CMPS 251
Assignment 1-Solution

Problem 1

Code

```
function [p pp]=horner(a,r)
n=length(a);
alpha=a(n);
beta=0;
for i=n-1:-1:1
    beta=alpha+r*beta;
    alpha=a(i)+r*alpha;
end
p=alpha;
pp=beta;
```

Hand Calculations

	2	0	9	-16	12	
-6		-12	72	-486	3012	
	2	-12	81	-502	3024	p(r)
		-12	144	-1350		
	2	-24	225	-1852	p'(r)	

	2	-3	-5	3	8	
2		4	2	-6	-6	
	2	1	-3	-3	2	p(r)
		4	10	14		
	2	5	7	11	p'(r)	

	3	0	-38	5	0	-1	
4		12	48	40	180	720	
	3	12	10	45	180	719	p(r)
		12	96	424	1876		
	3	24	106	469	2056	p'(r)	

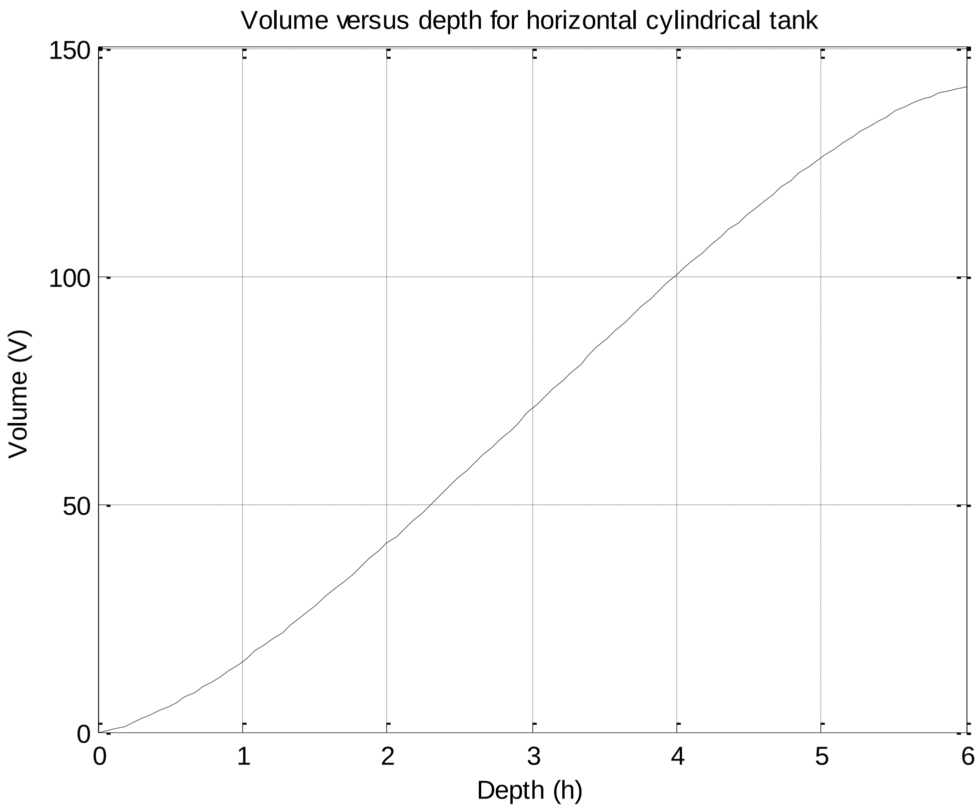
Problem 2

Code

```
function V = cylinder(r, L, h, plot_title)
V = (r^2*acos((r-h)./r)-(r-h).*sqrt(2*r*h-h.^2))*L;
plot(h,V,'k')
title(plot_title);
xlabel('Depth (h)');
ylabel('Volume (V)');
grid on;
```

Example

```
>> r=3;
>> L=5;
>> h=linspace(0,2*r);
>> V=cylinder(r,L,h,'Volume versus depth for horizontal cylindrical tank');
```



Problem 3

Code (x is taken as a vector to calculate in one shot for all values of x)

```
function [y n]=cosTaylor(x)
format long
xrad=x*pi/180;
n=zeros(1,length(x));
```

```
fprintf('x              y                          yexact                iterations
Error\n');
fprintf('-----------------------------------------------------------------------------
--------\n');
for k=1:length(x)
    E=1000;
    sum=0;
    while (E>0.000001)
    fact=factorial(2*n(k));
    sum=sum+(((-1)^n(k))*(xrad(k)^(2*n(k)))/fact);
    n(k)=n(k)+1;
    S(n(k))=sum;
        if n(k)>=2
        E=abs((S(n(k))-S(n(k)-1))/S(n(k)-1));
        end
    end
y(k)=sum;
yex(k)=cos(xrad(k));
abser(k)=abs(y(k)-yex(k));
fprintf('%3u \t %18.15f \t %18.15f \t %3u \t
%e\n',x(k),y(k),yex(k),n(k),abser(k));
end
```

