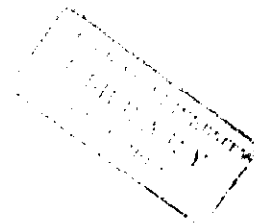


Spring 2000



# AMERICAN UNIVERSITY OF BEIRUT

Mathematics 251, Final Examination

Time = 1 hour and 50 minutes

**INSTRUCTIONS:** Please, print your name and ID number on both the booklet and the question/answer sheet.

You are allowed to use a calculator and one formula sheet.

For the multiple choice part, you CANNOT check more than one answer. If you mistakenly checked more than one answer, Please, correct it by writing "My answer is —" or you lose all points for that question. GOOD LUCK!

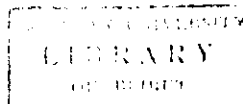
- Determine an approximation for  $e^{1/2}$ , using the third degree Taylor polynomial for  $e^x$  expanded about  $x = 0$ . (Round off your answer to 3 places after the decimal.)  
(A) 1.625 (B) 1.646 (C) 1.648 (D) 1.649 (E) 1.666

- You are to use the bisection algorithm to determine an approximation to  $\sqrt[3]{6}$ . Let  $r_n$  be the approximation of the root after the  $n^{\text{th}}$  steps. The algorithm is applied until the following condition is met:

$$\left| \frac{r_n - r_{n-1}}{r_n} \right| < 0.05. \quad (1)$$

Using starting values  $a = 1$  and  $b = 2$ , determine the number of iterations required

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10



- You are given the following values for the polynomial  $f(x)$ :

x	-1	0	2	3
f(x)	0	3	15	96

What is the coefficient of  $x^3$ ?

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Please turn to —

4. you are approximating  $\int_0^2 xe^x dx$  using two different methods. The approximation using one Simpson's rule is denoted by A. The approximation using the trapezoid rule with two equal subintervals is denoted by B. The exact value of the integral is denoted by C. Rank A, B, and C.

(A)  $A < B < C$  (B)  $A < C < B$  (C)  $B < A < C$  (D)  $C < A < B$  (E)  $C < B < A$

5. Romberg integration is used to approximate  $\int_0^1 \sqrt{1-x^2} dx$  until  $|R_{n,n} - R_{n,n-1}| < 0.005$ . Determine  $n$ .

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

(II) Written Questions. Please show all work!

1. The function  $f(x) = \cos(\pi x/2)$  is approximated by a quadratic spline,  $Q(x)$ , at nodes 0, 1, and 2. In addition to the standard properties of a quadratic spline, the function  $Q(x)$  satisfies the following conditions:  $Q'(0) = f'(0)$  and  $Q'(2) = f'(2)$ . Using  $Q(x)$ , approximate  $\cos(\pi/3)$ . (15 pts)

2. You are given the following system of three equations with three unknowns:

$$\begin{aligned} x_1 + 4x_2 + \alpha x_3 &= 6 \\ 2x_1 - x_2 + 2\alpha x_3 &= 3 \\ \alpha x_1 + 3x_2 + x_3 &= 5 \end{aligned}$$

- (a) Show that  $x_3 = 2/(1 + \alpha)$ , if  $\alpha$  does not equal -1 or +1. (10 pts)
- (b) If  $\alpha = -1$  or +1, discuss the solution(s) of the above system in both cases. (5 pts)
3. Given the following initial value problem (IVP):

$$\begin{aligned} x' &= -x + e^{-t} \\ x(-1) &= 0 \end{aligned}$$

- (a) Using Euler's method with  $h = 0.25$ , what is the approximate value of  $x(0)$ ? (10 pts)
- (b) What is the magnitude of the absolute error that has been incurred, knowing that the exact solution of the above (IVP) has the form of  $(1 + \alpha t)e^{-t}$ ? (10 pts)