text{ cw}$$

$$\tau_1 = R = 9.434, \quad \phi_s = 61^\circ - 45^\circ = 16^\circ \text{ cw}$$



3-9

(a)



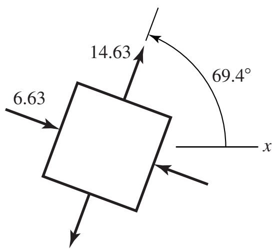
$$C = \frac{12-4}{2} = 4$$

$$CD = \frac{12+4}{2} = 8$$

$$R = \sqrt{8^2+7^2} = 10.63$$

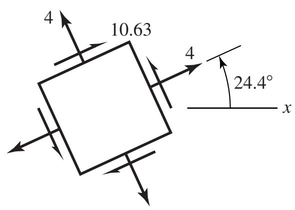
$$\sigma_1 = 4 + 10.63 = 14.63$$

$$\sigma_2 = 4 - 10.63 = -6.63$$

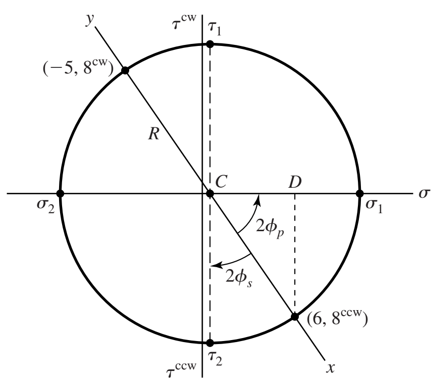


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{8}{7}\right] = 69.4^\circ \text{ ccw}$$

$$\tau_1 = R = 10.63, \quad \phi_s = 69.4^\circ - 45^\circ = 24.4^\circ \text{ ccw}$$



(b)



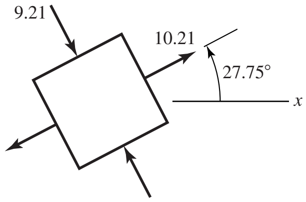
$$C = \frac{6-5}{2} = 0.5$$

$$CD = \frac{6+5}{2} = 5.5$$

$$R = \sqrt{5.5^2+8^2} = 9.71$$

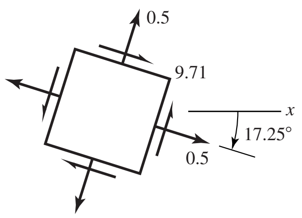
$$\sigma_1 = 0.5 + 9.71 = 10.21$$

$$\sigma_2 = 0.5 - 9.71 = -9.21$$

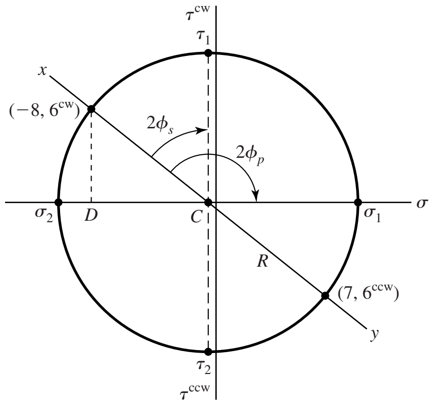


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{8}{5.5} = 27.75^\circ \text{ ccw}$$

$$\tau_1 = R = 9.71, \quad \phi_s = 45^\circ - 27.75^\circ = 17.25^\circ \text{ cw}$$



(c)



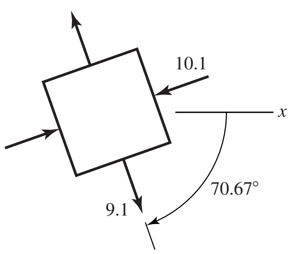
$$C = \frac{-8+7}{2} = -0.5$$

$$CD = \frac{8+7}{2} = 7.5$$

$$R = \sqrt{7.5^2 + 6^2} = 9.60$$

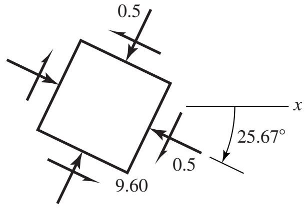
$$\sigma_1 = 9.60 - 0.5 = 9.10$$

$$\sigma_2 = -0.5 - 9.6 = -10.1$$

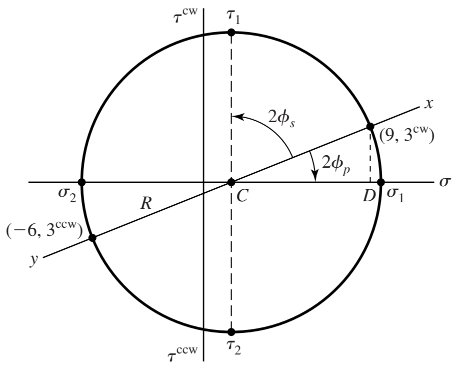


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{7.5}{6}\right] = 70.67^\circ \text{ cw}$$

$$\tau_1 = R = 9.60, \quad \phi_s = 70.67^\circ - 45^\circ = 25.67^\circ \text{ cw}$$



(d)



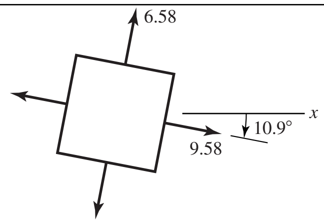
$$C = \frac{9-6}{2} = 1.5$$

$$CD = \frac{9+6}{2} = 7.5$$

$$R = \sqrt{7.5^2 + 3^2} = 8.078$$

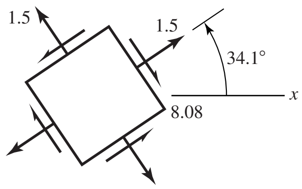
$$\sigma_1 = 1.5 + 8.078 = 9.58$$

$$\sigma_2 = 1.5 - 8.078 = -6.58$$



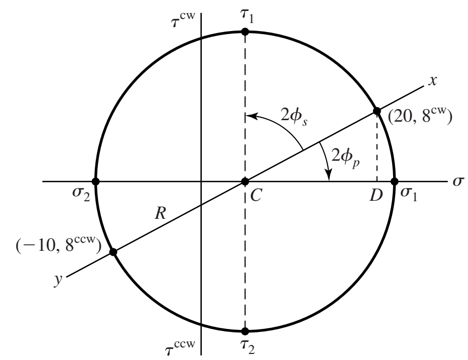
$$\phi_p = \frac{1}{2}\tan^{-1}\frac{3}{7.5} = 10.9^\circ \text{ cw}$$

$$\tau_1 = R = 8.078, \quad \phi_s = 45^\circ - 10.9^\circ = 34.1^\circ \text{ ccw}$$



3-10

(a)



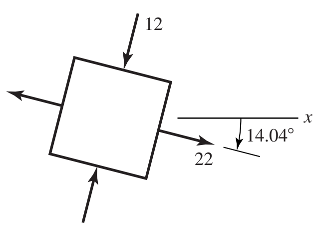
$$C = \frac{20 - 10}{2} = 5$$

$$CD = \frac{20 + 10}{2} = 15$$

$$R = \sqrt{15^2 + 8^2} = 17$$

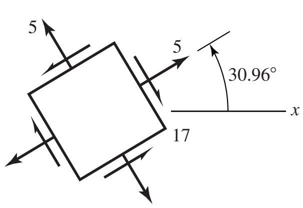
$$\sigma_1 = 5 + 17 = 22$$

$$\sigma_2 = 5 - 17 = -12$$

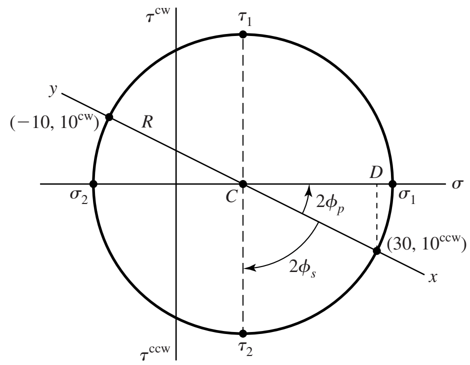


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{8}{15} = 14.04^\circ \text{ cw}$$

$$\tau_1 = R = 17, \quad \phi_s = 45^\circ - 14.04^\circ = 30.96^\circ \text{ ccw}$$



(b)



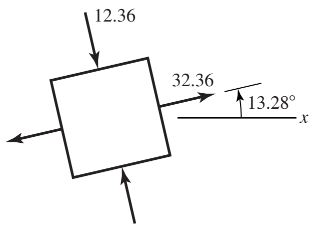
$$C = \frac{30 - 10}{2} = 10$$

$$CD = \frac{30 + 10}{2} = 20$$

$$R = \sqrt{20^2 + 10^2} = 22.36$$

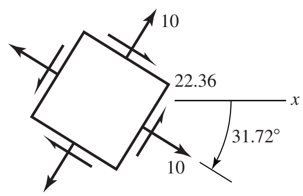
$$\sigma_1 = 10 + 22.36 = 32.36$$

$$\sigma_2 = 10 - 22.36 = -12.36$$

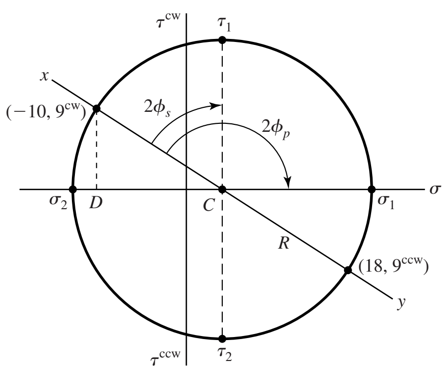


$$\phi_p = \frac{1}{2}\tan^{-1}\frac{10}{20} = 13.28^\circ \text{ ccw}$$

$$\tau_1 = R = 22.36, \quad \phi_s = 45^\circ - 13.28^\circ = 31.72^\circ \text{ cw}$$



(c)



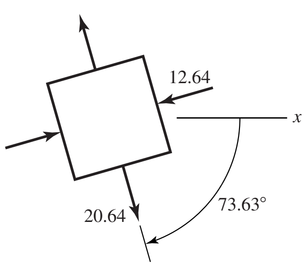
$$C = \frac{-10 + 18}{2} = 4$$

$$CD = \frac{10 + 18}{2} = 14$$

$$R = \sqrt{14^2 + 9^2} = 16.64$$

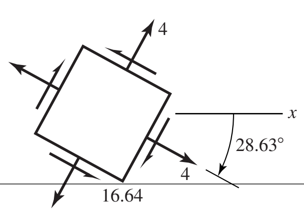
$$\sigma_1 = 4 + 16.64 = 20.64$$

$$\sigma_2 = 4 - 16.64 = -12.64$$

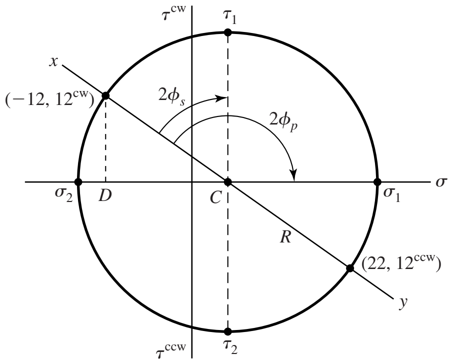


$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{14}{9}\right] = 73.63^\circ \text{ cw}$$

$$\tau_1 = R = 16.64, \quad \phi_s = 73.63^\circ - 45^\circ = 28.63^\circ \text{ cw}$$



(d)



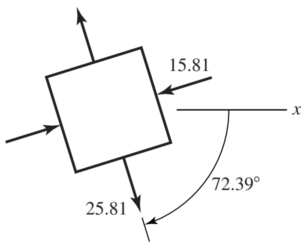
$$C = \frac{-12+22}{2} = 5$$

$$CD = \frac{12+22}{2} = 17$$

$$R = \sqrt{17^2 + 12^2} = 20.81$$

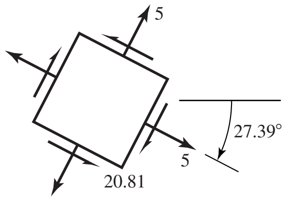
$$\sigma_1 = 5 + 20.81 = 25.81$$

$$\sigma_2 = 5 - 20.81 = -15.81$$



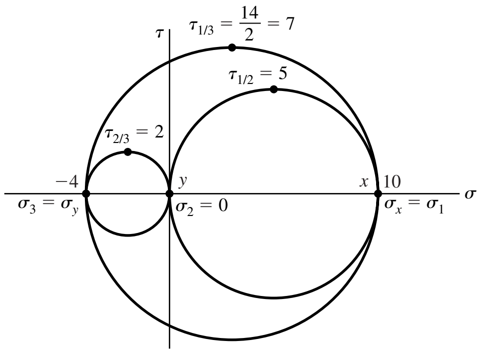
$$\phi_p = \frac{1}{2}\left[90 + \tan^{-1}\frac{17}{12}\right] = 72.39^\circ \text{ cw}$$

$$\tau_1 = R = 20.81, \quad \phi_s = 72.39^\circ - 45^\circ = 27.39^\circ \text{ cw}$$

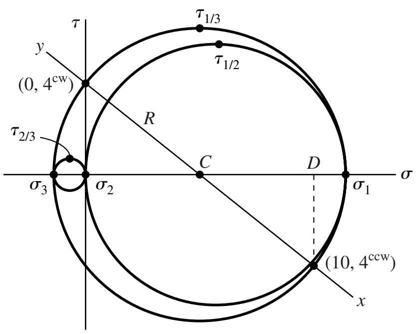


3-11

(a)



(b)



$$C = \frac{0+10}{2} = 5$$

$$CD = \frac{10-0}{2} = 5$$

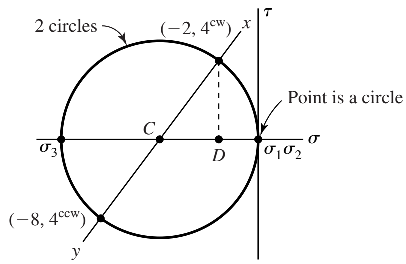
$$R = \sqrt{5^2 + 4^2} = 6.40$$

$$\sigma_1 = 5 + 6.40 = 11.40$$

$$\sigma_2 = 0, \quad \sigma_3 = 5 - 6.40 = -1.40$$

$$\tau_{1/3} = R = 6.40, \quad \tau_{1/2} = \frac{11.40}{2} = 5.70, \quad \tau_{2/3} = \frac{1.40}{2} = 0.70$$

