AMERICAN UNIVERSITY OF BEIRUT
Faculty of Arts and Sciences
Mathematics Department

MATH 251
FINAL EXAM
SPRING 2002/03
Closed Book, Two hours

GIVE YOUR ANSWERS ON THE QUESTION SHEET
SUBMIT WITH BOOKLET

STUDENT NAME	
ID NUMBER	
Section	

1. Consider the function $f(x) = e^{-x} - x^3$.

   (a) Prove that this function has one single positive root r on $(-\infty, \infty)$? (**Hint**: You may use an intersection of 2 curves argument)

   (b) Sketch on THE BOOKLET, the function $f(x)$ on $(0, 2)$.

   (c) Using the **bisection method** on $[0, 2]$, give with justification, the minimum number of iterations n_0, required to find r with a relative error of 0.5×10^{-5}.

$n_0 =$

   (d) Based on the function f, give, with justification, the formulae of the iterative schemes that generate the sequence of approximations, x_0, x_1, ... for respectively Newton's method and secant method.

Newton's method
$x_n =$ x_{n-1}
Secant method
$x_n =$

IN WHAT FOLLOWS CARRY ALL YOUR COMPUTATIONS WITH AT LEAST 7 FIGURES

(e) Give the sequence of 3 approximations obtained by applying 3 iterations of Newton's method with $x_0 = 0.5$.

x_1:	
x_2:	
x_3:	

(f) Give the sequence of 2 approximations obtained by applying 2 iterations of the secant method with $x_0 = 0$ and $x_1 = 1$.

x_2:	
x_3:	

2. To perform Gauss elimination for the following **quadridiagonal, diagonnaly dominant matrix**

$$\begin{pmatrix} d_1 & u_1 & v_1 & 0 & .. & .. & 0 & 0 \\ l_1 & d_2 & u_2 & v_2 & 0 & .. & .. & 0 \\ 0 & l_2 & d_3 & u_3 & v_3 & 0 & .. & 0 \\ 0 & 0 & l_3 & d_4 & u_4 & v_4 & .. & 0 \\ \vdots & . & . & . & . & . & . & . \\ 0 & .. & 0 & l_i & d_i & . & . & . \\ \vdots & . & . & . & . & . & . & . \\ \vdots & . & . & . & l_{n-3} & d_{n-2} & u_{n-2} & v_{n-2} \\ 0 & . & . & . & . & l_{n-2} & d_{n-1} & u_{n-1} \\ 0 & .. & .. & 0 & .. & 0 & l_{n-1} & d_n \end{pmatrix}$$

One uses the following algorithm:

`for i from 1 to n-1`

$l_i = l_i/d_i$

$d_{i+1} = d_{i+1} - l_i * u_i$

`if i<n-1`

$u_{i+1} = u_{i+1} - l_i * v_i$

end

end

Give the exact number of **floating point operations** needed to perform this algorithm.

3. Prove that for $k = 1, 2, ...,$ $x^{2^k} = (x^{2^{k-1}})^2$, then give with justification the minimum number of arithmetic operations (additions, subtractions and multiplications) to compute, using nested multiplication, the following polynomials

Polynomial $p(x)$	Minimum number of arithmetic operations
$x^{2^k} : k = 1, 2, ...$	
$3(x-1)^{32} + (x-1)^{64}$	
$6x^3 + 9x^7 + 3x^{15} - x^{31}$	

4. A function f is given on a set of **uniformly spaced data:** $\{x_i | i = 0, .., n\}$, where n is a positive integer and $h = x_{i+1} - x_i$:

$\mathbf{x_i}$	1.8	1.9	2.0	2.1	2.2
$\mathbf{f(x_i)}$	3.1268	3.2871	3.4556	3.6328	3.8190

Compute, the approximations of the definite integral $\int_{1.8}^{2.2} f(x)dx$, using the mid-point rectangular, trapezoidal and Simpson rules followed by Romberg integration, for $h_0 = 0.4, 0.2, 0.1$.

	Mid-point	Trapezoidal	Simpson rule	First Romberg
h	$M(h)$	$T(h)$	$S(h)$	$R1(h)$
0.4				No fill
0.2				
0.1	No fill		No fill	No fill

5. Let x, $y \in \mathbb{F}_s$ be two positive numbers in $\mathbb{F}_s$, the **IEEE simple precision number system**, with

$$x = m_x \times 2^{e_x} \text{ and } y = m_y \times 2^{e_y},$$

$$m_x = 1+a_1 2^{-1}+a_2 2-2+...+a_p 2^{-p} \text{ and } m_y = 1+b_1 2^{-1}+b_2 2-2+...+b_p 2^{-p},$$

and

$$a_i, b_i \in \{0,1\}, \forall i = 1,..p,$$

$$e_x, e_y \in S = \{n : -l_1 \leq n \leq l_2\},$$

where S is a set of integers.

(a) Specify the set S and the integer p.

$p=$
$l_1=$
$l_2=$

(b) Given x, specify for what values of y, one has ("absorption" of y by x), i.e. $x \oplus y = x$.