AMERICAN UNIVERSITY OF BEIRUT
Mathematics 251, Final Exam
June 22, 2002

Time = 1 Hour and 50 Minutes

You are allowed to use one standard size formula sheet and a calculator. Please, print your name and your ID number on both the question sheet and the booklet.

Part I: Multiple choice. Please, choose one and only one answer. Each question carries a weight of 10 points.

1. Let $f(x) = e^x$. Find a third-degree Taylor polynomial for $f(x)$ expanded about $x_0 = 1$ ~~and~~ to approximate $e^{0.99}$ using this Taylor polynomial. What is the expected degree of accuracy?
   (a) $10^{-5}/24$ (b) $e/24$ (c) $10^{-4}e/24$ (d) $10^{-4}e$ (e) none of the above.

2. One would like to approximate a root of the equation $x^3 + 4x^2 - 10 = 0$ in the interval [1,2] using the bisection method. What is the minimum number of iterations that are needed to achieve and accuracy of 10^{-5} (rounded)?
   (a) 17 b) 34 (c) 100 (d) 120 (e) none of the above.

3. You are given the following table of data:

x	0	1	2
f(x)	0	0.10	0.75

   Let A be the estimate of f(1.5) through polynomial interpolation. Later, the value of the function at x=3 became available and was given by f(3)=0.95. Let B be the new estimate of f(1.5) using the new updated data. What is the value of A-B?
   (a) 1/16 (b) 3/16 (c) 1/4 (d) 2 (e) none of the above.

4. You are to approximate $I = \int_0^1 sin(\pi x)dx$ by two methods. Method 1 uses the composite trapezoid rule with two equally divided subintervals, call it I_1 with error E_1. Method 2 uses the composite trapezoid rule with three equally divided subintervals, call it I_2 with error E_2. Determine $E_1 - E_2$.
   (a) $\frac{\sqrt{3}}{3} - \frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{1}{6}$ (e) none of the above.

5. Approximate $\int_0^1 (1+x^2)^{-1}dx$ by using the Romberg method. What is the value of $|R(2,1) - R(1,1)|$ (round your answer to the third decimal place)?
(a) 0.000 (b) 0.001 (c) 0.002 (d) 0.003 (e) none of the above

6. $S(x)$ is a natural cubic spline for the function $f(x) = sin(\pi x/2) + cos(\pi x/2)$ at knots $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$. What is the coefficient of x^3 in $S(x)$ when $0 \leq x \leq 1$?
(a) -1/3 (b) -1/2 (c) 0 (d) 1/2 (e) none of the above.

Part II: (written questions) Please, show all work!

7. Consider the following linear system of equations:

$$\begin{array}{rcrcrcrcr} x_1 & + & x_2 & & & + & 3x_4 & = & 4 \\ 2x_1 & + & x_2 & - & x_3 & + & 2x_4 & = & 1 \\ 3x_1 & - & x_2 & - & x_3 & + & 2x_4 & = & -3 \\ -x_1 & + & 2x_2 & + & 3x_3 & - & x_4 & = & 4 \end{array}$$

(a) Write the above system of equations in itis matrix form, that is, $A\underline{X} = \underline{b}$. (5 pts)

(b) Find a Lower triangular matrix L and an upper triangular matrix U such that A=LU. (10 pts)

(c) Find the solution of the above system. (5 pts)

8. Consider the following initial value problem:

$$\begin{aligned} x' &= 2xt^{-1} + t^2e^t \\ x(1) &= 0 \end{aligned}$$

(a) Show that $x(t) = t^2(e^t+k)$, where k is a constant to be determined, is an exact solution for the above initial value problem. (5 pts)

(b) Approximate x(1.01) by using one step Euler's method and then by using one step Runge-Kutta method of order 2. (10 pts)

(c) What method in part (b) gives the better approximation for $x(1.01)$? (5 pts)