(c)



$$C = \frac{-2-8}{2} = -5$$

$$CD = \frac{8-2}{2} = 3$$

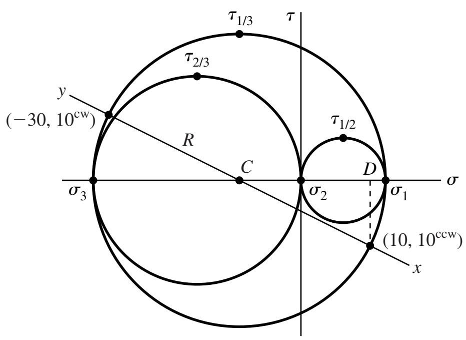
$$R = \sqrt{3^2 + 4^2} = 5$$

$$\sigma_1 = -5 + 5 = 0, \quad \sigma_2 = 0$$

$$\sigma_3 = -5 - 5 = -10$$

$$\tau_{1/3} = \frac{10}{2} = 5, \quad \tau_{1/2} = 0, \quad \tau_{2/3} = 5$$

(d)



$$C = \frac{10-30}{2} = -10$$

$$CD = \frac{10+30}{2} = 20$$

$$R = \sqrt{20^2 + 10^2} = 22.36$$

$$\sigma_1 = -10 + 22.36 = 12.36$$

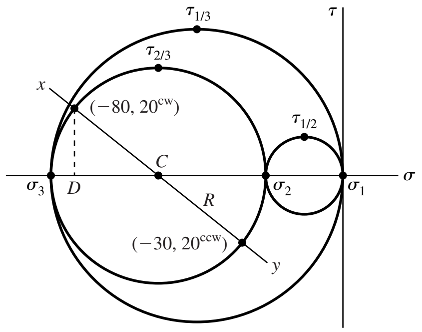
$$\sigma_2 = 0$$

$$\sigma_3 = -10 - 22.36 = -32.36$$

$$\tau_{1/3} = 22.36, \quad \tau_{1/2} = \frac{12.36}{2} = 6.18, \quad \tau_{2/3} = \frac{32.36}{2} = 16.18$$

3-12

(a)



$$C = \frac{-80-30}{2} = -55$$

$$CD = \frac{80-30}{2} = 25$$

$$R = \sqrt{25^2 + 20^2} = 32.02$$

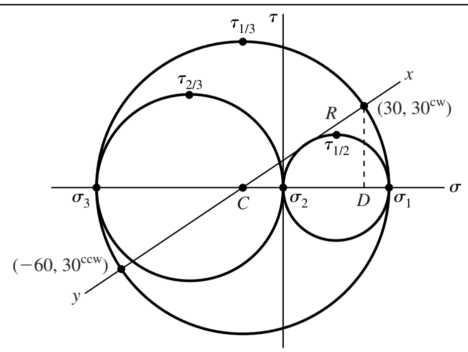
$$\sigma_1 = 0$$

$$\sigma_2 = -55 + 32.02 = -22.98 = -23.0$$

$$\sigma_3 = -55 - 32.0 = -87.0$$

$$\tau_{1/2} = \frac{23}{2} = 11.5, \quad \tau_{2/3} = 32.0, \quad \tau_{1/3} = \frac{87}{2} = 43.5$$

(b)



$$C = \frac{30 - 60}{2} = -15$$

$$CD = \frac{60 + 30}{2} = 45$$

$$R = \sqrt{45^2 + 30^2} = 54.1$$

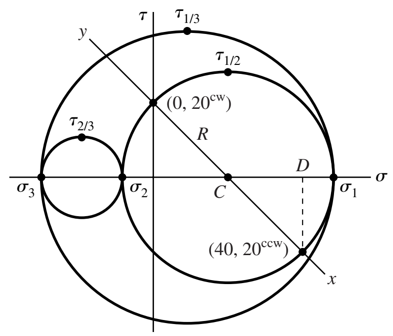
$$\sigma_1 = -15 + 54.1 = 39.1$$

$$\sigma_2 = 0$$

$$\sigma_3 = -15 - 54.1 = -69.1$$

$$\tau_{1/3} = \frac{39.1 + 69.1}{2} = 54.1, \quad \tau_{1/2} = \frac{39.1}{2} = 19.6, \quad \tau_{2/3} = \frac{69.1}{2} = 34.6$$

(c)



$$C = \frac{40 + 0}{2} = 20$$

$$CD = \frac{40 - 0}{2} = 20$$

$$R = \sqrt{20^2 + 20^2} = 28.3$$

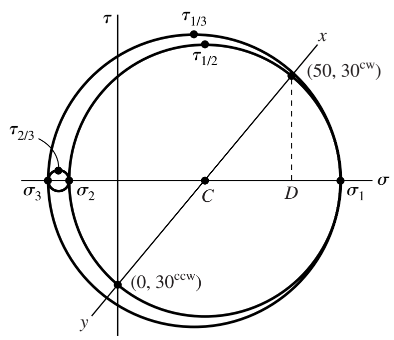
$$\sigma_1 = 20 + 28.3 = 48.3$$

$$\sigma_2 = 20 - 28.3 = -8.3$$

$$\sigma_3 = \sigma_z = -30$$

$$\tau_{1/3} = \frac{48.3 + 30}{2} = 39.1, \quad \tau_{1/2} = 28.3, \quad \tau_{2/3} = \frac{30 - 8.3}{2} = 10.9$$

(d)



$$C = \frac{50}{2} = 25$$

$$CD = \frac{50}{2} = 25$$

$$R = \sqrt{25^2 + 30^2} = 39.1$$

$$\sigma_1 = 25 + 39.1 = 64.1$$

$$\sigma_2 = 25 - 39.1 = -14.1$$

$$\sigma_3 = \sigma_z = -20$$

$$\tau_{1/3} = \frac{64.1 + 20}{2} = 42.1, \quad \tau_{1/2} = 39.1, \quad \tau_{2/3} = \frac{20 - 14.1}{2} = 2.95$$

3-13

$$\sigma = \frac{F}{A} = \frac{2000}{(\pi/4)(0.5^2)} = 10\,190 \text{ psi} = 10.19 \text{ kpsi} \quad Ans.$$

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 10\,190\frac{72}{30(10^6)} = 0.024\,46 \text{ in} \quad Ans.$$

$$\epsilon_1 = \frac{\delta}{L} = \frac{0.024\,46}{72} = 340(10^{-6}) = 340\mu \quad Ans.$$

From Table A-5, $\nu = 0.292$

$$\epsilon_2 = -\nu\epsilon_1 = -0.292(340) = -99.3\mu \quad Ans.$$

$$\Delta d = \epsilon_2 d = -99.3(10^{-6})(0.5) = -49.6(10^{-6}) \text{ in} \quad Ans.$$

3-14 From Table A-5, $E = 71.7$ GPa

$$\delta = \sigma\frac{L}{E} = 135(10^6)\frac{3}{71.7(10^9)} = 5.65(10^{-3}) \text{ m} = 5.65 \text{ mm} \quad Ans.$$

3-15 With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simultaneously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\epsilon_x & -\nu \\ E\epsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}} = \frac{E\epsilon_x + \nu E\epsilon_y}{1-\nu^2} = \frac{E(\epsilon_x + \nu\epsilon_y)}{1-\nu^2}$$

Likewise,

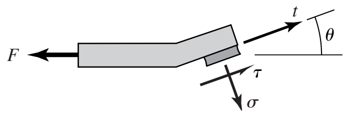
$$\sigma_y = \frac{E(\epsilon_y + \nu\epsilon_x)}{1-\nu^2}$$

From Table A-5, $E = 207$ GPa and $\nu = 0.292$. Thus,

$$\sigma_x = \frac{E(\epsilon_x + \nu\epsilon_y)}{1-\nu^2} = \frac{207(10^9)[0.0021 + 0.292(-0.000\,67)]}{1-0.292^2}(10^{-6}) = 431 \text{ MPa} \quad Ans.$$

$$\sigma_y = \frac{207(10^9)[-0.000\,67 + 0.292(0.0021)]}{1-0.292^2}(10^{-6}) = -12.9 \text{ MPa} \quad Ans.$$

3-16 The engineer has assumed the stress to be uniform. That is,



$$\sum F_t = -F\cos\theta + \tau A = 0 \quad \Rightarrow \quad \tau = \frac{F}{A}\cos\theta$$

When failure occurs in shear

$$S_{su} = \frac{F}{A}\cos\theta$$

The uniform stress assumption is common practice but is not exact. If interested in the details, see p. 570 of 6th edition.

3-17 From Eq. (3-15)

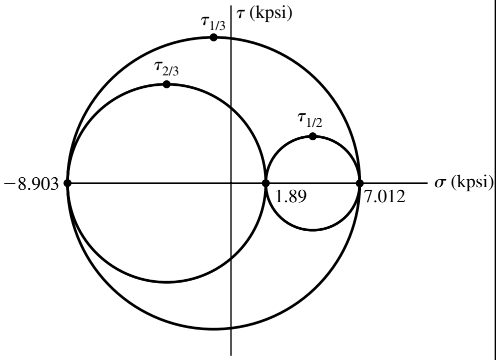
$$\sigma^3 - (-2+6-4)\sigma^2 + [-2(6) + (-2)(-4) + 6(-4) - 3^2 - 2^2 - (-5)^2]\sigma$$
$$- [-2(6)(-4) + 2(3)(2)(-5) - (-2)(2)^2 - 6(-5)^2 - (-4)(3)^2] = 0$$
$$\sigma^3 - 66\sigma + 118 = 0$$

Roots are: 7.012, 1.89, −8.903 kpsi *Ans.*

$$\tau_{1/2} = \frac{7.012 - 1.89}{2} = 2.56 \text{ kpsi}$$

$$\tau_{2/3} = \frac{8.903 + 1.89}{2} = 5.40 \text{ kpsi}$$

$$\tau_{max} = \tau_{1/3} = \frac{8.903 + 7.012}{2} = 7.96 \text{ kpsi} \quad \textit{Ans.}$$



Note: For Probs. 3-17 to 3-19, one can also find the eigenvalues of the matrix

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z \end{bmatrix}$$

for the principal stresses

3-18 From Eq. (3-15)

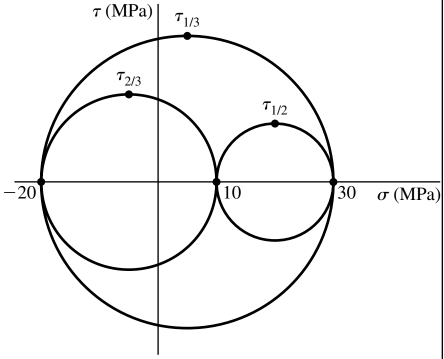
$$\sigma^3 - (10+0+10)\sigma^2 + \left[10(0) + 10(10) + 0(10) - 20^2 - \left(-10\sqrt{2}\right)^2 - 0^2\right]\sigma$$
$$- \left[10(0)(10) + 2(20)\left(-10\sqrt{2}\right)(0) - 10\left(-10\sqrt{2}\right)^2 - 0(0)^2 - 10(20)^2\right] = 0$$
$$\sigma^3 - 20\sigma^2 - 500\sigma + 6000 = 0$$

Roots are: 30, 10, −20 MPa *Ans.*

$$\tau_{1/2} = \frac{30 - 10}{2} = 10 \text{ MPa}$$

$$\tau_{2/3} = \frac{10 + 20}{2} = 15 \text{ MPa}$$

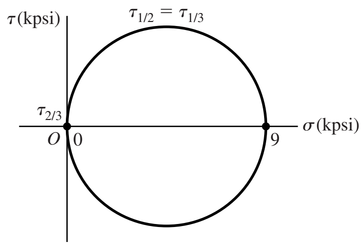
$$\tau_{max} = \tau_{1/3} = \frac{30 + 20}{2} = 25 \text{ MPa} \quad \textit{Ans.}$$



3-19 From Eq. (3-15)

$$\sigma^3 - (1+4+4)\sigma^2 + [1(4) + 1(4) + 4(4) - 2^2 - (-4)^2 - (-2)^2]\sigma$$
$$-[1(4)(4) + 2(2)(-4)(-2) - 1(-4)^2 - 4(-2)^2 - 4(2)^2] = 0$$
$$\sigma^3 - 9\sigma^2 = 0$$

Roots are: 9, 0, 0 kpsi



$$\tau_{2/3} = 0, \quad \tau_{1/2} = \tau_{1/3} = \tau_{\max} = \frac{9}{2} = 4.5 \text{ kpsi} \quad \textit{Ans.}$$

3-20

(a) $R_1 = \dfrac{c}{l}F \quad M_{\max} = R_1 a = \dfrac{ac}{l}F$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{ac}{l}F \quad \Rightarrow \quad F = \frac{\sigma bh^2 l}{6ac} \quad \textit{Ans.}$$

(b) $\dfrac{F_m}{F} = \dfrac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2(l_m/l)}{(a_m/a)(c_m/c)} = \dfrac{1(s)(s)^2(s)}{(s)(s)} = s^2 \quad \textit{Ans.}$

For equal stress, the model load varies by the square of the scale factor.

3-21

$$R_1 = \frac{wl}{2}, \quad M_{\max}|_{x=l/2} = \frac{w}{2}\frac{l}{2}\left(l - \frac{l}{2}\right) = \frac{wl^2}{8}$$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{wl^2}{8} = \frac{3Wl}{4bh^2} \quad \Rightarrow \quad W = \frac{4}{3}\frac{\sigma bh^2}{l} \quad \textit{Ans.}$$

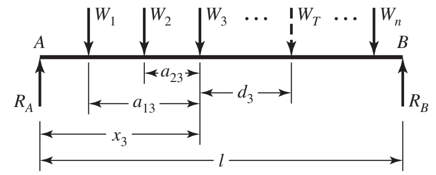
$$\frac{W_m}{W} = \frac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2}{l_m/l} = \frac{1(s)(s)^2}{s} = s^2 \quad \textit{Ans.}$$

$$\frac{w_m l_m}{wl} = s^2 \quad \Rightarrow \quad \frac{w_m}{w} = \frac{s^2}{s} = s \quad \textit{Ans.}$$

For equal stress, the model load w varies linearily with the scale factor.

3-22

(a) Can solve by iteration *or* derive equations for the general case.



