Math 241, Introduction to Abstract Algebra – Fall 2011–2012
Course website: `http://people.aub.edu.lb/~kmakdisi/`
Problem set 3, due Friday, October 21 at the beginning of class

Exercises from Fraleigh:

Section 6, exercises 53, 56. (Hint for exercise 56 part a: if $H = \langle h \rangle$ and $K = \langle k \rangle$, what can you say about the element $hk \in G$? As for exercise 56 part b, one possible hint is to do exercise A3.1 first. Another possible hint is to think about prime factorizations and to use the previous exercise 53.)

Section 8, exercises 1, 4, 5 (also express both τ and $\sigma^{-1}\tau\sigma$ as products of cycles), 6, 7, 8, 16, 21, 46.

Section 9, exercises 7, 10, 13, 39.

Additional Exercises (also required):

Exercise A3.1: Let $a, b \in \mathbf{Z}^+$.

a) Show that $a\mathbf{Z} \cap b\mathbf{Z}$ is a subgroup of $\mathbf{Z}$, not equal to $\{0\}$.

b) We thus know that there exists $m > 0$ such that $a\mathbf{Z} \cap b\mathbf{Z} = m\mathbf{Z}$. Explain why m is the least common multiple of a and b. We write $m = LCM(a, b)$.

c) Also explain why if c is any common multiple of a and b, then c is a multiple of m.

d) If $d = GCD(a, b)$ with $a = da'$, $b = db'$ and $GCD(a', b') = 1$, show that $LCM(a, b) = da'b' = ab/GCD(a, b)$. (Hint: what can you deduce if $da'x = db'y$? Why is this relevant to common multiples of a and b?)

Look at, but do not hand in:

Section 6, exercise 54.
Section 8, exercises 30–34, 40–43, 44, 45, 47.
Section 9, exercises 30, 34, 36, 37.