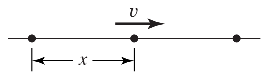
Chapter 1

Problems 1-1 through 1-4 are for student research.

1-5

(a) Point vehicles



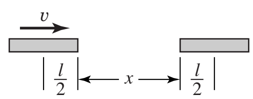
$$Q = \frac{\text{cars}}{\text{hour}} = \frac{v}{x} = \frac{42.1v - v^2}{0.324}$$

Seek stationary point maximum

$$\frac{dQ}{dv} = 0 = \frac{42.1 - 2v}{0.324} \quad \therefore v^* = 21.05\,\text{mph}$$

$$Q^* = \frac{42.1(21.05) - 21.05^2}{0.324} = 1368\,\text{cars/h} \quad \textit{Ans.}$$

(b)



$$Q = \frac{v}{x + l} = \left(\frac{0.324}{v(42.1) - v^2} + \frac{l}{v}\right)^{-1}$$

Maximize Q with $l = 10/5280$ mi

v	Q	
22.18	1221.431	
22.19	1221.433	
22.20	1221.435	←
22.21	1221.435	
22.22	1221.434	

$$\% \text{ loss of throughput} = \frac{1368 - 1221}{1221} = 12\% \quad \textit{Ans.}$$

(c) $\%$ increase in speed $\dfrac{22.2 - 21.05}{21.05} = 5.5\%$

Modest change in optimal speed *Ans.*

1-6 This and the following problem may be the student's first experience with a figure of merit.

- Formulate fom to reflect larger figure of merit for larger merit.
- Use a maximization optimization algorithm. When one gets into computer implementation and answers are not known, minimizing instead of maximizing is the largest error one can make.

$$\sum F_V = F_1 \sin\theta - W = 0$$
$$\sum F_H = -F_1 \cos\theta - F_2 = 0$$

From which

$$F_1 = W/\sin\theta$$
$$F_2 = -W\cos\theta/\sin\theta$$

$$\begin{aligned}
\text{fom} &= -\$ = -\text{¢}\gamma\ (\text{volume}) \\
&\doteq -\text{¢}\gamma(l_1 A_1 + l_2 A_2) \\
A_1 &= \frac{F_1}{S} = \frac{W}{S\sin\theta}, \quad l_2 = \frac{l_1}{\cos\theta} \\
A_2 &= \left|\frac{F_2}{S}\right| = \frac{W\cos\theta}{S\sin\theta} \\
\text{fom} &= -\text{¢}\gamma\left(\frac{l_2}{\cos\theta}\,\frac{W}{S\sin\theta} + \frac{l_2 W\cos\theta}{S\sin\theta}\right) \\
&= \frac{-\text{¢}\gamma W l_2}{S}\left(\frac{1+\cos^2\theta}{\cos\theta\sin\theta}\right)
\end{aligned}$$

Set leading constant to unity

$\theta°$	fom
0	$-\infty$
20	-5.86
30	-4.04
40	-3.22
45	-3.00
50	-2.87
54.736	-2.828
60	-2.886

$\theta^* = 54.736°$ *Ans.*

$\text{fom}^* = -2.828$

Alternative:

$$\frac{d}{d\theta}\left(\frac{1+\cos^2\theta}{\cos\theta\sin\theta}\right) = 0$$

And solve resulting transcendental for θ^*.

Check second derivative to see if a maximum, minimum, or point of inflection has been found. Or, evaluate fom on either side of θ^*.

1-7

(a) $x_1 + x_2 = X_1 + e_1 + X_2 + e_2$

$$\text{error} = e = (x_1 + x_2) - (X_1 + X_2)$$
$$= e_1 + e_2 \quad \textit{Ans.}$$

(b) $x_1 - x_2 = X_1 + e_1 - (X_2 + e_2)$

$$e = (x_1 - x_2) - (X_1 - X_2) = e_1 - e_2 \quad \textit{Ans.}$$

(c) $x_1 x_2 = (X_1 + e_1)(X_2 + e_2)$

$$e = x_1 x_2 - X_1 X_2 = X_1 e_2 + X_2 e_1 + e_1 e_2$$
$$\doteq X_1 e_2 + X_2 e_1 = X_1 X_2 \left(\frac{e_1}{X_1} + \frac{e_2}{X_2} \right) \quad \textit{Ans.}$$

(d) $\dfrac{x_1}{x_2} = \dfrac{X_1 + e_1}{X_2 + e_2} = \dfrac{X_1}{X_2}\left(\dfrac{1 + e_1/X_1}{1 + e_2/X_2}\right)$

$$\left(1 + \frac{e_2}{X_2}\right)^{-1} \doteq 1 - \frac{e_2}{X_2} \quad \text{and} \quad \left(1 + \frac{e_1}{X_1}\right)\left(1 - \frac{e_2}{X_2}\right) \doteq 1 + \frac{e_1}{X_1} - \frac{e_2}{X_2}$$

$$e = \frac{x_1}{x_2} - \frac{X_1}{X_2} \doteq \frac{X_1}{X_2}\left(\frac{e_1}{X_1} - \frac{e_2}{X_2}\right) \quad \textit{Ans.}$$