Find maximum moment under wheel W_3

$W_T = \sum W$ at centroid of W's

$$R_A = \frac{l - x_3 - d_3}{l} W_T$$

Under wheel 3

$$M_3 = R_A x_3 - W_1 a_{13} - W_2 a_{23} = \frac{(l - x_3 - d_3)}{l} W_T x_3 - W_1 a_{13} - W_2 a_{23}$$

For maximum, $\dfrac{dM_3}{dx_3} = 0 = (l - d_3 - 2x_3)\dfrac{W_T}{l} \quad \Rightarrow \quad x_3 = \dfrac{l - d_3}{2}$

substitute into M, $\Rightarrow \quad M_3 = \dfrac{(l - d_3)^2}{4l} W_T - W_1 a_{13} - W_2 a_{23}$

This means the midpoint of d_3 intersects the midpoint of the beam

For wheel i $\quad x_i = \dfrac{l - d_i}{2}, \quad M_i = \dfrac{(l - d_i)^2}{4l} W_T - \displaystyle\sum_{j=1}^{i-1} W_j a_{ji}$

Note for wheel 1: $\Sigma W_j a_{ji} = 0$

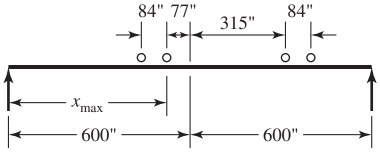
$$W_T = 104.4, \quad W_1 = W_2 = W_3 = W_4 = \frac{104.4}{4} = 26.1\ \text{kip}$$

Wheel 1: $\quad d_1 = \dfrac{476}{2} = 238\ \text{in}, \quad M_1 = \dfrac{(1200 - 238)^2}{4(1200)}(104.4) = 20\,128\ \text{kip}\cdot\text{in}$

Wheel 2: $\quad d_2 = 238 - 84 = 154\ \text{in}$

$$M_2 = \frac{(1200 - 154)^2}{4(1200)}(104.4) - 26.1(84) = 21\,605\ \text{kip}\cdot\text{in} = M_{\max}$$

Check if all of the wheels are on the rail



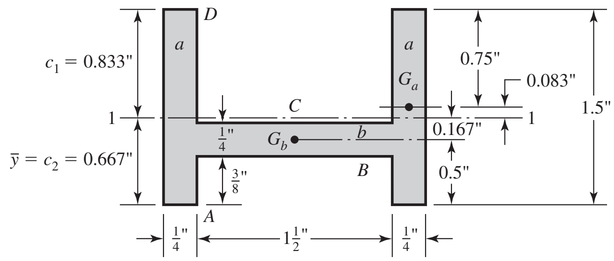
(b) $x_{\max} = 600 - 77 = 523\ \text{in}$

(c) See above sketch.

(d) inner axles

3-23

(a)



$$A_a = A_b = 0.25(1.5) = 0.375\text{ in}^2$$

$$A = 3(0.375) = 1.125\text{ in}^2$$

$$\bar{y} = \frac{2(0.375)(0.75) + 0.375(0.5)}{1.125} = 0.667\text{ in}$$

$$I_a = \frac{0.25(1.5)^3}{12} = 0.0703\text{ in}^4$$

$$I_b = \frac{1.5(0.25)^3}{12} = 0.001\,95\text{ in}^4$$

$$I_1 = 2[0.0703 + 0.375(0.083)^2] + [0.001\ 95 + 0.375(0.167)^2] = 0.158\text{ in}^4 \quad \textit{Ans.}$$

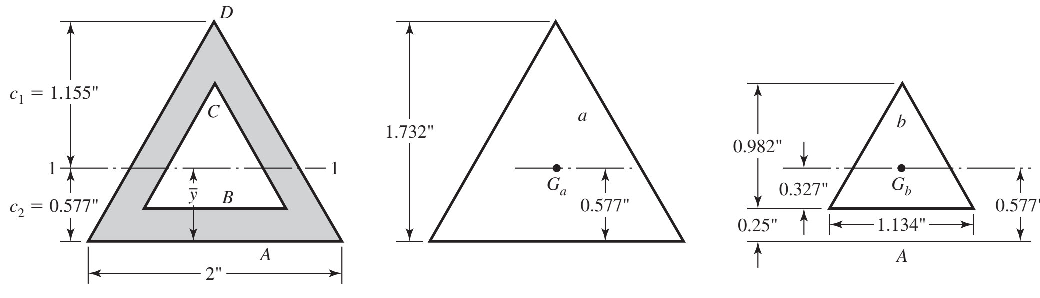
$$\sigma_A = \frac{10\,000(0.667)}{0.158} = 42(10)^3\text{ psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(0.667 - 0.375)}{0.158} = 18.5(10)^3\text{ psi} \quad \textit{Ans.}$$

$$\sigma_C = \frac{10\,000(0.167 - 0.125)}{0.158} = 2.7(10)^3\text{ psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(0.833)}{0.158} = -52.7(10)^3\text{ psi} \quad \textit{Ans.}$$

(b)



Here we treat the hole as a negative area.

$$A_a = 1.732\text{ in}^2$$

$$A_b = 1.134\left(\frac{0.982}{2}\right) = 0.557\text{ in}^2$$

$$A = 1.732 - 0.557 = 1.175\ \text{in}^2$$

$$\bar{y} = \frac{1.732(0.577) - 0.557(0.577)}{1.175} = 0.577\ \text{in} \quad \textit{Ans.}$$

$$I_a = \frac{bh^3}{36} = \frac{2(1.732)^3}{36} = 0.289\ \text{in}^4$$

$$I_b = \frac{1.134(0.982)^3}{36} = 0.0298\ \text{in}^4$$

$$I_1 = I_a - I_b = 0.289 - 0.0298 = 0.259\ \text{in}^4 \quad \textit{Ans.}$$

because the centroids are coincident.

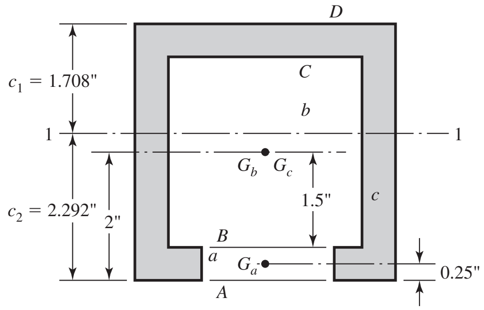
$$\sigma_A = \frac{10\,000(0.577)}{0.259} = 22.3(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_B = \frac{10\,000(0.327)}{0.259} = 12.6(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_C = -\frac{10\,000(0.982 - 0.327)}{0.259} = -25.3(10)^3\ \text{psi} \quad \textit{Ans.}$$

$$\sigma_D = -\frac{10\,000(1.155)}{0.259} = -44.6(10)^3\ \text{psi} \quad \textit{Ans.}$$

(c) Use two negative areas.



$$A_a = 1\ \text{in}^2, \quad A_b = 9\ \text{in}^2, \quad A_c = 16\ \text{in}^2, \quad A = 16 - 9 - 1 = 6\ \text{in}^2;$$

$$\bar{y}_a = 0.25\ \text{in}, \quad \bar{y}_b = 2.0\ \text{in}, \quad \bar{y}_c = 2\ \text{in}$$

$$\bar{y} = \frac{16(2) - 9(2) - 1(0.25)}{6} = 2.292\ \text{in} \quad \textit{Ans.}$$

$$c_1 = 4 - 2.292 = 1.708\ \text{in}$$

$$I_a = \frac{2(0.5)^3}{12} = 0.020\,83\ \text{in}^4$$

$$I_b = \frac{3(3)^3}{12} = 6.75\ \text{in}^4$$

$$I_c = \frac{4(4)^3}{12} = 21.333\ \text{in}^4$$

$$I_1 = [21.333 + 16(0.292)^2] - [6.75 + 9(0.292)^2] - [0.020\,83 + 1(2.292 - 0.25)^2] = 10.99\ \text{in}^4 \quad Ans.$$

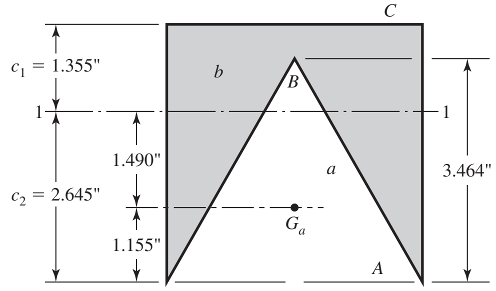
$$\sigma_A = \frac{10\,000(2.292)}{10.99} = 2086\ \text{psi} \quad Ans.$$

$$\sigma_B = \frac{10\,000(2.292 - 0.5)}{10.99} = 1631\ \text{psi} \quad Ans.$$

$$\sigma_C = -\frac{10\,000(1.708 - 0.5)}{10.99} = -1099\ \text{psi} \quad Ans.$$

$$\sigma_D = -\frac{10\,000(1.708)}{10.99} = -1554\ \text{psi} \quad Ans.$$

(d) Use *a* as a negative area.



$$A_a = 6.928\ \text{in}^2, \quad A_b = 16\ \text{in}^2, \quad A = 9.072\ \text{in}^2;$$

$$\bar{y}_a = 1.155\ \text{in}, \quad \bar{y}_b = 2\ \text{in}$$

$$\bar{y} = \frac{2(16) - 1.155(6.928)}{9.072} = 2.645\ \text{in} \quad Ans.$$

$$c_1 = 4 - 2.645 = 1.355\ \text{in}$$

$$I_a = \frac{bh^3}{36} = \frac{4(3.464)^3}{36} = 4.618\ \text{in}^4$$

$$I_b = \frac{4(4)^3}{12} = 21.33\ \text{in}^4$$

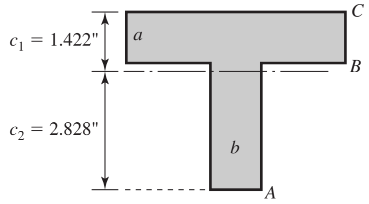
$$I_1 = [21.33 + 16(0.645)^2] - [4.618 + 6.928(1.490)^2] = 7.99\ \text{in}^4 \quad Ans.$$

$$\sigma_A = \frac{10\,000(2.645)}{7.99} = 3310\ \text{psi} \quad Ans.$$

$$\sigma_B = -\frac{10\,000(3.464 - 2.645)}{7.99} = -1025\ \text{psi} \quad Ans.$$

$$\sigma_C = -\frac{10\,000(1.355)}{7.99} = -1696\ \text{psi} \quad Ans.$$

(e)

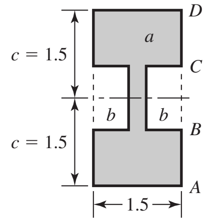


$$A_a = 6(1.25) = 7.5\ \text{in}^2$$
$$A_b = 3(1.5) = 4.5\ \text{in}^2$$
$$A = A_c + A_b = 12\ \text{in}^2$$
$$\bar{y} = \frac{3.625(7.5) + 1.5(4.5)}{12} = 2.828\ \text{in} \quad \textit{Ans.}$$

$$I = \frac{1}{12}(6)(1.25)^3 + 7.5(3.625 - 2.828)^2 + \frac{1}{12}(1.5)(3)^3 + 4.5(2.828 - 1.5)^2$$
$$= 17.05\ \text{in}^4 \quad \textit{Ans.}$$

$$\sigma_A = \frac{10\,000(2.828)}{17.05} = 1659\ \text{psi} \quad \textit{Ans.}$$
$$\sigma_B = -\frac{10\,000(3 - 2.828)}{17.05} = -101\ \text{psi} \quad \textit{Ans.}$$
$$\sigma_C = -\frac{10\,000(1.422)}{17.05} = -834\ \text{psi} \quad \textit{Ans.}$$

(f)



Let a = total area

$$A = 1.5(3) - 1(1.25) = 3.25\ \text{in}^2$$
$$I = I_a - 2I_b = \frac{1}{12}(1.5)(3)^3 - \frac{1}{12}(1.25)(1)^3$$
$$= 3.271\ \text{in}^4 \quad \textit{Ans.}$$

$$\sigma_A = \frac{10\,000(1.5)}{3.271} = 4586\ \text{psi}, \quad \sigma_D = -4586\ \text{psi}$$
$$\textit{Ans.}$$
$$\sigma_B = \frac{10\,000(0.5)}{3.271} = 1529\ \text{psi}, \quad \sigma_C = -1529\ \text{psi}$$

3-24

(a) The moment is maximum and constant between A and B

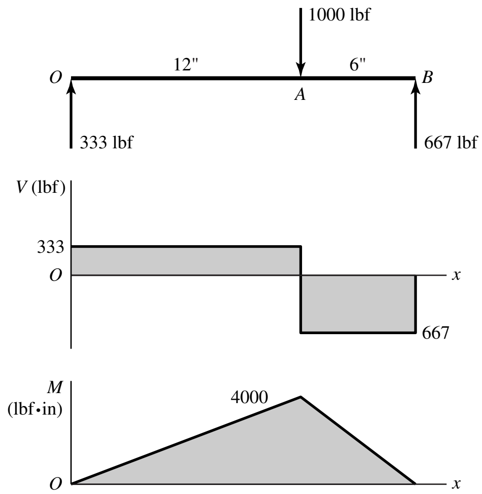
$$M = -50(20) = -1000\ \text{lbf}\cdot\text{in}, \quad I = \frac{1}{12}(0.5)(2)^3 = 0.3333\ \text{in}^4$$
$$\rho = \left|\frac{EI}{M}\right| = \frac{1.6(10^6)(0.3333)}{1000} = 533.3\ \text{in}$$
$$(x, y) = (30, -533.3)\ \text{in} \quad \textit{Ans.}$$

(b) The moment is maximum and constant between A and B

$$M = 50(5) = 250\ \text{lbf}\cdot\text{in}, \quad I = 0.3333\ \text{in}^4$$
$$\rho = \frac{1.6(10^6)(0.3333)}{250} = 2133\ \text{in} \quad \textit{Ans.}$$
$$(x, y) = (20, 2133)\ \text{in} \quad \textit{Ans.}$$

3-25

(a)



$$I = \frac{1}{12}(0.75)(1.5)^3 = 0.2109\,\text{in}^4$$

$$A = 0.75(1.5) = 1.125\,\text{in}$$

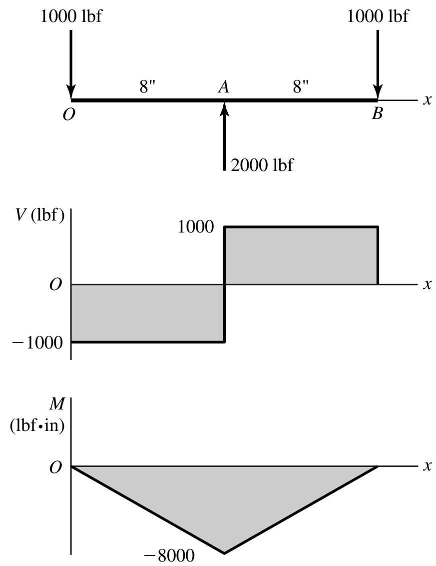
M_{max} is at A. At the bottom of the section,

$$\sigma_{\text{max}} = \frac{Mc}{I} = \frac{4000(0.75)}{0.2109} = 14\,225\,\text{psi} \quad \textit{Ans.}$$

Due to V, τ_{max} constant is between A and B at $y = 0$

$$\tau_{\text{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{667}{1.125} = 889\,\text{psi} \quad \textit{Ans.}$$

(b)



$$I = \frac{1}{12}(1)(2)^3 = 0.6667\,\text{in}^4$$

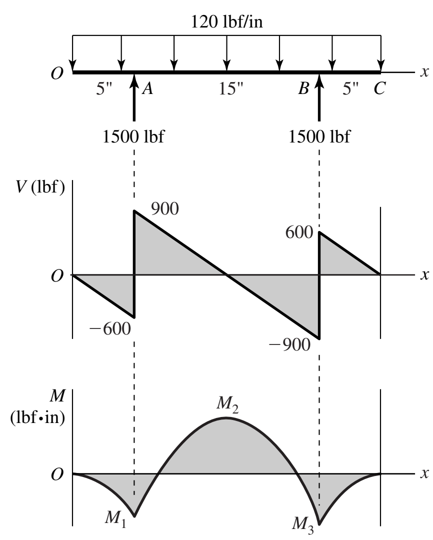
M_{max} is at A at the top of the beam

$$\sigma_{\text{max}} = \frac{8000(1)}{0.6667} = 12\,000\,\text{psi} \quad \textit{Ans.}$$

$$|V_{\text{max}}| = 1000\,\text{lbf} \quad \text{from } O \text{ to } B \text{ at } y = 0$$

$$\tau_{\text{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{1000}{(2)(1)} = 750\,\text{psi} \quad \textit{Ans.}$$

