

AMERICAN UNIVERSITY OF BEIRUT

MATH 251. Final Exam

June 15, 2001

Time = 1 Hour and 50 Minutes

Part I: Multiple choice (each question carries a weight of 10 points)
(VIP: Checking more than one answer is considered as a wrong answer!)

1. Given a function $f(x)$ which has derivative of infinite order. What does the following expression.

$$\frac{(1+h)}{4h^2}f(x+2h) - \frac{1}{2h^2}f(x) + \frac{(1-h)}{4h^2}f(x-2h), \quad (1)$$

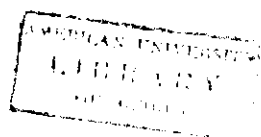
estimate?

- (a) $f'(x)$ (b) $f''(x)$ (c) $f'(x) + f''(x)$ (d) $f'(x) - f''(x)$ (e) none of the above
2. The integral $\int_0^1 2^x dx$ is to be approximated by using Romberg method. What is the value of n such that $|R(n, n) - R(n, n-1)| < 0.005$.
(a) 1 (b) 2 (c) 3 (d) 4 (e) none of the above
3. You are to obtain an approximate solution to the equation: $x^2 - 4x + 1 = 0$. One iteration by the Newton's method is performed, using the initial approximation $x_0 = 0$, resulting in a first approximation, x_1 . One iteration of the Secant method is then performed, using x_0 and x_1 as the initial approximations, producing an approximate solution, x_2 . Determine x_2
(a) $3/8$ (b) $31/60$ (c) $-1/2$ (d) 1 (e) none of the above
4. Given the following table of data:

x	1	2	3	4	5
$f(x)$	-2	1	1	2	3

Using Lagrange interpolating polynomial for the above data, what is the value of $l_4(-2)$?

- (a) 1 (b) 5 (c) 15 (d) 35 (e) none of the above



5. Refer to the data of the previous question. You are to approximate $f(x)$ by a quadratic spline, $Q_i(x)$, for $i = 0, 1, 2, 3$. What is the coefficient of x^2 in $Q_3(x)$?
 (a) -7 (b) -6 (c) 3 (d) 7 (e) none of the above
6. Refer to the data in previous question (4). The Least Squares method is used to approximate a linear relationship to $f(x)$, that is, $a + bx$. Using this approximation, what is the value of $f(3.5)$?
 (a) -0.65 (b) 0.15 (c) 1.55 (d) 2.65 (e) none of the above

Part II: Written Questions (Please show all work!)

7. Consider the following Initial Value Problem (IVP):

$$\begin{aligned} x' &= -x + 2 + e^{-t} \\ x(0) &= 2 \end{aligned}$$

- (a) Find the approximate value of $x(1)$ using Euler's method and $h = 0.5$. (5pts)
- (b) Find the approximate value of $x(1)$ using Runge-Kutta method of order 2 and $h = 0.5$. (5 pts)
- (c) It is known that the exact solution solution to the above IVP is of the form $x = \alpha + \beta t e^{-t}$. Determine the values of α and β and then compute the absolute error for the above approximations, respectively. (10 pts)
8. You are given the following system of equations:

$$\begin{aligned} \alpha x_1 + 3x_2 + x_3 &= 5 \\ x_1 + x_2 + \alpha x_3 &= 6 \\ 2x_1 - x_2 + 2\alpha x_3 &= 3 \end{aligned}$$

where α is a real number.

- (a) Use the Naive Gaussian Elimination method to solve the above system. (10 pts)
- (b) Discuss the solution(s) of the above system when $\alpha = 1$ and $\alpha = -1$. (10 pts)