# AMERICAN UNIVERSITY OF BEIRUT <br> Faculty of Arts and Sciences <br> Computer Science Department <br> CMPS 251 <br> MID TERM EXAMINATION <br> SUMMER 2003-2004 <br> Closed Book, One hour 15 minutes 

SUBMIT THE QUESTION SHEET WITH BOOKLET (ONLY NON-PROGRAMMABLE AND NON-GRAPHIC CALCULATORS ARE ALLOWED)

| STUDENT NAME |  |
| :--- | :--- |
| ID NUMBER |  |

1. Let $x \in \mathbb{F} \equiv \mathbb{F}\left(b, p, e_{\min }, e_{\max }\right)$, with $x= \pm m \times b^{e}$.
-(5 points) Fill in the bounds on $m$
$\qquad$
and if $m_{2}=\operatorname{succ}\left(m_{1}\right)$, both $m_{1}$ and $m_{2}$ mantissas in $\mathbb{F}$, find $m_{2}-m_{1}$.
-(10 points)Fill in the missing statements in the following MATLAB program that generates the positive elements of a floating-point system $\mathbb{F}\left(b, p, e_{\text {min }}, e_{\text {max }}\right)$.
function $x=f$ loat ( $b, p, e m i n$, emax)
$\mathrm{x}=0$;
epsm=b^(-p+1);
\%M represents all possible values taken by the mantissa
M=. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ;
$\mathrm{E}=\mathrm{b}^{\wedge} \mathrm{emin}$;
for
$\mathrm{x}=\left[\begin{array}{ll}\mathrm{x} & \mathrm{M} * \mathrm{E}\end{array}\right]$;
E=E*. . . . . . . . . ;
end
-(5 points) How many floating-point operations (additions and multiplications) would be required to execute the above program.
2. Consider the floating-point system $\mathbb{F}=\mathbb{F}(10,6,-4,5)$. This system uses rounding to the closest.
(a) Fill in the following table. (10 points)

| Values of following parameters and elements in IEEE single precision system |  |
| :--- | :--- |
| $x_{\min }$ |  |
| $x_{\max }$ |  |
| $\epsilon_{M}$ (epsilon machine) |  |
| Representation of $\frac{1}{7}$ |  |
| $\operatorname{succ}\left(\frac{1}{7}\right)$ |  |

(b) (10 points) Convert $x=(52.225)_{10}$ into octal form? Give then the hexadecimal form of the internal IEEE single precision floating point representation of $x$, using rounding to the closest.

| Conversion of $x=(52.225)_{10}$ | to octal and IEEE hexadecimal form |
| :--- | :--- |
| Corresponding octal form |  |
| Corresponding IEEE hexadecimal form |  |

3. Consider the function $f(x)=e^{-x}-3 x$.
(a) (5 points) Show that this function has one root $r$ on $(-\infty, \infty)$. Graph this function on the interval $[-1,1]$
(b) (5 points) Find the least number of iterations that provide an approxximation to $r$ within 5 significant figures using the bisection method.

> Number of iterations :

## IN WHAT FOLLOWS CARRY ALL YOUR COMPUTATIONS WITH AT LEAST 5 FIGURES

(c) (10 points) Compute the following iterations:
-Give the sequence of 2 approximations obtained by applying 3 iterations of the bisection method.

| $x_{1}:$ |  |
| :--- | :--- |
| $x_{2}:$ |  |

-Give the iteration function $r_{n}=g\left(r_{n-1}\right)$ of Newton's method :

Then compute the sequence of 2 approximations obtained by applying 2 iterations of Newton's method with $x_{0}=0.5$.

| $x_{1}:$ |  |
| :---: | :--- |
| $x_{2}:$ |  |

-Give the iteration function $r_{n}=g\left(r_{n-1}, r_{n-2}\right)$ of the secant method:

Then give the sequence of 2 approximations obtained by applying 2 iterations of the secant method with $x_{0}=1$ and $x_{1}=0.5$.

| $x_{1}:$ |  |
| :---: | :--- |
| $x_{2}:$ |  |