(c)



$$I = \frac{1}{12}(0.75)(2)^3 = 0.5\,\text{in}^4$$

$$M_1 = -\frac{1}{2}600(5) = -1500\ \text{lbf}\cdot\text{in} = M_3$$

$$M_2 = -1500 + \frac{1}{2}(900)(7.5) = 1875\ \text{lbf}\cdot\text{in}$$

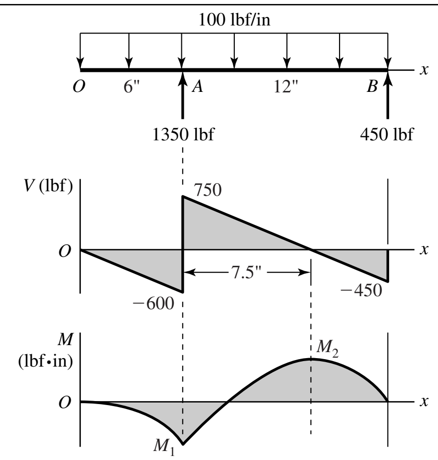
M_{max} is at span center. At the bottom of the beam,

$$\sigma_{\text{max}} = \frac{1875(1)}{0.5} = 3750\,\text{psi} \quad \textit{Ans.}$$

At A and B at $y = 0$

$$\tau_{\text{max}} = \frac{3}{2}\frac{900}{(0.75)(2)} = 900\,\text{psi} \quad \textit{Ans.}$$

(d)



$$I = \frac{1}{12}(1)(2)^3 = 0.6667\ \text{in}^4$$

$$M_1 = -\frac{600}{2}(6) = -1800\ \text{lbf}\cdot\text{in}$$

$$M_2 = -1800 + \frac{1}{2}750(7.5) = 1013\ \text{lbf}\cdot\text{in}$$

At A, top of beam

$$\sigma_{\max} = \frac{1800(1)}{0.6667} = 2700\ \text{psi} \quad \textit{Ans.}$$

At A, $y = 0$

$$\tau_{\max} = \frac{3}{2}\frac{750}{(2)(1)} = 563\ \text{psi} \quad \textit{Ans.}$$

3-26

$$M_{\max} = \frac{wl^2}{8} \quad \Rightarrow \quad \sigma_{\max} = \frac{wl^2c}{8I} \quad \Rightarrow \quad w = \frac{8\sigma I}{cl^2}$$

(a) $l = 12(12) = 144\ \text{in}$, $I = (1/12)(1.5)(9.5)^3 = 107.2\ \text{in}^4$

$$w = \frac{8(1200)(107.2)}{4.75(144^2)} = 10.4\ \text{lbf/in} \quad \textit{Ans.}$$

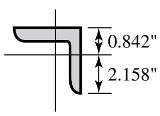
(b) $l = 48\ \text{in}$, $I = (\pi/64)(2^4 - 1.25^4) = 0.6656\ \text{in}^4$

$$w = \frac{8(12)(10^3)(0.6656)}{1(48)^2} = 27.7\ \text{lbf/in} \quad \textit{Ans.}$$

(c) $l = 48\ \text{in}$, $I \doteq (1/12)(2)(3^3) - (1/12)(1.625)(2.625^3) = 2.051\ \text{in}^4$

$$w = \frac{8(12)(10^3)(2.051)}{1.5(48)^2} = 57.0\ \text{lbf/in} \quad \textit{Ans.}$$

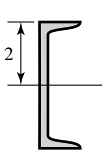
(d) $l = 72$ in; Table A-6, $I = 2(1.24) = 2.48\ \text{in}^4$



$$c_{\max} = 2.158''$$

$$w = \frac{8(12)(10^3)(2.48)}{2.158(72)^2} = 21.3\ \text{lbf/in} \quad \textit{Ans.}$$

(e) $l = 72$ in; Table A-7, $I = 3.85\ \text{in}^4$

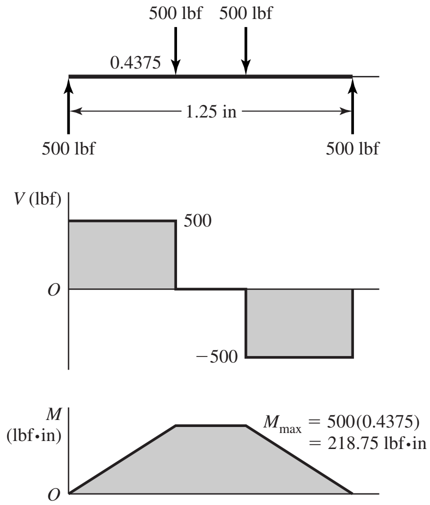


$$w = \frac{8(12)(10^3)(3.85)}{2(72^2)} = 35.6\ \text{lbf/in} \quad \textit{Ans.}$$

(f) $l = 72\ \text{in}$, $I = (1/12)(1)(4^3) = 5.333\ \text{in}^4$

$$w = \frac{8(12)(10^3)(5.333)}{(2)(72)^2} = 49.4\ \text{lbf/in} \quad \textit{Ans.}$$

3-27 **(a)** Model (c)



$$I = \frac{\pi}{64}(0.5^4) = 3.068(10^{-3})\ \text{in}^4$$

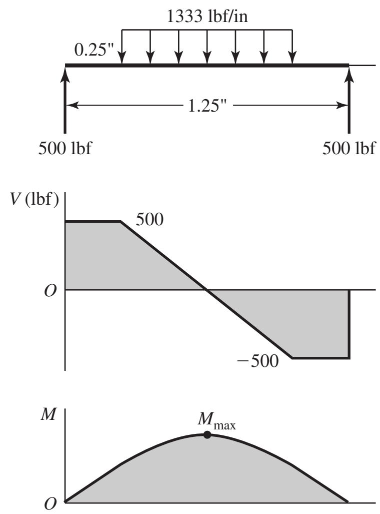
$$A = \frac{\pi}{4}(0.5^2) = 0.1963\ \text{in}^2$$

$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068(10^{-3})}$$

$$= 17\,825\ \text{psi} = 17.8\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400\ \text{psi} \quad \textit{Ans.}$$

(b) Model (d)



$$M_{\max} = 500(0.25) + \frac{1}{2}(500)(0.375)$$

$$= 218.75\ \text{lbf} \cdot \text{in}$$

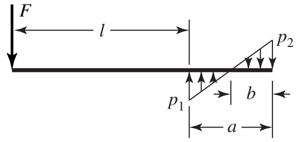
$$V_{\max} = 500\ \text{lbf}$$

Same M and V

$$\therefore \sigma = 17.8\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = 3400\ \text{psi} \quad \textit{Ans.}$$

3-28



$$q = -F\langle x\rangle^{-1} + p_1\langle x-l\rangle^0 - \frac{p_1+p_2}{a}\langle x-l\rangle^1 + \text{ terms for } x > l+a$$

$$V = -F + p_1\langle x-l\rangle^1 - \frac{p_1+p_2}{2a}\langle x-l\rangle^2 + \text{ terms for } x > l+a$$

$$M = -Fx + \frac{p_1}{2}\langle x-l\rangle^2 - \frac{p_1+p_2}{6a}\langle x-l\rangle^3 + \text{ terms for } x > l+a$$

At $x = (l+a)^+$, $V = M = 0$, terms for $x > l + a = 0$

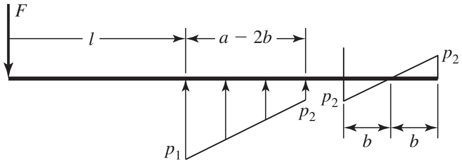
$$-F + p_1 a - \frac{p_1+p_2}{2a}a^2 = 0 \quad \Rightarrow \quad p_1 - p_2 = \frac{2F}{a} \qquad (1)$$

$$-F(l+a)+\frac{p_1 a^2}{2}-\frac{p_1+p_2}{6a}a^3=0 \quad \Rightarrow \quad 2p_1-p_2=\frac{6F(l+a)}{a^2} \tag{2}$$

From (1) and (2) $$p_1=\frac{2F}{a^2}(3l+2a), \quad p_2=\frac{2F}{a^2}(3l+a) \tag{3}$$

From similar triangles $$\frac{b}{p_2}=\frac{a}{p_1+p_2} \quad \Rightarrow \quad b=\frac{ap_2}{p_1+p_2} \tag{4}$$

$M_{\max}$ occurs where $V=0$



$$x_{\max}=l+a-2b$$

$$M_{\max}=-F(l+a-2b)+\frac{p_1}{2}(a-2b)^2-\frac{p_1+p_2}{6a}(a-2b)^3$$

$$=-Fl-F(a-2b)+\frac{p_1}{2}(a-2b)^2-\frac{p_1+p_2}{6a}(a-2b)^3$$

Normally $M_{\max}=-Fl$
The fractional increase in the magnitude is

$$\Delta=\frac{F(a-2b)-(p_1/2)(a-2b)^2-[(p_1+p_2)/6a](a-2b)^3}{Fl} \tag{5}$$

For example, consider $F=1500$ lbf, $a=1.2$ in, $l=1.5$ in

(3) $$p_1=\frac{2(1500)}{1.2^2}[3(1.5)+2(1.2)]=14\,375 \text{ lbf/in}$$

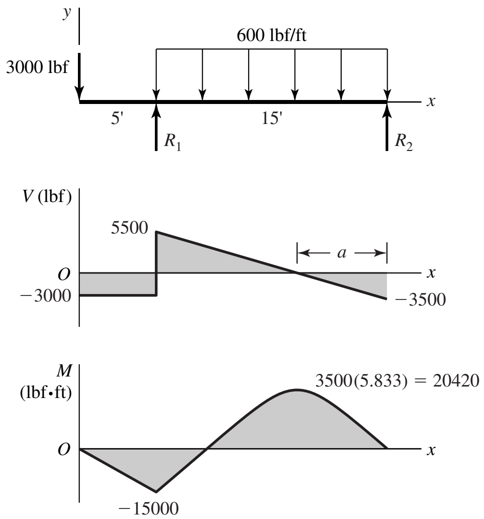
$$p_2=\frac{2(1500)}{1.2^2}[3(1.5)+1.2]=11\,875 \text{ lbf/in}$$

(4) $$b=1.2(11\,875)/(14\,375+11\,875)=0.5429 \text{ in}$$

Substituting into (5) yields

$$\Delta=0.036\,89 \quad \text{or} \quad 3.7\% \text{ higher than } -Fl$$

3-29

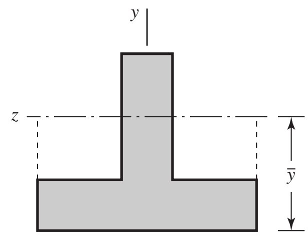


$$R_1 = \frac{600(15)}{2} + \frac{20}{15}3000 = 8500 \text{ lbf}$$

$$R_2 = \frac{600(15)}{2} - \frac{5}{15}3000 = 3500 \text{ lbf}$$

$$a = \frac{3500}{600} = 5.833 \text{ ft}$$

(a)



$$\bar{y} = \frac{1(12) + 5(12)}{24} = 3 \text{ in}$$

$$I_z = \frac{1}{3}[2(5^3) + 6(3^3) - 4(1^3)] = 136 \text{ in}^4$$

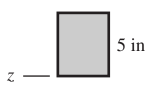
At $x = 5$ ft, $\quad y = -3$ in, $\quad \sigma_x = -\dfrac{-15000(12)(-3)}{136} = -3970$ psi

$y = 5$ in, $\quad \sigma_x = -\dfrac{-15000(12)5}{136} = 6620$ psi

At $x = 14.17$ ft, $\quad y = -3$ in, $\quad \sigma_x = -\dfrac{20420(12)(-3)}{136} = 5405$ psi

$y = 5$ in, $\quad \sigma_x = -\dfrac{20420(12)5}{136} = -9010$ psi

Max tension = 6620 psi *Ans.*
Max compression = −9010 psi *Ans.*
(b) $V_{max} = 5500$ lbf

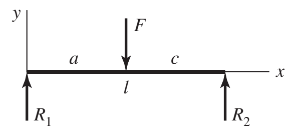


$$Q_{n.a.} = \bar{y}A = 2.5(5)(2) = 25 \text{ in}^3$$

$$\tau_{\underset{v}{max}} = \frac{VQ}{Ib} = \frac{5500(25)}{136(2)} = 506 \text{ psi} \quad \textit{Ans.}$$

(c) $\tau_{max} = \dfrac{|\sigma_{max}|}{2} = \dfrac{9010}{2} = 4510$ psi *Ans.*

3-30



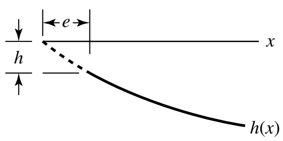
$$R_1 = \frac{c}{l}F$$

$$M = \frac{c}{l}Fx \quad 0 \le x \le a$$

$$\sigma = \frac{6M}{bh^2} = \frac{6(c/l)Fx}{bh^2} \quad \Rightarrow \quad h = \sqrt{\frac{6cFx}{bl\sigma_{max}}} \quad 0 \le x \le a \quad Ans.$$

3-31 From Prob. 3-30, $R_1 = \frac{c}{l}F = V, \quad 0 \le x \le a$

$$\tau_{max} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{(c/l)F}{bh} \qquad \therefore\ h = \frac{3}{2}\frac{Fc}{lb\tau_{max}} \quad Ans.$$

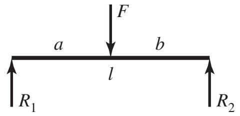


From Prob. 3-30 $= \sqrt{\dfrac{6Fcx}{lb\sigma_{max}}}$ sub in $x = e$ and equate to h above

$$\frac{3}{2}\frac{Fc}{lb\tau_{max}} = \sqrt{\frac{6Fce}{lb\sigma_{max}}}$$

$$e = \frac{3}{8}\frac{Fc\sigma_{max}}{lb\tau^2_{max}} \quad Ans.$$

3-32



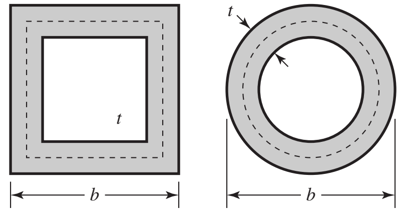
$$R_1 = \frac{b}{l}F$$

$$M = \frac{b}{l}Fx$$

$$\sigma_{max} = \frac{32M}{\pi d^3} = \frac{32}{\pi d^3}\frac{b}{l}Fx$$

$$d = \left[\frac{32}{\pi}\frac{bFx}{l\sigma_{max}}\right]^{1/3} \quad 0 \le x \le a \quad Ans.$$

3-33



Square: $A_m = (b - t)^2$

$$T_{sq} = 2A_m t\tau_{all} = 2(b - t)^2 t\tau_{all}$$

Round: $A_m = \pi(b - t)^2/4$

$$T_{rd} = 2\pi(b - t)^2 t\tau_{all}/4$$

Ratio of torques

$$\frac{T_{sq}}{T_{rd}} = \frac{2(b-t)^2 t\tau_{all}}{\pi(b-t)^2 t\tau_{all}/2} = \frac{4}{\pi} = 1.27$$

Twist per unit length
square:

$$\theta_{sq} = \frac{2G\theta_1 t}{t\tau_{all}}\left(\frac{L}{A}\right)_m = C\left|\frac{L}{A}\right|_m = C\frac{4(b-t)}{(b-t)^2}$$

Round:

$$\theta_{rd} = C\left(\frac{L}{A}\right)_m = C\frac{\pi(b-t)}{\pi(b-t)^2/4} = C\frac{4(b-t)}{(b-t)^2}$$

Ratio equals 1, twists are the same.

