أَلجَامِعَة أَلأَميركيّة في بَيروت

American University of Beirut

Math. 241

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(1) Define the following terms.

(i) Group.
(ii) Subgroup.
(iii) Normal subgroup.
(iv) Index of a subgroup in a group.
(v) Group homomorphism.
(vi) Ring.
(vii) Ring homomorphism.
(viii) Ideal of a ring.
(ix) Principal ideal domain.
(x) Field.
(xi) Characteristic of a field.

(2) State, without proof, the following theorems.

(i) Cayley's theorem.
(ii) Cauchy's theorem.
(iii) The first part of Sylow's theorem.

(3) How many automorphisms does the group $\mathbb{Z}_{143}$ have?

(4) Find the remainder of dividing 100003^{120} by 143. A stupid method, such as using the calculator, is worse than a wrong answer.

(5) Show that if G is a group and $a, b \in G$, then $(ab)^{-1} = b^{-1}a^{-1}$.

(6) State and prove Lagrange's theorem.

(7) Show that if R is a finite commutative integral domain, then R is a field.

(8) Define the field of fractions of a commutative integral domain. Carry out the construction of the field of fractions, without showing the details of the proof.

(9) Give an example of a division ring which is not commutative. Remember the quaternions?

(10) Let R be a ring and let $e \in R$ be an element for which $e = e^2$. Show that

$$(ex - exe)^2 = (xe - exe)^2 = 0$$

for all $x \in R$. Deduce that if R is a ring in which $x^2 = x$ for all $x \in R$ then R is commutative.