1-8

(a)

$$x_1 = \sqrt{5} = 2.236\ 067\ 977\ 5$$
$$X_1 = 2.23 \quad \text{3-correct digits}$$
$$x_2 = \sqrt{6} = 2.449\ 487\ 742\ 78$$
$$X_2 = 2.44 \quad \text{3-correct digits}$$
$$x_1 + x_2 = \sqrt{5} + \sqrt{6} = 4.685\ 557\ 720\ 28$$
$$e_1 = x_1 - X_1 = \sqrt{5} - 2.23 = 0.006\ 067\ 977\ 5$$
$$e_2 = x_2 - X_2 = \sqrt{6} - 2.44 = 0.009\ 489\ 742\ 78$$
$$e = e_1 + e_2 = \sqrt{5} - 2.23 + \sqrt{6} - 2.44 = 0.015\ 557\ 720\ 28$$

$$\text{Sum} = x_1 + x_2 = X_1 + X_2 + e$$
$$= 2.23 + 2.44 + 0.015\ 557\ 720\ 28$$
$$= 4.685\ 557\ 720\ 28 \text{ (Checks)} \quad \textit{Ans.}$$

(b) $X_1 = 2.24, \quad X_2 = 2.45$

$$e_1 = \sqrt{5} - 2.24 = -0.003\ 932\ 022\ 50$$
$$e_2 = \sqrt{6} - 2.45 = -0.000\ 510\ 257\ 22$$
$$e = e_1 + e_2 = -0.004\ 442\ 279\ 72$$
$$\text{Sum} = X_1 + X_2 + e$$
$$= 2.24 + 2.45 + (-0.004\ 442\ 279\ 72)$$
$$= 4.685\ 557\ 720\ 28 \quad \textit{Ans.}$$

1-9

(a) $\sigma = 20(6.89) = 137.8$ MPa

(b) $F = 350(4.45) = 1558$ N $= 1.558$ kN

(c) $M = 1200$ lbf · in $(0.113) = 135.6$ N · m

(d) $A = 2.4(645) = 1548$ mm^2

(e) $I = 17.4$ in^4 $(2.54)^4 = 724.2$ cm^4

(f) $A = 3.6(1.610)^2 = 9.332$ km^2

(g) $E = 21(1000)(6.89) = 144.69(10^3)$ MPa $= 144.7$ GPa

(h) $v = 45$ mi/h $(1.61) = 72.45$ km/h

(i) $V = 60$ in^3 $(2.54)^3 = 983.2$ cm$^3 = 0.983$ liter

1-10

(a) $l = 1.5/0.305 = 4.918$ ft $= 59.02$ in

(b) $\sigma = 600/6.89 = 86.96$ kpsi

(c) $p = 160/6.89 = 23.22$ psi

(d) $Z = 1.84(10^5)/(25.4)^3 = 11.23$ in^3

(e) $w = 38.1/175 = 0.218$ lbf/in

(f) $\delta = 0.05/25.4 = 0.00197$ in

(g) $v = 6.12/0.0051 = 1200$ ft/min

(h) $\epsilon = 0.0021$ in/in

(i) $V = 30/(0.254)^3 = 1831$ in^3

1-11

(a) $\sigma = \dfrac{200}{15.3} = 13.1\ \text{MPa}$

(b) $\sigma = \dfrac{42(10^3)}{6(10^{-2})^2} = 70(10^6)\ \text{N/m}^2 = 70\ \text{MPa}$

(c) $y = \dfrac{1200(800)^3(10^{-3})^3}{3(207)10^9(64)10^3(10^{-3})^4} = 1.546(10^{-2})\ \text{m} = 15.5\ \text{mm}$

(d) $\theta = \dfrac{1100(250)(10^{-3})}{79.3(10^9)(\pi/32)(25)^4(10^{-3})^4} = 9.043(10^{-2})\ \text{rad} = 5.18^\circ$

1-12

(a) $\sigma = \dfrac{600}{20(6)} = 5\ \text{MPa}$

(b) $I = \dfrac{1}{12}8(24)^3 = 9216\ \text{mm}^4$

(c) $I = \dfrac{\pi}{64}32^4(10^{-1})^4 = 5.147\ \text{cm}^4$

(d) $\tau = \dfrac{16(16)}{\pi(25^3)(10^{-3})^3} = 5.215(10^6)\ \text{N/m}^2 = 5.215\ \text{MPa}$

1-13

(a) $\tau = \dfrac{120(10^3)}{(\pi/4)(20^2)} = 382\ \text{MPa}$

(b) $\sigma = \dfrac{32(800)(800)(10^{-3})}{\pi(32)^3(10^{-3})^3} = 198.9(10^6)\ \text{N/m}^2 = 198.9\,\text{MPa}$

(c) $Z = \dfrac{\pi}{32(36)}(36^4 - 26^4) = 3334\ \text{mm}^3$

(d) $k = \dfrac{(1.6)^4\,(10^{-3})^4(79.3)(10^9)}{8(19.2)^3(10^{-3})^3(32)} = 286.8\,\text{N/m}$