Note the weight ratio is

$$\frac{W_{sq}}{W_{rd}} = \frac{\rho l(b-t)^2}{\rho l\pi(b-t)(t)} = \frac{b-t}{\pi t} \qquad \text{thin-walled assumes } b \geq 20t$$

$$= \frac{19}{\pi} = 6.04 \qquad \text{with } b = 20t$$

$$= 2.86 \qquad \text{with } b = 10t$$

3-34 $l = 40$ in, $\tau_{all} = 11\,500$ psi, $G = 11.5(10^6)$ psi, $t = 0.050$ in

$$r_m = r_i + t/2 = r_i + 0.025 \quad \text{for } r_i > 0$$
$$= 0 \quad \text{for } r_i = 0$$

$$A_m = (1-0.05)^2 - 4\left(r_m^2 - \frac{\pi}{4}r_m^2\right) = 0.95^2 - (4-\pi)r_m^2$$

$$L_m = 4(1 - 0.05 - 2r_m + 2\pi r_m/4) = 4[0.95 - (2-\pi/2)r_m]$$

Eq. (3-45): $T = 2A_m t\tau = 2(0.05)(11\,500)A_m = 1150A_m$

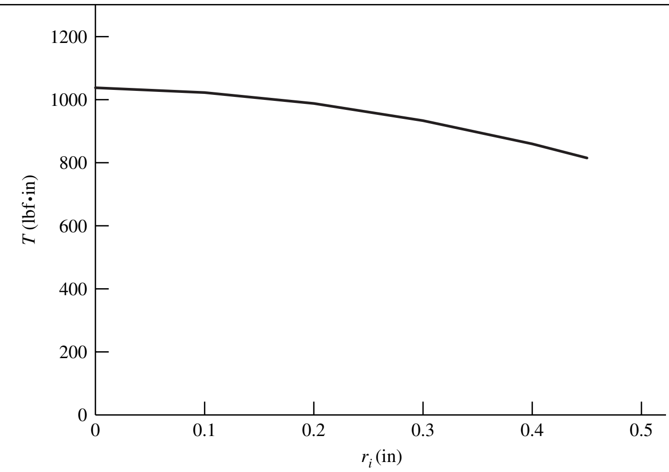
Eq. (3-46):

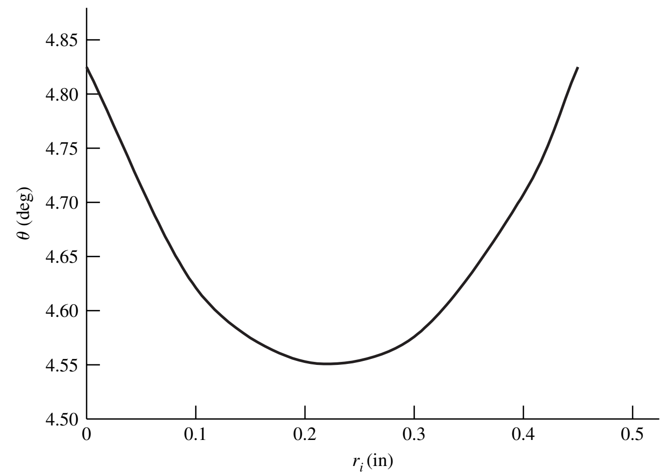
$$\theta(\text{deg}) = \theta_1 l\frac{180}{\pi} = \frac{TL_m l}{4GA_m^2 t}\frac{180}{\pi} = \frac{TL_m(40)}{4(11.5)(10^6)A_m^2(0.05)}\frac{180}{\pi}$$

$$= 9.9645(10^{-4})\frac{TL_m}{A_m^2}$$

Equations can then be put into a spreadsheet resulting in:

r_i	r_m	A_m	L_m	r_i	T(lbf · in)	r_i	θ(deg)
0	0	0.902 5	3.8	0	1037.9	0	4.825
0.10	0.125	0.889 087	3.585 398	0.10	1022.5	0.10	4.621
0.20	0.225	0.859 043	3.413 717	0.20	987.9	0.20	4.553
0.30	0.325	0.811 831	3.242 035	0.30	933.6	0.30	4.576
0.40	0.425	0.747 450	3.070 354	0.40	859.6	0.40	4.707
0.45	0.475	0.708 822	2.984 513	0.45	815.1	0.45	4.825





Torque carrying capacity reduces with r_i. However, this is based on an assumption of uniform stresses which is not the case for small r_i. Also note that weight also goes down with an increase in r_i.

3-35 From Eq. (3-47) where θ_1 is the same for each leg.

$$T_1 = \frac{1}{3}G\theta_1 L_1 c_1^3, \quad T_2 = \frac{1}{3}G\theta_1 L_2 c_2^3$$

$$T = T_1 + T_2 = \frac{1}{3}G\theta_1\left(L_1 c_1^3 + L_2 c_2^3\right) = \frac{1}{3}G\theta_1 \sum L_i c_i^3 \quad \textit{Ans.}$$

$$\tau_1 = G\theta_1 c_1, \quad \tau_2 = G\theta_1 c_2$$

$$\tau_{\max} = G\theta_1 c_{\max} \quad \textit{Ans.}$$

3-36

(a) $\tau_{\max} = G\theta_1 c_{\max}$

$$G\theta_1 = \frac{\tau_{\max}}{c_{\max}} = \frac{12\,000}{1/8} = 9.6(10^4) \text{ psi/in}$$

$$T_{1/16} = \frac{1}{3}G\theta_1(Lc^3)_{1/16} = \frac{1}{3}(9.6)(10^4)(5/8)(1/16)^3 = 4.88 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{1/8} = \frac{1}{3}(9.6)(10^4)(5/8)(1/8)^3 = 39.06 \text{ lbf} \cdot \text{in} \quad Ans.$$

$$\tau_{1/16} = 9.6(10^4)1/16 = 6000 \text{ psi}, \quad \tau_{1/8} = 9.6(10^4)1/8 = 12\,000 \text{ psi} \quad Ans.$$

(b) $$\theta_1 = \frac{9.6(10^4)}{12(10^6)} = 87(10^{-3}) \text{ rad/in} = 0.458°\text{/in} \quad Ans.$$

3-37 *Separate strips:* For each 1/16 in thick strip,

$$T = \frac{Lc^2\tau}{3} = \frac{(1)(1/16)^2(12\,000)}{3} = 15.625 \text{ lbf} \cdot \text{in}$$

$$\therefore T_{\max} = 2(15.625) = 31.25 \text{ lbf} \cdot \text{in} \quad Ans.$$

For each strip,

$$\theta = \frac{3Tl}{Lc^3G} = \frac{3(15.625)(12)}{(1)(1/16)^3(12)(10^6)} = 0.192 \text{ rad} \quad Ans.$$

$$k_t = T/\theta = 31.25/0.192 = 162.8 \text{ lbf} \cdot \text{in/rad} \quad Ans.$$

Solid strip: From Eq. (3-47),

$$T_{\max} = \frac{Lc^2\tau}{3} = \frac{1(1/8)^2 12\,000}{3} = 62.5 \text{ lbf} \cdot \text{in} \quad Ans.$$

$$\theta = \theta_1 l = \frac{\tau l}{Gc} = \frac{12\,000(12)}{12(10^6)(1/8)} = 0.0960 \text{ rad} \quad Ans.$$

$$k_l = 62.5/0.0960 = 651 \text{ lbf} \cdot \text{in/rad} \quad Ans.$$

3-38 $\tau_{\text{all}} = 60$ MPa, H = 35 kW

(a) $n = 2000$ rpm

Eq. (4-40) $$T = \frac{9.55H}{n} = \frac{9.55(35)10^3}{2000} = 167.1 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad d = \left(\frac{16T}{\pi \tau_{\max}}\right)^{1/3} = \left[\frac{16(167.1)}{\pi(60)10^6}\right]^{1/3} = 24.2(10^{-3}) \text{ m} = 24.2 \text{ mm} \quad Ans.$$

(b) $n = 200$ rpm $\quad \therefore T = 1671 \text{ N} \cdot \text{m}$

$$d = \left[\frac{16(1671)}{\pi(60)10^6}\right]^{1/3} = 52.2(10^{-3}) \text{ m} = 52.2 \text{ mm} \quad Ans.$$

3-39 $\tau_{\text{all}} = 110$ MPa, $\theta = 30°$, $d = 15$ mm, $l = ?$

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3$$

$$\theta = \frac{Tl}{JG}\left(\frac{180}{\pi}\right)$$

$$l = \frac{\pi}{180}\frac{JG\theta}{T} = \frac{\pi}{180}\left[\frac{\pi}{32}\frac{d^4G\theta}{(\pi/16)\,\tau d^3}\right] = \frac{\pi}{360}\frac{dG\theta}{\tau}$$

$$= \frac{\pi}{360}\frac{(0.015)(79.3)(10^9)(30)}{110(10^6)} = 2.83\text{ m}\quad \textit{Ans.}$$

3-40 $d = 3$ in, replaced by 3 in hollow with $t = 1/4$ in

(a) $$T_{\text{solid}} = \frac{\pi}{16}\tau(3^3)\quad T_{\text{hollow}} = \frac{\pi}{32}\tau\frac{(3^4 - 2.5^4)}{1.5}$$

$$\%\Delta T = \frac{(\pi/16)(3^3) - (\pi/32)\,[(3^4 - 2.5^4)/1.5]}{(\pi/16)(3^3)}(100) = 48.2\%\quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd^2 = k(3^2),\quad W_{\text{hollow}} = k(3^2 - 2.5^2)$

$$\%\Delta W = \frac{k(3^2) - k(3^2 - 2.5^2)}{k(3^2)}(100) = 69.4\%\quad \textit{Ans.}$$

3-41 $T = 5400\text{ N}\cdot\text{m},\ \tau_{\text{all}} = 150\text{ MPa}$

(a) $$\tau = \frac{Tc}{J}\quad\Rightarrow\quad 150(10^6) = \frac{5400(d/2)}{(\pi/32)[d^4 - (0.75d)^4]} = \frac{4.023(10^4)}{d^3}$$

$$d = \left(\frac{4.023(10^4)}{150(10^6)}\right)^{1/3} = 6.45(10^{-2})\,\text{m} = 64.5\text{ mm}$$

From Table A-17, the next preferred size is $d = 80$ mm; $ID = 60$ mm *Ans.*

(b) $$J = \frac{\pi}{32}(0.08^4 - 0.06^4) = 2.749(10^{-6})\text{ mm}^4$$

$$\tau_i = \frac{5400(0.030)}{2.749(10^{-6})} = 58.9(10^6)\text{ Pa} = 58.9\text{ MPa}\quad \textit{Ans.}$$

3-42

(a) $$T = \frac{63\,025H}{n} = \frac{63\,025(1)}{5} = 12\,605\text{ lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d_C^3}\quad\Rightarrow\quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(12\,605)}{\pi(14\,000)}\right]^{1/3} = 1.66\text{ in}\quad \textit{Ans.}$$

From Table A-17, select 1 3/4 in

$$\tau_{\text{start}} = \frac{16(2)(12\,605)}{\pi(1.75^3)} = 23.96(10^3)\text{ psi} = 23.96\text{ kpsi}$$

(b) design activity

3-43 $\omega = 2\pi n/60 = 2\pi(8)/60 = 0.8378$ rad/s

$$T = \frac{H}{\omega} = \frac{1000}{0.8378} = 1194 \text{ N}\cdot\text{m}$$

$$d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(1194)}{\pi(75)(10^6)}\right]^{1/3} = 4.328(10^{-2}) \text{ m} = 43.3 \text{ mm}$$

From Table A-17, select 45 mm *Ans.*

3-44 $s = \sqrt{A}, \quad d = \sqrt{4A/\pi}$

Square: Eq. (3-43) with $b = c$

$$\tau_{\max} = \frac{4.8T}{c^3}$$

$$(\tau_{\max})_{sq} = \frac{4.8T}{(A)^{3/2}}$$

Round:

$$(\tau_{\max})_{rd} = \frac{16}{\pi}\frac{T}{d^3} = \frac{16T}{\pi(4A/\pi)^{3/2}} = \frac{3.545T}{(A)^{3/2}}$$

$$\frac{(\tau_{\max})_{sq}}{(\tau_{\max})_{rd}} = \frac{4.8}{3.545} = 1.354$$

Square stress is 1.354 times the round stress *Ans.*

3-45 $s = \sqrt{A}, \quad d = \sqrt{4A/\pi}$

Square: Eq. (3-44) with $b = c$, $\beta = 0.141$

$$\theta_{sq} = \frac{Tl}{0.141c^4G} = \frac{Tl}{0.141(A)^{4/2}G}$$

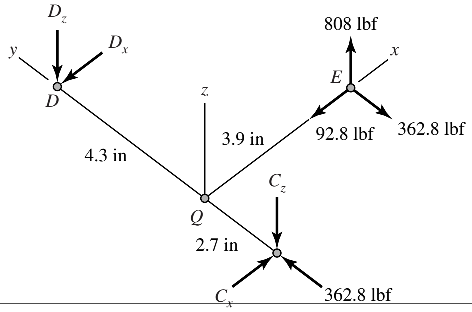
Round:

$$\theta_{rd} = \frac{Tl}{JG} = \frac{Tl}{(\pi/32)\,(4A/\pi)^{4/2}\,G} = \frac{6.2832Tl}{(A)^{4/2}G}$$

$$\frac{\theta_{sq}}{\theta_{rd}} = \frac{1/0.141}{6.2832} = 1.129$$

Square has greater θ by a factor of 1.13 *Ans.*

3-46



$$\left(\sum M_D\right)_z = 7C_x - 4.3(92.8) - 3.9(362.8) = 0$$

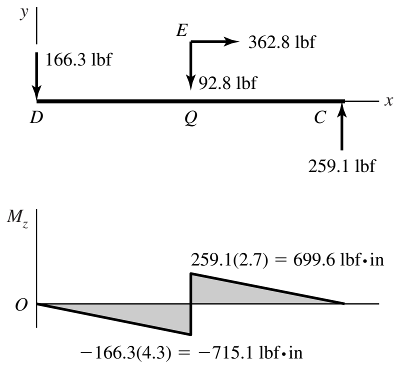
$$C_x = 259.1 \text{ lbf}$$

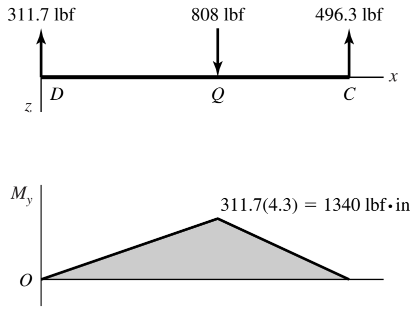
$$\left(\sum M_C\right)_z = -7D_x - 2.7(92.8) + 3.9(362.8) = 0$$

$$D_x = 166.3 \text{ lbf}$$

$$\left(\sum M_D\right)_x \quad \Rightarrow \quad C_z = \frac{4.3}{7} 808 = 496.3 \text{ lbf}$$

$$\left(\sum M_C\right)_x \quad \Rightarrow \quad D_z = \frac{2.7}{7} 808 = 311.7 \text{ lbf}$$





$$\text{Torque} : \underset{x=4.3^+_{\text{in}}}{T} = 808(3.9) = 3151 \text{ lbf} \cdot \text{in}$$

$$\text{Bending } Q : \underset{x=4.3^+_{\text{in}}}{M} = \sqrt{699.6^2 + 1340^2} = 1512 \text{ lbf} \cdot \text{in}$$