Output

```
x             y                       yexact                iterations      Error
----------------------------------------------------------------------------
  0       1.000000000000000        1.000000000000000          2    0.000000e+00
 15       0.965925825742181        0.965925826289068          4    5.468872e-10
 30       0.866025404210352        0.866025403784439          5    4.259136e-10
 45       0.707106781071925        0.707106781186548          6    1.146229e-10
 60       0.499999996390943        0.500000000000000          6    3.609057e-09
 75       0.258819045596420        0.258819045102521          7    4.938989e-10
 90       0.000000000000000        0.000000000000000         15    1.869282e-17
105      -0.258819045867583       -0.258819045102521          8    7.650619e-10
120      -0.500000006458324       -0.500000000000000          8    6.458324e-09
135      -0.707106780415388       -0.707106781186547          9    7.711598e-10
150      -0.866025398664009       -0.866025403784439          9    5.120430e-09
165      -0.965925826910442       -0.965925826289068         10    6.213735e-10
180      -1.000000003529080       -1.000000000000000         10    3.529080e-09
195      -0.965925843719891       -0.965925826289068         10    1.743082e-08
210      -0.866025401551483       -0.866025403784439         11    2.232955e-09
225      -0.707106771034927       -0.707106781186548         11    1.015162e-08
240      -0.500000001336014       -0.500000000000000         12    1.336014e-09
255      -0.258819050807114       -0.258819045102521         12    5.704594e-09
270       0.000000000000001       -0.000000000000000         21    1.486629e-15
285       0.258819048229428        0.258819045102521         13    3.126907e-09
300       0.500000011825080        0.500000000000000         13    1.182508e-08
315       0.707106779502986        0.707106781186547         14    1.683561e-09
330       0.866025397611516        0.866025403784438         14    6.172923e-09
345       0.965925804932024        0.965925826289068         14    2.135704e-08
360       1.000000003196771        1.000000000000000         15    3.196771e-09
```

Problem 4

$$\ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$$

To calculate the Mclaurin Series, you can use the Taylor series of $\ln(1+x)$ to obtain that of $\ln(1-x)$ or you can derive it as explained in the slides by calculating $f^k(x)$ and $f^k(0)$ using the fact that (ln(u))'=u'/u.

$$f(x) = \ln\left(\frac{1+x}{1-x}\right) \Rightarrow f(0) = 0$$

$$f'(x) = \frac{1}{x+1} - \frac{1}{x-1} \Rightarrow f'(0) = 2$$

$$f''(x) = -\frac{1}{(x+1)^2} + \frac{1}{(x-1)^2} \Rightarrow f''(0) = 0$$

$$\vdots$$

$$f^k(x) = \frac{(-1)^{k-1}(k-1)!}{(x+1)^k} + \frac{(-1)^k(k-1)!}{(x-1)^k} \Rightarrow \begin{cases} f^k(0) = 0 \text{ if k is even} \\ f^k(0) = 2(k-1)! \text{ if k is odd} \end{cases}$$

Finally, the Mclaurin series of $\ln\left(\frac{1+x}{1-x}\right)$ can be written as

$$f(x) = \sum_{k=0}^{\infty} \frac{f^k(0)}{k!} x^k = 2\sum_{n=0}^{\infty} \frac{(2n+1-1)!}{(2n+1)!} x^{2n+1} = 2\sum_{n=0}^{\infty} \frac{(2n)!}{(2n+1)!} x^{2n+1} = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, \ |x| < 1$$

Problem 5

For both series $a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$ and $\lim_{n\to\infty} a_n = 0$

1- $\frac{1}{(n+1)^2+3} < 0.001 \Rightarrow (n+1)^2 + 3 > 1000 \Rightarrow (n+1)^2 > 997 \Rightarrow n \geq 31$

2- $\frac{1}{\ln(\ln(n+1+3))} < 0.001 \Rightarrow \ln(\ln(n+4)) > 1000 \Rightarrow n > -4 + e^{e^{1000}} \sim 10^{10^{433.9322662145524}}$

We can clearly note that for the second series, an infinite number of terms is needed to achieve an error less than 0.001. For the first series, 31 terms are enough.