AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Computer Science Department

CMPS 251
FINAL EXAM
FALL 2003-2004

Closed Book, Two hours

GIVE YOUR ANSWERS ON THE QUESTION SHEET
SUBMIT WITH BOOKLET

STUDENT NAME	
ID NUMBER	

1. In this problem, we consider the following table for the function $f(x)$. All computations shall be carried out **with 8 significant figures**.

x_i	y_i
0.000	4.000000000000000
0.125	3.938461538461539
0.250	3.764705882352941
0.375	3.506849315068493
0.500	3.200000000000000
0.625	2.876404494382022
0.750	2.560000000000000
0.875	2.265486725663717
1.000	2.000000000000000

(a) Write the polynomial of degree 3 $p(x)$ that would best approximate $f(0.3)$. Find $p(0.3)$.

(b) Using the central difference formula to approximate $f'(0.5)$, followed by Richardson's extrapolation find the best approximation to $f'(0.5)$. For that purpose, fill out the following table.

h	$\phi_{c,h}$	$\phi_{c,h}^{(1)}$	$\phi_{c,h}^{(2)}$
0.5		$\times$	$\times$
		$\times$	$\times$
0.25			$\times$
			$\times$
0.125			

(c) Using the Mid-point, trapezoidal and Simpson's rules followed by Romberg integrations fill out the following table used to approximate $I = \int_0^1 f(x)dx$

h	M_h	T_h	S_h	$R_h^{(1)}$	$R_h^{(2)}$
h_0				× ×	× ×
$\frac{h_0}{2}$					× ×
$\frac{h_0}{4}$					
$\frac{h_0}{8}$	× ×		× ×	× ×	× ×

2. Suppose a real number L is appoximated by $\phi(h)$ such that:

$$L = \phi(h) + c_1 h^3 + c_2 h^5 + c_3 h^7 + ...,$$

where the coefficients $\{c_i\}$ are independent from h. What combination of $\phi(h)$ and $\phi(\frac{h}{2})$ would give a better approximation $\phi^1(h)$ to L than $\phi(h)$? What is the order α of the approximation of L by $\phi^1(h)$, (i.e. $L = \phi^1(h) + O(h^\alpha)$)?

3. Loss of significant figures may result in the computation of the following functions of the variable x for certain values of x. Specify these values then propose alternative functions that would remedy the loss of significant figures. (If necessary you may use Taylor's series).

(a) $f(x) = x + \sqrt{x^2 - 1}$

(b) $g(x) = x - \sin(x)$

4. To perform Naive Gauss elimination for the following **quadridiagonal matrix**

$$\begin{pmatrix}
d_1 & u_1 & v_1 & 0 & .. & .. & 0 & 0 \\
l_1 & d_2 & u_2 & v_2 & 0 & .. & .. & 0 \\
0 & l_2 & d_3 & u_3 & v_3 & 0 & .. & 0 \\
0 & 0 & l_3 & d_4 & u_4 & v_4 & .. & 0 \\
\vdots & . & . & . & . & . & . & . \\
0 & .. & 0 & l_i & d_i & . & . & . \\
\vdots & . & . & . & . & . & . & . \\
\vdots & . & . & . & l_{n-3} & d_{n-2} & u_{n-2} & v_{n-2} \\
0 & . & . & . & . & l_{n-2} & d_{n-1} & u_{n-1} \\
0 & .. & .. & 0 & .. & 0 & l_{n-1} & d_n
\end{pmatrix}$$

One uses the following algorithm:

`for i from 1 to n-1`

  $l_i = l_i / d_i$

  $d_{i+1} = d_{i+1} - l_i * u_i$

  `if i<n-1`

    $u_{i+1} = u_{i+1} - l_i * v_i$

  end

end

Give the exact number of **floating point operations** needed to perform this algorithm.

5. Give with justification the minimum number of arithmetic operations (additions, subtractions and multiplications) to compute, using nested multiplication, the following polynomials

Polynomial $p(x)$	Minimum number of arithmetic operations
$(x-2)^{17}+(x-2)^{31}$	
$4x^5-6x^{12}+2x^{17}-x^{33}$	