Torque:

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3151)}{\pi(1.25^3)} = 8217 \text{ psi}$$

Bending:

$$\sigma_b = \pm\frac{32(1512)}{\pi(1.25^3)} = \pm 7885 \text{ psi}$$

Axial:

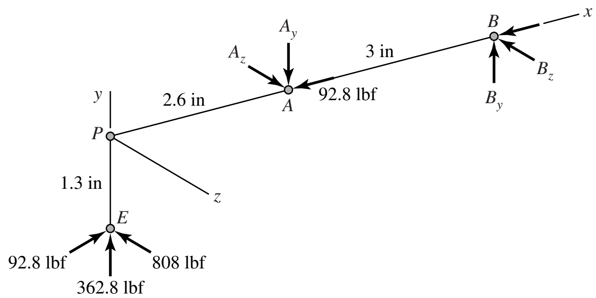
$$\sigma_a = -\frac{F}{A} = -\frac{362.8}{(\pi/4)(1.25^2)} = -296 \text{ psi}$$

$$|\sigma_{\max}| = 7885 + 296 = 8181 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{8181}{2}\right)^2 + 8217^2} = 9179 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_{\substack{\max\\ \text{tens.}}} = \frac{7885 - 296}{2} + \sqrt{\left(\frac{7885 - 296}{2}\right)^2 + 8217^2} = 12\,845 \text{ psi} \quad \textit{Ans.}$$

3-47



$$\left(\sum M_B\right)_z = -5.6(362.8) + 1.3(92.8) + 3A_y = 0$$

$$A_y = 637.0 \text{ lbf}$$

$$\left(\sum M_A\right)_z = -2.6(362.8) + 1.3(92.8) + 3B_y = 0$$

$$B_y = 274.2 \text{ lbf}$$

$$\left(\sum M_B\right)_y = 0 \quad \Rightarrow \quad A_z = \frac{5.6}{3}808 = 1508.3 \text{ lbf}$$

$$\left(\sum M_A\right)_y = 0 \quad \Rightarrow \quad B_z = \frac{2.6}{3}808 = 700.3 \text{ lbf}$$

Torsion: $T = 808(1.3) = 1050 \text{ lbf}\cdot\text{in}$

$$\tau = \frac{16(1050)}{\pi(1^3)} = 5348 \text{ psi}$$

Bending: $M_p = 92.8(1.3) = 120.6 \text{ lbf}\cdot\text{in}$

$$M_A = 3\sqrt{B_y^2 + B_z^2} = 3\sqrt{274.2^2 + 700.3^2}$$

$$= 2256 \text{ lbf}\cdot\text{in} = M_{\max}$$

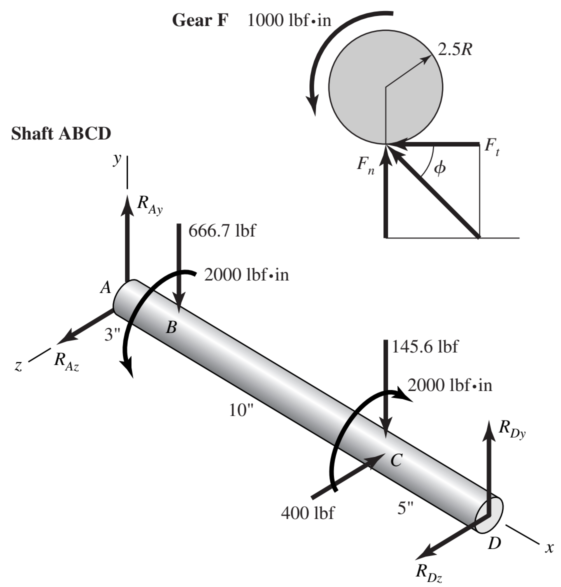
$$\sigma_b = \pm\frac{32(2256)}{\pi(1^3)} = \pm 22\,980 \text{ psi}$$

$$\text{Axial: } \underset{\text{inAP}}{\sigma} = -\frac{92.8}{(\pi/4)1^2} = -120 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-22980 - 120}{2}\right)^2 + 5348^2} = 12\,730 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_{\substack{\max\\ \text{tens}}} = \frac{22980 - 120}{2} + \sqrt{\left(\frac{22980 - 120}{2}\right)^2 + 5348^2} = 24\,049 \text{ psi} \quad \textit{Ans.}$$

3-48



$$F_t = \frac{1000}{2.5} = 400\,\text{lbf}$$

$$F_n = 400 \tan 20 = 145.6 \text{ lbf}$$

Torque at C $\quad T_C = 400(5) = 2000 \text{ lbf} \cdot \text{in}$

$$P = \frac{2000}{3} = 666.7\,\text{lbf}$$

$$\sum (M_A)_z = 0 \quad \Rightarrow \quad 18R_{Dy} - 145.6(13) - 666.7(3) = 0 \quad \Rightarrow \quad R_{Dy} = 216.3 \text{ lbf}$$

$$\sum (M_A)_y = 0 \quad \Rightarrow \quad -18R_{Dz} + 400(13) = 0 \quad \Rightarrow \quad R_{Dz} = 288.9 \text{ lbf}$$

$$\sum F_y = 0 \quad \Rightarrow \quad R_{Ay} + 216.3 - 666.7 - 145.6 = 0 \quad \Rightarrow \quad R_{Ay} = 596.0 \text{ lbf}$$

$$\sum F_z = 0 \quad \Rightarrow \quad R_{Az} + 288.9 - 400 = 0 \quad \Rightarrow \quad R_{Az} = 111.1 \text{ lbf}$$

$$M_B = 3\sqrt{596^2 + 111.1^2} = 1819 \text{ lbf} \cdot \text{in}$$

$$M_C = 5\sqrt{216.3^2 + 288.9^2} = 1805 \text{ lbf} \cdot \text{in}$$

$\therefore$ Maximum stresses occur at B. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(1819)}{\pi(1.25^3)} = 9486 \text{ psi}$$

$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(2000)}{\pi(1.25^3)} = 5215 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{9486}{2} + \sqrt{\left(\frac{9486}{2}\right)^2 + 5215^2} = 11\,792 \text{ psi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = 7049 \text{ psi} \quad \textit{Ans.}$$

3-49 $r = d/2$

(a) For top, $\theta = 90^\circ$,

$$\sigma_r = \frac{\sigma}{2}[1 - 1 + (1 - 1)(1 - 3)\cos 180] = 0 \quad \textit{Ans.}$$

$$\sigma_\theta = \frac{\sigma}{2}[1 + 1 - (1 + 3)\cos 180] = 3\sigma \quad Ans.$$

$$\tau_{r\theta} = -\frac{\sigma}{2}(1 - 1)(1 + 3)\sin 180 = 0 \quad Ans.$$

For side, $\theta = 0^\circ$,

$$\sigma_r = \frac{\sigma}{2}[1 - 1 + (1 - 1)(1 - 3)\cos 0] = 0 \quad Ans.$$

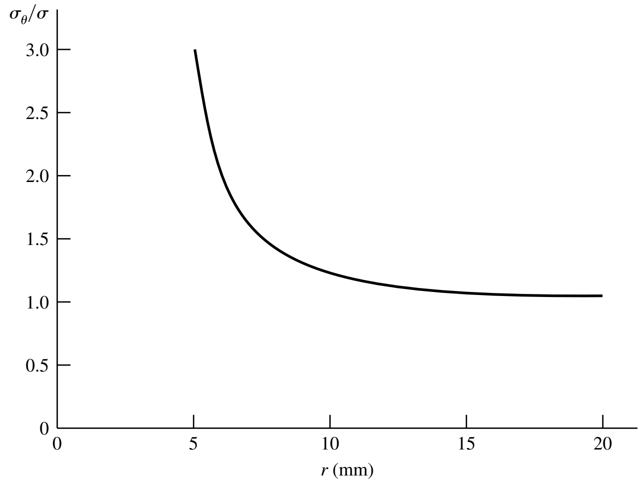
$$\sigma_\theta = \frac{\sigma}{2}[1 + 1 - (1 + 3)\cos 0] = -\sigma \quad Ans.$$

$$\tau_{r\theta} = -\frac{\sigma}{2}(1 - 1)(1 + 3)\sin 0 = 0 \quad Ans.$$

(b)

$$\sigma_\theta/\sigma = \frac{1}{2}\left[1 + \frac{100}{4r^2} - \left(1 + \frac{3}{16}\frac{10^4}{r^4}\right)\cos 180\right] = \frac{1}{2}\left(2 + \frac{25}{r^2} + \frac{3}{16}\frac{10^4}{r^4}\right)$$

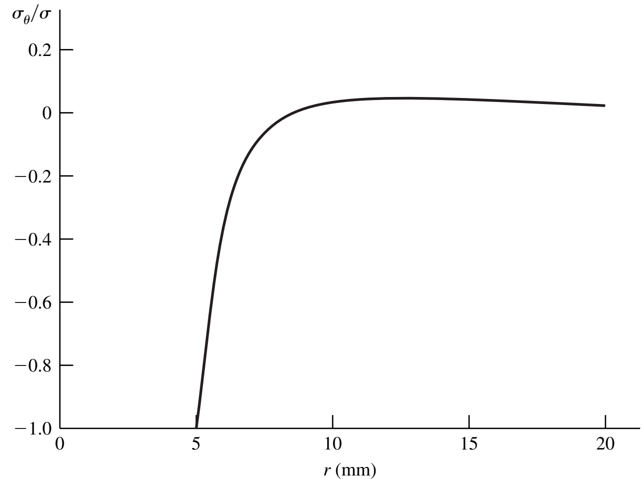
r	σ_θ/σ
5	3.000
6	2.071
7	1.646
8	1.424
9	1.297
10	1.219
11	1.167
12	1.132
13	1.107
14	1.088
15	1.074
16	1.063
17	1.054
18	1.048
19	1.042
20	1.037



(c)

$$\sigma_\theta/\sigma = \frac{1}{2}\left[1 + \frac{100}{4r^2} - \left(1 + \frac{3}{16}\frac{10^4}{r^4}\right)\cos 0\right] = \frac{1}{2}\left(\frac{25}{r^2} - \frac{3}{16}\frac{10^4}{r^4}\right)$$

r	σ_θ/σ
5	−1.000
6	−0.376
7	−0.135
8	−0.034
9	0.011
10	0.031
11	0.039
12	0.042
13	0.041
14	0.039
15	0.037
16	0.035
17	0.032
18	0.030
19	0.027
20	0.025



3-50

$$D/d = \frac{1.5}{1} = 1.5$$

$$r/d = \frac{1/8}{1} = 0.125$$

Fig. A-15-8: $K_{ts} \doteq 1.39$

Fig. A-15-9: $K_t \doteq 1.60$

$$\sigma_A = K_t\frac{Mc}{I} = \frac{32K_tM}{\pi d^3} = \frac{32(1.6)(200)(14)}{\pi(1^3)} = 45\,630 \text{ psi}$$

$$\tau_A = K_{ts}\frac{Tc}{J} = \frac{16K_{ts}T}{\pi d^3} = \frac{16(1.39)(200)(15)}{\pi(1^3)} = 21\,240 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_A}{2} + \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \frac{45.63}{2} + \sqrt{\left(\frac{45.63}{2}\right)^2 + 21.24^2}$$

$$= 54.0 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{45.63}{2}\right)^2 + 21.24^2} = 31.2 \text{ kpsi} \quad \textit{Ans.}$$

3-51 As shown in Fig. 3-32, the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (3-50)

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad Ans.$$

$$\sigma_{r,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \quad Ans.$$

3-52 If $p_i = 0$, Eq. (3-49) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o/r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,\max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2} \quad Ans.$$

For σ_r, we have

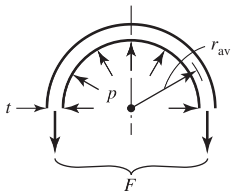
$$\sigma_r = \frac{-p_o r_o^2 + r_i^2 r_o^2 p_o/r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,\max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o \quad Ans.$$

3-53



$$F = pA = \pi r_{av}^2 p$$

$$\sigma_1 = \sigma_2 = \frac{F}{A_{wall}} = \frac{\pi r_{av}^2 p}{2\pi r_{av} t} = \frac{p r_{av}}{2t} \quad Ans.$$

3-54 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$ where σ_l is intermediate in value. From Prob. 4-50

$$\tau_{\max} = \frac{1}{2}(\sigma_{t,\,\max} - \sigma_{r,\,\max})$$

$$\tau_{\max} = \frac{p_i}{2}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} + 1\right)$$

Now solve for p_i using $r_o = 75$ mm, $r_i = 69$ mm, and $\tau_{\max} = 25$ MPa. This gives $p_i = 3.84$ MPa *Ans.*

3-55 Given $r_o = 5$ in, $r_i = 4.625$ in and referring to the solution of Prob. 3-54,

$$\tau_{\max} = \frac{350}{2}\left[\frac{(5)^2 + (4.625)^2}{(5)^2 - (4.625)^2} + 1\right]$$

$$= 2\,424 \text{ psi} \quad \textit{Ans.}$$

3-56 From Table A-20, $S_y = 57$ kpsi; also, $r_o = 0.875$ in and $r_i = 0.625$ in

From Prob. 3-52

$$\sigma_{t,\max} = -\frac{2p_o r_o^2}{r_o^2 - r_i^2}$$

Rearranging

$$p_o = \frac{\left(r_o^2 - r_i^2\right)(0.8S_y)}{2r_o^2}$$

Solving, gives $p_o = 11\,200$ psi *Ans.*

3-57 From Table A-20, $S_y = 390$ MPa; also $r_o = 25$ mm, $r_i = 20$ mm.

