CMPS 251-Numerical Computing
Chapter 4: Practice Problems
Not Due

Reading Material:
- Cheney & Kincaid: Sections 4.1-4.3

Notes: Try to solve as many problems as possible. These are important for the exam.

Problem 1

Find the polynomial of degree 2 that interpolates at the datapoints $(0, 1)$, $(1, 2)$, and $(4, 2)$. You should get $p_2(t) = -\frac{1}{4}t^2 + \frac{5}{4}t + 1$. Try it using Lagrange polynomials and monomials.

Problem 2

For each function listed below, use divided difference tables to construct the Newton interpolating polynomial for the set of nodes specified.

(a) $f(x) = \sqrt{x}, \quad x_i = 0, 1, 4.$

(b) $f(x) = \ln x, \quad x_i = 1, 3/2, 2.$

(c) $f(x) = \sin(\pi x), \quad x_i = 0, 1/4, 1/2, 3/4, 1.$

(d) $f(x) = \log_2(x), \quad x_i = 1, 2, 4.$

(e) $f(x) = \sin(\pi x), \quad x_i = -1, 0, 1.$

Problem 3

What is the error in quadratic interpolation to $f(x) = \sqrt{x}$, using equally spaced nodes on the interval $[\frac{1}{4}, 1]$? What about a quartic interpolation?

Problem 4

Prove that

$$f''(x_0) = \frac{f(x_0 - h) - 2f(x_0) + f(x_0 + h)}{h^2} + ch^2 + \mathcal{O}(h^4)$$

and find c. This is known as the three point central difference for the second derivative.

Problem 5

Find the second degree polynomial which interpolates $f(x) = \frac{8}{x}$ on the interval $[1, 4]$ at the points $1, 2, 4$ and find an approximation to $f(3)$ using the polynomial. Find a bound on the error $|f(3) - p_2(3)|$ using Theorem 1 in the slides. Compare it to the true value.

Problem 6

Consider $f(x) = e^{-x}\sin(x/2)$. Approximate $f''(1.4)$ using

- Three point backward difference $(h = 0.2)$

- Three point forward difference ($h = 0.2$)
- Three point central difference ($h = 0.2$)
- Three point central difference ($h = 0.1$)

Problem 7

Approximate $f(x) = 1/x$ using the three points $(2, 0.5)$, $(2.5, 0.4)$, $(4, 0.25)$. Use Neville's algorithm.

Problem 8

Given the data points

x	4	3.9	3.8	3.7
y	-0.06604	-0.02724	0.01282	0.05383

Determine the root of $f(x) = 0$ by Neville's method.

Problem 9

Let $f(x) = 3xe^x - \cos(x)$. Use the following data and the appropriate difference equation to approximate $f''(1.3)$ with $h = 0.1$ and $h = 0.01$.

x	1.20	1.29	1.30	1.31	1.40
y	11.59006	13.78176	14.04276	14.30741	16.86187

Problem 10

Analyze the round-off errors for the formula

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2} f''(\xi_0)$$

Suppose than in evaluating $f(x_0 + h)$ and $f(x_0)$, we encounter the errors $e(x_0 + h)$ and $e(x_0)$, respectively. And for instance the true value $f(x_0)$ is related to the computed value $\tilde{f}(x_0)$ by $f(x_0) = \tilde{f}(x_0) + e(x_0)$. Assume a machine precision ϵ.

Problem 11

Use Richardson extrapolation and calculate $D(1, 1)$ for the the following function

$$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2, \quad h = 0.5, \quad x = 0.5$$

Use the two point central difference equation to estimate the derivatives.

Problem 12

Given the data points

x	0	0	1	1
y	0	1	0	1
$f(x,y)$	5	4	3	6

Find the bivariate polynomial interpolating these points. Use Lagrange interpolation.

Problem 13

Find an approximation to $f'(x_1)$ by differentiating the quadratic polynomial which interpolates f at the points $x_0 = x_1 - h_1$, x_1, $x_2 = x_1 + h_2$. What does the formula reduce to when $h_1 = h_2 = h$?