From Prob. 3-51

$$\sigma_{t,\max} = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad \text{therefore} \quad p_i = 0.8S_y\left(\frac{r_o^2 - r_i^2}{r_o^2 + r_i^2}\right)$$

solving gives $p_i = 68.5$ MPa *Ans.*

3-58 Since σ_t and σ_r are both positive and $\sigma_t > \sigma_r$

$$\tau_{\max} = (\sigma_t)_{\max}/2$$

where σ_t is max at r_i

Eq. (3-55) for $r = r_i = 0.375$ in

$$(\sigma_t)_{\max} = \frac{0.282}{386}\left[\frac{2\pi(7200)}{60}\right]^2\left(\frac{3+0.292}{8}\right)$$

$$\times\left[0.375^2 + 5^2 + \frac{(0.375^2)(5^2)}{0.375^2} - \frac{1+3(0.292)}{3+0.292}(0.375^2)\right] = 8556 \text{ psi}$$

$$\tau_{\max} = \frac{8556}{2} = 4278 \text{ psi} \quad \textit{Ans.}$$

Radial stress: $$\sigma_r = k\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Maxima: $$\frac{d\sigma_r}{dr} = k\left(2\frac{r_i^2 r_o^2}{r^3} - 2r\right) = 0 \quad \Rightarrow \quad r = \sqrt{r_i r_o} = \sqrt{0.375(5)} = 1.3693 \text{ in}$$

$$(\sigma_r)_{\max} = \frac{0.282}{386}\left[\frac{2\pi(7200)}{60}\right]^2\left(\frac{3+0.292}{8}\right)\left[0.375^2 + 5^2 - \frac{0.375^2(5^2)}{1.3693^2} - 1.3693^2\right]$$

$$= 3656 \text{ psi} \quad \textit{Ans.}$$

3-59

$$\omega = 2\pi(2069)/60 = 216.7 \text{ rad/s},$$
$$\rho = 3320\,\text{kg/m}^3,\ \nu = 0.24,\ r_i = 0.0125\,\text{m},\ r_o = 0.15\,\text{m};$$

use Eq. (3-55)

$$\sigma_t = 3320(216.7)^2\left(\frac{3+0.24}{8}\right)\left[(0.0125)^2 + (0.15)^2 + (0.15)^2 - \frac{1+3(0.24)}{3+0.24}(0.0125)^2\right](10)^{-6}$$

$$= 2.85 \text{ MPa} \quad \textit{Ans.}$$

3-60

$$\rho = \frac{(6/16)}{386(1/16)(\pi/4)(6^2 - 1^2)} = 5.655(10^{-4})\ \text{lbf}\cdot\text{s}^2/\text{in}^4$$

$\tau_{\max}$ is at bore and equals $\dfrac{\sigma_t}{2}$

Eq. (3-55)

$$(\sigma_t)_{\max} = 5.655(10^{-4})\left[\frac{2\pi(10\,000)}{60}\right]^2\left(\frac{3+0.20}{8}\right)\left[0.5^2 + 3^2 + 3^2 - \frac{1+3(0.20)}{3+0.20}(0.5)^2\right]$$

$$= 4496 \text{ psi}$$

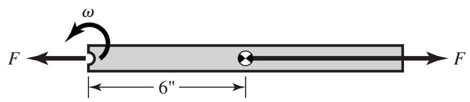
$$\tau_{\max} = \frac{4496}{2} = 2248 \text{ psi} \quad \textit{Ans.}$$

3-61

$$\omega = 2\pi(3000)/60 = 314.2 \text{ rad/s}$$

$$m = \frac{0.282(1.25)(12)(0.125)}{386}$$

$$= 1.370(10^{-3}) \text{ lbf} \cdot \text{s}^2/\text{in}$$



$$F = m\omega^2 r = 1.370(10^{-3})(314.2^2)(6)$$

$$= 811.5 \text{ lbf}$$

$$A_{nom} = (1.25 - 0.5)(1/8) = 0.093\,75 \text{ in}^2$$

$$\sigma_{nom} = \frac{811.5}{0.093\,75} = 8656 \text{ psi} \quad \textit{Ans.}$$

Note: Stress concentration Fig. A-15-1 gives $K_t \doteq 2.25$ which increases σ_{max} and fatigue.

3-62 to 3-67

$$\nu = 0.292, \quad E = 30 \text{ Mpsi (207 GPa)}, \quad r_i = 0$$

$$R = 0.75 \text{ in (20 mm)}, \quad r_o = 1.5 \text{ in (40 mm)}$$

Eq. (3-57)

$$p_{psi} = \frac{30(10^6)\delta}{0.75^3}\left[\frac{(1.5^2 - 0.75^2)(0.75^2 - 0)}{2(1.5^2 - 0)}\right] = 1.5(10^7)\delta \quad (1)$$

$$p_{Pa} = \frac{207(10^9)\delta}{0.020^3}\left[\frac{(0.04^2 - 0.02^2)(0.02^2 - 0)}{2(0.04^2 - 0)}\right] = 3.881(10^{12})\delta \quad (2)$$

3-62

$$\delta_{max} = \frac{1}{2}[40.042 - 40.000] = 0.021 \text{ mm} \quad \textit{Ans.}$$

$$\delta_{min} = \frac{1}{2}[40.026 - 40.025] = 0.0005 \text{ mm} \quad \textit{Ans.}$$

From (2)

$$p_{max} = 81.5 \text{ MPa}, \quad p_{min} = 1.94 \text{ MPa} \quad \textit{Ans.}$$

3-63

$$\delta_{max} = \frac{1}{2}(1.5016 - 1.5000) = 0.0008 \text{ in} \quad \textit{Ans.}$$

$$\delta_{min} = \frac{1}{2}(1.5010 - 1.5010) = 0 \quad \textit{Ans.}$$

Eq. (1) $\quad p_{max} = 12\,000 \text{ psi}, \quad p_{min} = 0 \quad \textit{Ans.}$

3-64

$$\delta_{\max} = \frac{1}{2}(40.059 - 40.000) = 0.0295\,\text{mm} \quad Ans.$$

$$\delta_{\min} = \frac{1}{2}(40.043 - 40.025) = 0.009\ \text{mm} \quad Ans.$$

Eq. (2) $\quad p_{\max} = 114.5\,\text{MPa}, \quad p_{\min} = 34.9\ \text{MPa} \quad Ans.$

3-65

$$\delta_{\max} = \frac{1}{2}(1.5023 - 1.5000) = 0.001\,15\,\text{in} \quad Ans.$$

$$\delta_{\min} = \frac{1}{2}(1.5017 - 1.5010) = 0.000\,35\ \text{in} \quad Ans.$$

Eq. (1) $\quad p_{\max} = 17\,250\,\text{psi} \quad p_{\min} = 5250\ \text{psi} \quad Ans.$

3-66

$$\delta_{\max} = \frac{1}{2}(40.076 - 40.000) = 0.038\,\text{mm} \quad Ans.$$

$$\delta_{\min} = \frac{1}{2}(40.060 - 40.025) = 0.0175\ \text{mm} \quad Ans.$$

Eq. (2) $\quad p_{\max} = 147.5\,\text{MPa} \quad p_{\min} = 67.9\ \text{MPa} \quad Ans.$

3-67

$$\delta_{\max} = \frac{1}{2}(1.5030 - 1.500) = 0.0015\,\text{in} \quad Ans.$$

$$\delta_{\min} = \frac{1}{2}(1.5024 - 1.5010) = 0.0007\ \text{in} \quad Ans.$$

Eq. (1) $\quad p_{\max} = 22\,500\,\text{psi} \quad p_{\min} = 10\,500\ \text{psi} \quad Ans.$

3-68

$$\delta = \frac{1}{2}(1.002 - 1.000) = 0.001\,\text{in} \quad r_i = 0, \quad R = 0.5\,\text{in}, \quad r_o = 1\,\text{in}$$

$$\nu = 0.292, \quad E = 30\,\text{Mpsi}$$

Eq. (3-57)

$$p = \frac{30(10^6)(0.001)}{0.5^3}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{2(1^2 - 0)}\right] = 2.25(10^4)\ \text{psi} \quad Ans.$$

Eq. (3-50) for outer member at $r_i = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(2.25)(10^4)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 37\,500\ \text{psi} \quad Ans.$$

Inner member, from Prob. 3-52

$$(\sigma_t)_i = -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right) = -\frac{2.25(10^4)(0.5^2)}{0.5^2 - 0}\left(1 + \frac{0}{0.5^2}\right) = -22\,500 \text{ psi} \quad Ans.$$

3-69

$$\nu_i = 0.292, \quad E_i = 30(10^6) \text{ psi}, \quad \nu_o = 0.211, \quad E_o = 14.5(10^6) \text{ psi}$$

$$\delta = \frac{1}{2}(1.002 - 1.000) = 0.001 \text{ in}, \quad r_i = 0, \quad R = 0.5, \quad r_o = 1$$

Eq. (3-56)

$$0.001 = \left[\frac{0.5}{14.5(10^6)}\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2} + 0.211\right) + \frac{0.5}{30(10^6)}\left(\frac{0.5^2 + 0}{0.5^2 - 0} - 0.292\right)\right] p$$

$$p = 13\,064 \text{ psi} \quad Ans.$$

Eq. (3-50) for outer member at $r_i = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(13\,064)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 21\,770 \text{ psi} \quad Ans.$$

Inner member, from Prob. 3-52

$$(\sigma_t)_i = -\frac{13\,064(0.5^2)}{0.5^2 - 0}\left(1 + \frac{0}{0.5^2}\right) = -13\,064 \text{ psi} \quad Ans.$$

3-70

$$\delta_{\max} = \frac{1}{2}(1.003 - 1.000) = 0.0015 \text{ in} \quad r_i = 0, \quad R = 0.5 \text{ in}, \quad r_o = 1 \text{ in}$$

$$\delta_{\min} = \frac{1}{2}(1.002 - 1.001) = 0.0005 \text{ in}$$

Eq. (3-57)

$$p_{\max} = \frac{30(10^6)(0.0015)}{0.5^3}\left[\frac{(1^2 - 0.5^2)(0.5^2 - 0)}{2(1^2 - 0)}\right] = 33\,750 \text{ psi} \quad Ans.$$

Eq. (3-50) for outer member at $r = 0.5$ in

$$(\sigma_t)_o = \frac{0.5^2(33\,750)}{1^2 - 0.5^2}\left(1 + \frac{1^2}{0.5^2}\right) = 56\,250 \text{ psi} \quad Ans.$$

For inner member, from Prob. 3-52, with $r = 0.5$ in

$$(\sigma_t)_i = -33\,750 \text{ psi} \quad Ans.$$

For $\delta_{\min}$ all answers are $0.0005/0.0015 = 1/3$ of above answers *Ans.*

3-71

$$\nu_i = 0.292, \quad E_i = 30 \text{ Mpsi}, \quad \nu_o = 0.334, \quad E_o = 10.4 \text{ Mpsi}$$

$$\delta_{\max} = \frac{1}{2}(2.005 - 2.000) = 0.0025 \text{ in}$$

$$\delta_{\min} = \frac{1}{2}(2.003 - 2.002) = 0.0005 \text{ in}$$

$$0.0025 = \left[\frac{1.0}{10.4(10^6)}\left(\frac{2^2+1^2}{2^2-1^2} + 0.334\right) + \frac{1.0}{30(10^6)}\left(\frac{1^2+0}{1^2-0} - 0.292\right)\right] p_{\max}$$

$$p_{\max} = 11\,576 \text{ psi} \quad \textit{Ans.}$$

Eq. (3-50) for outer member at $r = 1$ in

$$(\sigma_t)_o = \frac{1^2(11\,576)}{2^2 - 1^2}\left(1 + \frac{2^2}{1^2}\right) = 19\,293 \text{ psi} \quad \textit{Ans.}$$

Inner member from Prob. 3-52 with $r = 1$ in

$$(\sigma_t)_i = -11\,576 \text{ psi} \quad \textit{Ans.}$$

For $\delta_{\min}$ all above answers are $0.0005/0.0025 = 1/5$ *Ans.*

3-72

(a) Axial resistance

Normal force at fit interface

$$N = pA = p(2\pi Rl) = 2\pi pRl$$

Fully-developed friction force

$$F_{ax} = fN = 2\pi fpRl \quad \textit{Ans.}$$

(b) Torsional resistance at fully developed friction is

$$T = fRN = 2\pi fpR^2 l \quad \textit{Ans.}$$

3-73 $d = 1$ in, $r_i = 1.5$ in, $r_o = 2.5$ in.

From Table 3-4, for $R = 0.5$ in,

$$r_c = 1.5 + 0.5 = 2 \text{ in}$$

$$r_n = \frac{0.5^2}{2\left(2 - \sqrt{2^2 - 0.5^2}\right)} = 1.968\,245\,8 \text{ in}$$

$$e = r_c - r_n = 2.0 - 1.968\,245\,8 = 0.031\,754 \text{ in}$$

$$c_i = r_n - r_i = 1.9682 - 1.5 = 0.4682 \text{ in}$$

$$c_o = r_o - r_n = 2.5 - 1.9682 = 0.5318 \text{ in}$$

$$A = \pi d^2/4 = \pi(1)^2/4 = 0.7854 \text{ in}^2$$

$$M = Fr_c = 1000(2) = 2000 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65)

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{1000}{0.7854} + \frac{2000(0.4682)}{0.7854(0.031\,754)(1.5)} = 26\,300 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{1000}{0.7854} - \frac{2000(0.5318)}{0.7854(0.031\,754)(2.5)} = -15\,800 \text{ psi} \quad Ans.$$

3-74 Section AA:

$D = 0.75$ in, $r_i = 0.75/2 = 0.375$ in, $r_o = 0.75/2 + 0.25 = 0.625$ in

From Table 3-4, for $R = 0.125$ in,

$$r_c = (0.75 + 0.25)/2 = 0.500 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.5 - \sqrt{0.5^2 - 0.125^2}\right)} = 0.492\,061\,5 \text{ in}$$

$$e = 0.5 - r_n = 0.007\,939 \text{ in}$$

$$c_o = r_o - r_n = 0.625 - 0.492\,06 = 0.132\,94 \text{ in}$$

$$c_i = r_n - r_i = 0.492\,06 - 0.375 = 0.117\,06 \text{ in}$$

$$A = \pi(0.25)^2/4 = 0.049\,087$$

$$M = Fr_c = 100(0.5) = 50 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{100}{0.049\,09} + \frac{50(0.117\,06)}{0.049\,09(0.007\,939)(0.375)} = 42\,100 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{100}{0.049\,09} - \frac{50(0.132\,94)}{0.049\,09(0.007\,939)(0.625)} = -25\,250 \text{ psi} \quad Ans.$$

Section BB: Abscissa angle θ of line of radius centers is

$$\theta = \cos^{-1}\left(\frac{r_2 + d/2}{r_2 + d + D/2}\right)$$

$$= \cos^{-1}\left(\frac{0.375 + 0.25/2}{0.375 + 0.25 + 0.75/2}\right) = 60^\circ$$

$$M = F\frac{D + d}{2}\cos\theta = 100(0.5)\cos 60^\circ = 25 \text{ lbf} \cdot \text{in}$$

$$r_i = r_2 = 0.375 \text{ in}$$

$$r_o = r_2 + d = 0.375 + 0.25 = 0.625 \text{ in}$$

$$e = 0.007\,939 \text{ in} \quad \text{(as before)}$$

$$\sigma_i = \frac{F\cos\theta}{A} - \frac{Mc_i}{Aer_i}$$

$$= \frac{100\cos 60^\circ}{0.049\,09} - \frac{25(0.117\,06)}{0.049\,09(0.007\,939)0.375} = -19\,000 \text{ psi} \quad Ans.$$

$$\sigma_o = \frac{100\cos 60^\circ}{0.049\,09} + \frac{25(0.132\,94)}{0.049\,09(0.007\,939)0.625} = 14\,700 \text{ psi} \quad Ans.$$

On section BB, the shear stress due to the shear force is zero at the surface.

3-75 $r_i = 0.125$ in, $r_o = 0.125 + 0.1094 = 0.2344$ in

From Table 3-4 for $h = 0.1094$

$$r_c = 0.125 + 0.1094/2 = 0.1797 \text{ in}$$
$$r_n = 0.1094/\ln(0.2344/0.125) = 0.174\,006 \text{ in}$$
$$e = r_c - r_n = 0.1797 - 0.174\,006 = 0.005\,694 \text{ in}$$
$$c_i = r_n - r_i = 0.174\,006 - 0.125 = 0.049\,006 \text{ in}$$
$$c_o = r_o - r_n = 0.2344 - 0.174\,006 = 0.060\,394 \text{ in}$$
$$A = 0.75(0.1094) = 0.082\,050 \text{ in}^2$$
$$M = F(4 + h/2) = 3(4 + 0.1094/2) = 12.16 \text{ lbf} \cdot \text{in}$$
$$\sigma_i = -\frac{3}{0.082\,05} - \frac{12.16(0.0490)}{0.082\,05(0.005\,694)(0.125)} = -10\,240 \text{ psi} \quad \textit{Ans.}$$
$$\sigma_o = -\frac{3}{0.082\,05} + \frac{12.16(0.0604)}{0.082\,05(0.005\,694)(0.2344)} = 6670 \text{ psi} \quad \textit{Ans.}$$

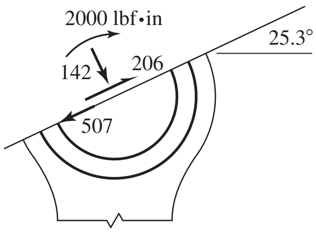
3-76 Find the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$.

$$F_x = F_{1x} + F_{2x} = 250\cos 60^\circ + 333\cos 0^\circ = 458 \text{ lbf}$$
$$F_y = F_{1y} + F_{2y} = 250\sin 60^\circ + 333\sin 0^\circ = 216.5 \text{ lbf}$$
$$F = (458^2 + 216.5^2)^{1/2} = 506.6 \text{ lbf}$$

This is the pin force on the lever which acts in a direction

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{216.5}{458} = 25.3^\circ$$

On the 25.3° surface from $\mathbf{F}_1$



$$F_t = 250\cos(60^\circ - 25.3^\circ) = 206 \text{ lbf}$$
$$F_n = 250\sin(60^\circ - 25.3^\circ) = 142 \text{ lbf}$$
$$r_c = 1 + 3.5/2 = 2.75 \text{ in}$$
$$A = 2[0.8125(0.375) + 1.25(0.375)] = 1.546\,875 \text{ in}^2$$

The denominator of Eq. (3-63), given below, has four additive parts.

$$r_n = \frac{A}{\int (dA/r)}$$

For $\int dA/r$, add the results of the following equation for each of the four rectangles.

$$\int_{r_i}^{r_o} \frac{b\,dr}{r} = b \ln \frac{r_o}{r_i}, \qquad b = \text{width}$$

$$\int \frac{dA}{r} = 0.375 \ln \frac{1.8125}{1} + 1.25 \ln \frac{2.1875}{1.8125} + 1.25 \ln \frac{3.6875}{3.3125} + 0.375 \ln \frac{4.5}{3.6875}$$

$$= 0.666\,810\,6$$

$$r_n = \frac{1.546\,875}{0.666\,810\,6} = 2.3198 \text{ in}$$

$$e = r_c - r_n = 2.75 - 2.3198 = 0.4302 \text{ in}$$

$$c_i = r_n - r_i = 2.320 - 1 = 1.320 \text{ in}$$

$$c_o = r_o - r_n = 4.5 - 2.320 = 2.180 \text{ in}$$

Shear stress due to 206 lbf force is zero at inner and outer surfaces.

$$\sigma_i = -\frac{142}{1.547} + \frac{2000(1.32)}{1.547(0.4302)(1)} = 3875 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{142}{1.547} - \frac{2000(2.18)}{1.547(0.4302)(4.5)} = -1548 \text{ psi} \quad \textit{Ans.}$$

3-77

$$A = (6 - 2 - 1)(0.75) = 2.25 \text{ in}^2$$

$$r_c = \frac{6+2}{2} = 4 \text{ in}$$

Similar to Prob. 3-76,

$$\int \frac{dA}{r} = 0.75 \ln \frac{3.5}{2} + 0.75 \ln \frac{6}{4.5} = 0.635\,473\,4 \text{ in}$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{2.25}{0.635\,473\,4} = 3.5407 \text{ in}$$

$$e = 4 - 3.5407 = 0.4593 \text{ in}$$

$$\sigma_i = \frac{5000}{2.25} + \frac{20\,000(3.5407 - 2)}{2.25(0.4593)(2)} = 17\,130 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{5000}{2.25} - \frac{20\,000(6 - 3.5407)}{2.25(0.4593)(6)} = -5710 \text{ psi} \quad \textit{Ans.}$$

3-78

$$A = \int_{r_i}^{r_o} b\,dr = \int_2^6 \frac{2}{r}\,dr = 2 \ln \frac{6}{2}$$

$$= 2.197\,225 \text{ in}^2$$

$$r_c = \frac{1}{A}\int_{r_i}^{r_o} br\,dr = \frac{1}{2.197\,225}\int_2^6 \frac{2r}{r}\,dr$$

$$= \frac{2}{2.197\,225}(6-2) = 3.640\,957 \text{ in}$$

$$r_n = \frac{A}{\int_{r_i}^{r_o}(b/r)\,dr} = \frac{2.197\,225}{\int_2^6 (2/r^2)\,dr}$$

$$= \frac{2.197\,225}{2[1/2 - 1/6]} = 3.295\,837 \text{ in}$$

$$e = R - r_n = 3.640\,957 - 3.295\,837 = 0.345\,12$$

$$c_i = r_n - r_i = 3.2958 - 2 = 1.2958 \text{ in}$$

$$c_o = r_o - r_n = 6 - 3.2958 = 2.7042 \text{ in}$$

$$\sigma_i = \frac{20\,000}{2.197} + \frac{20\,000(3.641)(1.2958)}{2.197(0.345\,12)(2)} = 71\,330 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{20\,000}{2.197} - \frac{20\,000(3.641)(2.7042)}{2.197(0.345\,12)(6)} = -34\,180 \text{ psi} \quad \textit{Ans.}$$

3-79 $r_c = 12$ in, $M = 20(2+2) = 80$ kip · in

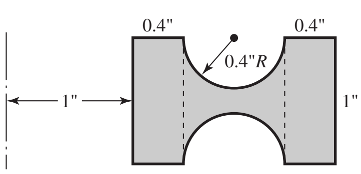
From statics book, $I = \dfrac{\pi}{4}a^3 b = \dfrac{\pi}{4}(2^3)1 = 2\pi \text{ in}^4$

Inside: $\sigma_i = \dfrac{F}{A} + \dfrac{My}{I}\dfrac{r_c}{r_i} = \dfrac{20}{2\pi} + \dfrac{80(2)}{2\pi}\dfrac{12}{10} = 33.7$ kpsi *Ans.*

Outside: $\sigma_o = \dfrac{F}{A} - \dfrac{My}{I}\dfrac{r_c}{r_o} = \dfrac{20}{2\pi} - \dfrac{80(2)}{2\pi}\dfrac{12}{14} = -18.6$ kpsi *Ans.*

Note: A much more accurate solution (see the 7th edition) yields $\sigma_i = 32.25$ kpsi and $\sigma_o = -19.40$ kpsi

3-80



For rectangle, $\displaystyle\int \frac{dA}{r} = b \ln r_o/r_i$

For circle, $\dfrac{A}{\int(dA/r)} = \dfrac{r^2}{2\left(r_c - \sqrt{r_c^2 - r^2}\right)}$, $A_o = \pi r^2$

$$\therefore \int \frac{dA}{r} = 2\pi\left(r_c - \sqrt{r_c^2 - r^2}\right)$$

$$\sum \int \frac{dA}{r} = 1 \ln \frac{2.6}{1} - 2\pi \left(1.8 - \sqrt{1.8^2 - 0.4^2}\right) = 0.672\,723\,4$$

$$A = 1(1.6) - \pi(0.4^2) = 1.097\,345\,2 \text{ in}^2$$

$$r_n = \frac{1.097\,345\,2}{0.672\,723\,4} = 1.6312 \text{ in}$$

$$e = 1.8 - r_n = 0.1688 \text{ in}$$

$$c_i = 1.6312 - 1 = 0.6312 \text{ in}$$

$$c_o = 2.6 - 1.6312 = 0.9688 \text{ in}$$

$$M = 3000(5.8) = 17\,400 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{3}{1.0973} + \frac{17.4(0.6312)}{1.0973(0.1688)(1)} = 62.03 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{3}{1.0973} - \frac{17.4(0.9688)}{1.0973(0.1688)(2.6)} = -32.27 \text{ kpsi} \quad \textit{Ans.}$$

3-81 From Eq. (3-68)

$$a = KF^{1/3} = F^{1/3} \left\{ \frac{3}{8} \frac{2[(1-\nu^2)/E]}{2(1/d)} \right\}^{1/3}$$

Use $\nu = 0.292$, F in newtons, E in N/mm^2 and d in mm, then

$$K = \left\{ \frac{3}{8} \frac{[(1-0.292^2)/207\,000]}{1/25} \right\}^{1/3} = 0.0346$$

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi (KF^{1/3})^2}$$

$$= \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi (0.0346)^2}$$

$$= 399F^{1/3} \text{ MPa} = |\sigma_{\max}| \qquad \textit{Ans.}$$

$$\tau_{\max} = 0.3 p_{\max}$$

$$= 120F^{1/3} \text{ MPa} \qquad \textit{Ans.}$$

3-82 From Prob. 3-81,

$$K = \left\{ \frac{3}{8} \frac{2[(1-0.292^2)/207\,000]}{1/25 + 0} \right\}^{1/3} = 0.0436$$

$$p_{\max} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi (0.0436)^2} = 251F^{1/3}$$

and so, $\sigma_z = -251F^{1/3}$ MPa *Ans.*

$$\tau_{\max} = 0.3(251)F^{1/3} = 75.3F^{1/3} \text{ MPa} \quad \textit{Ans.}$$

$$z = 0.48a = 0.48(0.0436)18^{1/3} = 0.055 \text{ mm} \quad \textit{Ans.}$$

3-83 $\nu_1 = 0.334$, $E_1 = 10.4$ Mpsi, $l = 2$ in, $d_1 = 1$ in, $\nu_2 = 0.211$, $E_2 = 14.5$ Mpsi, $d_2 = -8$ in.

With $b = K_c F^{1/2}$, from Eq. (3-73),

$$K_c = \left(\frac{2}{\pi(2)} \frac{(1 - 0.334^2)/[10.4(10^6)] + (1 - 0.211^2)/[14.5(10^6)]}{1 - 0.125} \right)^{1/2}$$

$$= 0.000\,234\,6$$

Be sure to check σ_x for both ν_1 and ν_2. Shear stress is maximum in the aluminum roller. So,

$$\tau_{\max} = 0.3 p_{\max}$$

$$p_{\max} = \frac{4000}{0.3} = 13\,300 \text{ psi}$$

Since $p_{\max} = 2F/(\pi bl)$ we have

$$p_{\max} = \frac{2F}{\pi l K_c F^{1/2}} = \frac{2F^{1/2}}{\pi l K_c}$$

So,

$$F = \left(\frac{\pi l K_c p_{\max}}{2} \right)^2$$

$$= \left(\frac{\pi(2)(0.000\,234\,6)(13\,300)}{2} \right)^2$$

$$= 96.1 \text{ lbf} \quad Ans.$$

3-84 Good class problem

3-85 From Table A-5, $\nu = 0.211$

$$\frac{\sigma_x}{p_{\max}} = (1 + \nu) - \frac{1}{2} = (1 + 0.211) - \frac{1}{2} = 0.711$$

$$\frac{\sigma_y}{p_{\max}} = 0.711$$

$$\frac{\sigma_z}{p_{\max}} = 1$$

These are principal stresses

$$\frac{\tau_{\max}}{p_{\max}} = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}(1 - 0.711) = 0.1445$$

3-86 From Table A-5: $\nu_1 = 0.211$, $\nu_2 = 0.292$, $E_1 = 14.5(10^6)$ psi, $E_2 = 30(10^6)$ psi, $d_1 = 6$ in, $d_2 = \infty$, $l = 2$ in

(a) Eq. (3-73): $$b = \sqrt{\frac{2(800)}{\pi(2)}\,\frac{(1-0.211^2)/14.5(10^6) + (1-0.292^2)/[30(10^6)]}{1/6 + 1/\infty}}$$

$$= 0.012\,135 \text{ in}$$

$$p_{\max} = \frac{2(800)}{\pi(0.012\,135)(2)} = 20\,984 \text{ psi}$$

For $z = 0$ in,

$$\sigma_{x1} = -2\nu_1 p_{\max} = -2(0.211)20\,984 = -8855 \text{ psi in wheel}$$
$$\sigma_{x2} = -2(0.292)20\,984 = -12\,254 \text{ psi}$$

In plate

$$\sigma_y = -p_{\max} = -20\,984 \text{ psi}$$
$$\sigma_z = -20\,984 \text{ psi}$$

These are principal stresses.

(b) For $z = 0.010$ in,

$$\sigma_{x1} = -4177 \text{ psi} \quad \text{in wheel}$$
$$\sigma_{x2} = -5781 \text{ psi} \quad \text{in plate}$$
$$\sigma_y = -3604 \text{ psi}$$
$$\sigma_z = -16\,194 \text{ psi}$$